KEVIN'S WEBSURFER HANDBOOK V
FOR CRYSTAL RADIO

INDUCTANCE, INDUCTORS and MEASURING Q

Kevin Smith
2013

NOTES:

NOTES:

INTRODUCTION

Finally, I have cobbled together a handbood dedicated to that one crystal radio component virtually all builders fabricate for themselves, the coil. This has been a long time in comming together. As I have increased my knowledge of set design and theory, I have found a deeper need for good technical background reading. Inductors, as will quickly be found, are highly mathematical beasties. Their properties, those we most wish to know such as Quality (Q), Resistance, Losses, etc are devilishely difficult to measure. As such, I have not spent as much time digging into this as I have for diodes, antennas, grounds etc.

I gather together in this volume articles with the central theme of inductance. Some of these are introductory and have appeared in earlier handbooks, others are specialized and will be found here alone. I wish to include both theory as well as practical articles on their design and construction. As an addition, you will find two non-crystal radio projects which otherwise rely on induction and induction coils for their performamce. These describe a cool VLF-ULF coil. Hopefully this volume gives the crystal set designer / builder a good base of understanding for this chief set component.

Of general note, the web is a marvelous source of data and information. Many long-time crystal set builders, and many others have created dedicated sites to disseminate information and resources, to share their creations and knowledge. I am eternally in your debt. All of the material in this handbook is copyright for which I have not sought permission. Therefore this is not presented for publication or copy. It is only my personal resource. I encourage anyone finding this copy to pursue ON THE WEB the web pages identified within. I include the name of the author and web address of each section. I wish to sincerely thank every author presented for their excellent pages and ask forgiveness for my editing into this handbook.

Kevin Smith
2013

www.lessmiths.com/~kjsmith/crystal/cr0intro.shtml

By the way, the speakers sound terrific! I'm not sure how much of the improvement was from the litz wire, but it certainly didn't hurt the sound!

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I tried several of these methods, and none were satisfactory in producing a nice clean tinned wire end, without breaking or damaging the strands. However, I did eventually come up with a simple, fast and very effective method, so I thought I would share it with the other DIY'ers here, in case they are crazy enough to build something with litz wire.

A Dremel tool with a stainless steel wire wheel attachment works very well. I found that the stainless wheel works faster and lasts longer than the carbon steel wheel, with the added advantage that the small pieces of the wheel brush that wear off during the process are non-magnetic, so they won't attach to nearby driver magnets (don't need little pieces of wire inside my EMIMs and EMITs!). Simply remove ~ 1/2" of the insulation, then untwist the various strands from each other (the Cardas Litz has 3 counterwound layers of different diameter wire to untwist, do them one at a time) and flatten them into a flat fan shape. Set the Dremel to about 1/2 of maximum speed and apply to the coated wire, ensuring that the wheel is ALWAYS turning toward the ends of the wire (otherwise they will entangle with the spinning wire wheel and snap off!). 10 seconds on each side of the fanned wire is very effective at removing the varnish coating without destroying the wires. Then re-fan the wire in a different direction and repeat the process 2 or 3 times to ensure that all the wires get stripped. You will be left with nice clean varnish-free wires, which can then be tinned as usual.

As a test of this method I measured the AC impedance of each of the wires of the same length at 10kHz using my LCR meter, and found that they all were very consistent, indicating that I was effectively and consistently removing the varnish and using all the wires in the bundle.

Litz Wire - Here's the Best Way I've Found to Prep
Maxamillion
http://www.audiokarma.org/forums/archive/index.php/t-282190.html

I just completed building outboard crossovers for my Infinity RSIIb speakers, and I was determined to wire the crossovers together and to the speakers using Cardas Copper Litz wire, due to its flatter AC impedance as frequencies increase. The Cardas Litz wire consists of many fine strands of wire, each individually insulated, which all need to be stripped and tinned together at each end in order to form a proper termination. Quite a job!

I searched the literature for methods and there were many, including:

1) The use of a solder pot at >800 degrees (too expensive to purchase one just for this project),

2) Using a hot soldering iron and lots of flux (very, very messy and very, very ineffective in my case!),

3) Scraping the coating off with an exacto knife, (not attempted due to the tedium factor),

4) Burning the coating off with a torch, followed by sanding (I found it works, but you end up oxidizing the wires and even incinerating some of the smaller strands),

5) Individually sanding each strand (I'd be doing this job for years as each wire has dozens of strands and I have 6 tweeters, 6 midranges and 4 woofers to wire up to 18 capacitors and 20 inductors!).

DESIGNING AN AIR-CORE INDUCTOR
by Kenneth A. Kuhn
http://www.kennethkuhn.com/students/crystal_radios/designing_inductors.pdf

Introduction
This chapter describes the mathematical process for designing an air-core inductor comprised of a single layer solenoid winding over a rigid coil form. Although the development of the mathematics is a bit complicated the final result is simple to apply. For practical reasons, this chapter will make use of English rather than metric units. A well designed and constructed air-core coil has better performance than those with ferrite cores. Ferrite acts as a flux multiplier and has the advantage that the physical size of the inductor can be reduced. That is very important for small radios and the chief reason ferrite is used. The price paid for small size is loss of performance but that loss is generally negligible in active radios. The loss is not bad for crystal radio performance and many good crystal radios have been built using ferrite core inductors. But ferrite is not required. Purists correctly argue that the coil should be air core as that is how early radios were built. A mediocre ferrite core inductor will work considerably better than a poorly designed air-core one and that has probably led to the popularity of ferrite as the process for designing good air-core inductors is not widely known. This chapter reveals those secrets.

Analytic equation
The classic equation (which you can find in any book or article about winding inductors) for calculating the inductance of a given single layer coil is (Reference 2):

$$L = \frac{r2*n2}{9*r + 10*l} \qquad \text{Eq. 1}$$

where:
L = inductance in microhenries
r = coil radius in inches (center of coil to center of conductor)
n = number of turns
l = coil length in inches (center of starting turn to center of ending turn)
This equation is generally accurate to around one percent for inductors of common dimensions. It is more convenient to work with coil diameter and Equation 1 can be written as:

$$L = \frac{d2*n2}{18*d + 40*l} \qquad \text{Eq. 2}$$

where d is the coil diameter in inches (center of conductor to center of conductor)
Example: What is the inductance of a coil has a diameter of 2.5 inches, a length of 2.33 inches, and has 72 turns?

$$L = \frac{2.5*2.5*72*72}{18*2.5 + 40*2.33} = 234 \text{ uH}$$

Development of design equations
Equations 1 and 2 are fine for determining the inductance of an existing coil but are very awkward to apply to the design of a desired coil as there are many variables. Any time there are a multitude of variables then the possibility of optimum combinations or relations should be explored. In the following development the number of variables is reduced by finding

When the coefficient of coupling, k is equal to 1, (unity) such that all the lines of flux of one coil cuts all of the turns of the other, the mutual inductance is equal to the geometric mean of the two individual inductances of the coils. So when the two inductances are equal and L_1 is equal to L_2, the mutual inductance that exists between the two coils can be defined as:

$$M = \sqrt{L_1 L_2} = L$$

Example No1

Two inductors whose self-inductances are given as 75mH and 55mH respectively, are positioned next to each other on a common magnetic core so that 75% of the lines of flux from the first coil are cutting the second coil. Calculate the total mutual inductance that exists between them.

$$M = k\sqrt{L_1 L_2}$$

$$M = 0.75\sqrt{75\text{mH} \times 55\text{mH}} = 48.2\text{mH}$$

In the next tutorial about Inductors, we look at connecting together *Inductors in Series* and the affect this combination has on the circuits mutual inductance, total inductance and their induced voltages.

relations to known constants. We first replace the coil length by a factor that relates it to the diameter.

l = k*d   Eq. 3

where
l = coil length in inches
k = a dimensionless constant
d = coil diameter in inches as before
Substituting Equation 3 into Equation 2 gives:

$$L = \frac{d2*n2}{18*d + 40*k*d}$$

which reduces to

$$L = \frac{d*n2}{18 + 40*k} \qquad \text{Eq. 4}$$

It can be shown that the value of k that minimizes the length of wire to wind the coil is 0.450. However, other research indicates (see Reference 1) that the value of k that minimizes coil losses is approximately 0.96 even though that value uses about twenty percent more wire. Factors contributing to coil losses include:

* Ohmic losses in the wire including skin-effect
* Dielectric losses in the coil form and nearby materials
* Dielectric losses in the insulation around the wire
* Induction losses in nearby materials

There are also losses caused by adjacent turns being too close together. It has been found (see Reference 1) that the optimum spacing (wire center to wire center) of adjacent turns is between about 1.3 to 2.0 times the diameter of the conductor. Coils for crystal radios are commonly wound using what is known as magnet wire (thin enamel insulation) and the turns are tightly wound next to each other corresponding to a spacing factor slightly greater than 1.0 (the thin insulation is of finite thickness). Although it is less than the optimum discussed it works well.

Without some special technique (such as a lathe) it can be very difficult to manually wind a coil with controlled spacing between the turns. One easy method for achieving a spacing factor of 2.0 is to wind two wires tightly side by side at the same time and then remove one of the windings when finished. Smaller spacing factors can be achieved using a smaller diameter wire for the spacer but the difficulty of controlling two wires will increase. It might occur to someone to use a wire with a thicker insulation so that a spacing is naturally formed with a tight winding. The problem with this method is that the insulation may increase dielectric losses and become self defeating –although this may be a small issue –be sure to try it before tossing the concept. This method can work great if Teflon wire is used as that is a very low-loss material and the internal wire strands are silver plated.

Equation 4 can be used to determine the optimum coil diameter for a given inductance and wire size. We note that the coil length is the number of turns divided by t (turns per inch of the wire). We also note that the coil length has previously been related to the coil diameter by the constant, k. Thus:

$$n = k*d*t \qquad \text{Eq. 5}$$

Substituting Equation 5 into Equation 4 gives:

However, the above equation assumes zero flux leakage and 100% magnetic coupling between the two coils, L_1 and L_2. In reality there will always be some loss due to leakage and position, so the magnetic coupling between the two coils can never reach or exceed 100%, but can become very close to this value in some special inductive coils. If some of the total magnetic flux links with the two coils, this amount of flux linkage can be defined as a fraction of the total possible flux linkage between the coils. This fractional value is called the **coefficient of coupling** and is given the letter k.

Coupling Coefficient

Generally, the amount of inductive coupling that exists between the two coils is expressed as a fractional number between 0 and 1 instead of a percentage (%) value, where 0 indicates zero or no inductive coupling, and 1 indicating full or maximum inductive coupling. In other words, if k = 1 the two coils are perfectly coupled, if k > 0.5 the two coils are said to be tightly coupled and if k < 0.5 the two coils are said to be loosely coupled. Then the equation above which assumes a perfect coupling can be modified to take into account this coefficient of coupling, k and is given as:

Coupling Factor Between Coils

$$k = \frac{M}{\sqrt{L_1 L_2}} \text{ or } M = k \times \sqrt{L_1 L_2} \quad H$$

the size, number of turns, relative position or orientation of the two coils. Because of this, we can write the mutual inductance between the two coils as: $M_{12} = M_{21} = M$.

Hopefully we remember from our tutorials on *Electromagnets* that the self inductance of each individual coil is given as:

$$L_1 = \frac{\mu_o \mu_r N_1^2 A}{\ell} \text{ and } L_2 = \frac{\mu_o \mu_r N_2^2 A}{\ell}$$

Then by cross-multiplying the two equations above, the mutual inductance that exists between the two coils can be expressed in terms of the self inductance of each coil.

$$M^2 = L_1 L_2$$

giving us a final and more common expression for the mutual inductance between two coils as:

Mutual Inductance Between Coils

$$M = \sqrt{L_1 L_2} \quad H$$


```
     k2*t2*d3
L = --------------                    Eq. 6
    18 + 40*k
```

We will use 0.96 for k and t will be that of the particular wire we have available. Solving Equation 6 for the optimum diameter gives:

d_optimum = 4*(L/t2)1/3 Eq. 7

Figure 1 shows a plot of this Equation 7 for common wire sizes. In all cases the turns are close-spaced. The lower curves are for common enamel insulated magnet wire. The two upper curves are for vinyl insulated house wire which can be considered if a large diameter coil form is available. To use the curves, select the desired inductance and the wire size that will be used. Look up the optimum coil form diameter and then use the closest practical form you have to that size. The optimum is broad so do not worry about being exactly on it. Note that the true diameter is the sum of the diameter of the coil form and the diameter of the wire since by definition the coil diameter is measured between opposite centers of the wire.

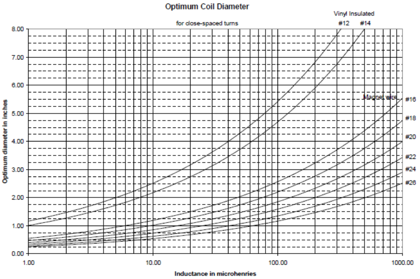


Figure 1: Optimum coil diameter

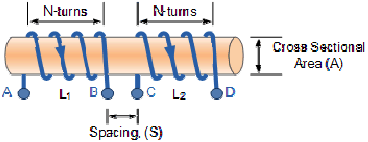
The following table provides typical values for t (turns per inch) for some common wire sizes:

Table 1: Wire data

Gauge	t	Comments
12	6.2	Vinyl insulated house wire
14	7.7	Vinyl insulated house wire
16	19	Enamel insulated magnet wire
18	24	ditto
20	31	ditto
22	39	ditto
24	50	ditto
26	62	ditto

The length of the winding will be the number of turns divided by the turns per inch of the wire. That is:

Mutual Induction



Here the current flowing in coil one, L_1 sets up a magnetic field around itself with some of these magnetic field lines passing through coil two, L_2 giving us mutual inductance. Coil one has a current of I_1 and N_1 turns while, coil two has N_2 turns. Therefore, the mutual inductance, M_{12} of coil two that exists with respect to coil one depends on their position with respect to each other and is given as:

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

Likewise, the flux linking coil one, L_1 when a current flows around coil two, L_2 is exactly the same as the flux linking coil two when the same current flows around coil one above, then the mutual inductance of coil one with respect of coil two is defined as M_{21}. This mutual inductance is true irrespective of

The mutual inductance that exists between the two coils can be greatly increased by positioning them on a common soft iron core or by increasing the number of turns of either coil as would be found in a transformer. If the two coils are tightly wound one on top of the other over a common soft iron core unity coupling is said to exist between them as any losses due to the leakage of flux will be extremely small. Then assuming a perfect flux linkage between the two coils the mutual inductance that exists between them can be given as.

$$M = \frac{\mu_o \mu_r N_1 N_2 A}{\ell}$$

- Where:
- μ_o is the permeability of free space ($4.\pi.10^{-7}$)
- μ_r is the relative permeability of the soft iron core
- N is in the number of coil turns
- A is in the cross-sectional area in m^2
- l is the coils length in meters

l = n/t Eq. 8

We now substitute Equation 8 into Equation 2 and solve for n

d2*n2 –18*d*L –40*(n/t)*L = 0 Eq. 9

t*d2*n2 –40*L *n –18*d*t*L = 0 Eq. 10

Solving for n gives:

$$n = \frac{20*L + sqrt(400*L2 + 18*t2*d3*L)}{t*d2} \qquad \text{Eq. 11}$$

Although a precise value (in inches) for the length of wire required can be calculated using trigonometry for a spiral, a very close value can be calculated as

w = pi*d*n Eq. 12

Remember that d is the sum of the coil form diameter and the diameter of the wire. This approximation assumes that the diameter of the wire is very small in comparison to that of the coil form. Also remember to allow an extra couple of inches for connecting leads at each end of the coil.

Example: A 300 uH coil is needed. The expected Q should be over 350. What coil diameters and wire sizes could possibly meet this?
Solution: Using Figure 2 it can be seen that wire sizes #12, #14, #16, #18, and #20 could achieve the required Q. Using Figure 1 the required coil form diameters are:

Optimum

Gauge	diameter
#12	7.75”
#14	6.75”
#16	3.75”
#18	3.15”
#20	2.65”

A piece of 4.5” OD PVC pipe is available and #14 electrical wire is available. From Table 1, #14 insulated wire will make about 7.7 turns per inch. Thus, the effective diameter is 4.5 plus 1/7.7 = 4.63 inches. Using Equation 11 the number of turns required is 86. Using Equation 8 the length of the winding is 11.2 inches. The length/diameter ratio is 2.4 which is a bit longer than the optimum of 0.96. The length of wire required is given by Equation 12 and is 1,251 inches. The length would have been 1,058 inches if the optimal diameter could have been used. This extra length will cause somewhat higher losses –it might still meet the desired spec though. This is about as far as I would go in rounding to an available coil form diameter.

Estimation of Inductor Q

All inductors have an equivalent series resistance loss as discussed earlier and is comprised of a number of components. We measure the quality factor or Q of the inductor by computing the ratio of inductive reactance at the frequency of interest to the series loss resistance as follows:

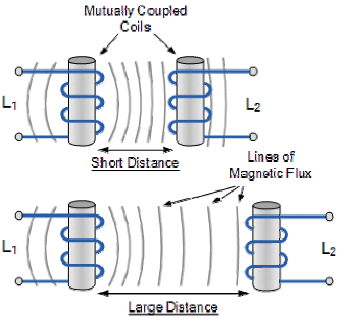
$$Q = \frac{XL}{Rs} \qquad \text{Eq. 13}$$

where
Q is the dimensionless “quality” factor of the inductor

magnetic flux generated by the first coil will interact with the coil turns of the second coil inducing a relatively large emf and therefore producing a large mutual inductance value.

Likewise, if the two coils are farther apart from each other or at different angles, the amount of induced magnetic flux from the first coil into the second will be weaker producing a much smaller induced emf and therefore a much smaller mutual inductance value. So the effect of mutual inductance is very much dependant upon the relative positions or spacing, (S) of the two coils and this is demonstrated below.

Mutual Inductance between Coils



Mutual Inductance of Two Coils
Wayne Storr
http://www.electronics-tutorials.ws/inductor/mutual-inductance.html

In the previous tutorial we saw that an inductor generates an induced emf within itself as a result of the changing magnetic field around its own turns, and when this emf is induced in the same circuit in which the current is changing this effect is called **Self-induction**, (L). However, when the emf is induced into an adjacent coil situated within the same magnetic field, the emf is said to be induced magnetically, inductively or by **Mutual induction**, symbol (M). Then when two or more coils are magnetically linked together by a common magnetic flux they are said to have the property of **Mutual Inductance**.

Mutual Inductance is the basic operating principal of transformers, motors, generators and any other electrical component that interacts with another magnetic field. Then we can define mutual induction as the current flowing in one coil induces an emf in an adjacent coil. But mutual inductance can also be a bad thing as "stray" or "leakage" inductance from a coil can interfere with the operation of another adjacent component by means of electromagnetic induction, so some form of electrical screening to a ground potential may be required.

The amount of mutual inductance that links one coil to another depends very much on the relative positioning of the two coils. If one coil is positioned next to the other coil so that their physical distance apart is small, then nearly nearly all of the

XL is the inductive reactance in ohms at the frequency of interest
Rs is the equivalent series resistance in ohms at the frequency of interest

Note that inductive reactance, XL, is calculated as

XL = 2*π*F*L

where
F is the frequency in Hz
L is the inductance in henries

The equivalent series resistance is the net of ohmic losses including skin effect, dielectric losses in distributed capacitance and coil structure, absorption losses by nearby conducting media, magnetic losses in nearby magnetic media, etc. With care these losses can be kept small but it takes very little loss to reduce the Q of an inductor from 400 to 200. The magnitude of Rs can be measured on sophisticated impedance equipment but it is hard to calculate the effect of all factors. Figure 2 shows an estimated value of Q at 1 MHz considering typical losses assuming the coil is wound optimally and is not disturbed by nearby lossy materials. Use the figure only as a guideline as your specific results may be better or worse. The expected Q at 540 kHz will be between about 50 to 70 percent of what is shown and the expected Q at 1.6 MHz will be around 1.2 to 1.5 times that shown.

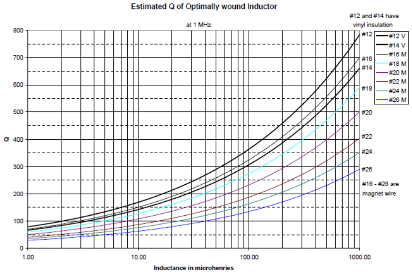


Figure 2: Estimated Q of Inductor

The Q we obtain from Equation 11 is for the unloaded coil (i.e. antenna and crystal detector not connected). The net loaded Q will typically be significantly smaller but ideally (as discussed in another chapter) would be in the general range of one hundred. Thus, we would like to start with an unloaded Q of several hundred. As can be seen in Figure 2 the Q of the inductor can be made higher by using larger diameter wire. From Figure 1 this also means using a large diameter coil form. This is a very important conclusion –high Q coils need to be physically large.

Type of wire

The only material to consider for the wire is copper. A variety of styles of copper wire is readily available. The most basic choice is between solid or stranded. Although a variety of arguments can be made for and against each, in practical terms you will not notice any difference in performance although one

The power delivered to an ideal transformer[2] on its primary circuit will match the power output on its secondary circuit and we can write $I_p V_p = I_s V_s$ or

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

[2] Here we neglect the resistances of the windings and the energy expended in magnetizing and demagnetizing the iron core.

$$E_s = -N_s \frac{\Delta\Phi}{\Delta t}$$

In the primary coil the induced emf is due to self-induction and is given by Faraday's law as

$$E_p = -N_p \frac{\Delta\Phi}{\Delta t}$$

The term $\Delta\Phi/\Delta t$ is the same in both equations since the same amount of magnetic flux Φ passes through both coils. Dividing the equations gives

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

If the resistances of the coils are negligible the terminal voltages V_s and V_p of the coils are nearly equal to the magnitudes of the emfs E_s and E_p. Hence we can write

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

This is called the ***transformer equation***.

or the other may have physical advantages for your particular construction method. Avoid wires that are plated as those will have higher losses since skin-effect will cause most of the conduction to be in the plating which has higher resistance than copper. Avoid wires with rubber or cheap plastic insulations as dielectric losses will be higher. An exception is silver plated Teflon wire as that has the best conductivity and the lowest dielectric losses –but it is expensive.

For use in low to medium frequency inductors there is a special wire called Litzengrad or just Litz for short. It is designed to minimize skin-effect losses and is made by assembling many strands of enamel insulated magnet wire together to form a wire that has a large surface area. Litz wire is not easy to find and tends to be expensive. If you are going to use Litz wire then make sure that other losses as previously discussed are minimized. Otherwise Litz wire will make little if any difference and will be wasted effort and expense. Avoid belief in a variety of myths about skin-effect. Although it is true that skin-effect is more severe on large diameter conductors, a larger diameter still conducts better than a smaller diameter at any frequency. This can be seen in Figure 3 which shows the frequency dependence of the resistance per meter factor of some common wire sizes.

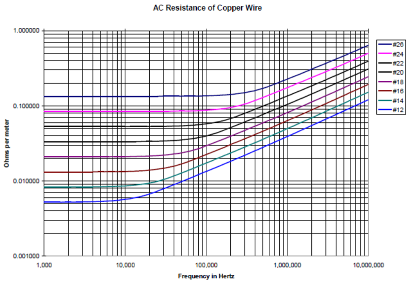


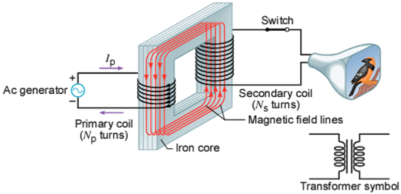
Figure 3: AC Resistance of Copper Wire

Coil Forms

From a loss standpoint air is the best coil form material there is. The obvious problem is that air has no structural strength. However, there are methods used by commercial inductor companies that employ a minimal structure so that the coil form is around 99 percent air. Manually, you can achieve the effect by first winding the coil using large diameter solid copper wire (i.e. #18, #16, #14, etc.) on a rigid coil form and then carefully sliding the winding off of the form. The stiff wire will retain the shape and you can easily space the turns to the optimal discussed previously. You will need a few supports to keep the whole thing from being too loose.

A popular coil form is some kind of cardboard tube that you have salvaged from a variety of sources such as used for paper towels or shipping tubes. These are great if you are using small diameter wire as small wire will not self support. Plastic pipe

Transformers



A ***transformer*** is a device for increasing or decreasing an AC voltage. It consists of an iron core with two coil wrappings: a primary coil with N_p turns and a secondary coil with N_s turns. See the figure at the right.

Assume an AC source, such as an AC generator, produces an alternating current I_p in the primary coil. The primary coil creates a changing magnetic flux Φ in the core which links the turns of both the primary and secondary coils.[1] In the secondary coil the induced emf arises from mutual induction and is given by Faraday's law as

[1] The details of the operation of a transformer, which also involve the magnetization and demagnetization of the iron core, is complex. Here we assume, for simplicity, a transformer operating in steady-state with a purely resistive load.

Assume that the solenoid carries a current I. Then the magnetic flux in the solenoid is

$$\Phi = \mu_0 \frac{NI}{l} A. \;\; L = \frac{N\Phi}{I} = \frac{N}{\cancel{I}}\mu_0 \frac{N\cancel{I}}{l} A$$

$$L = \mu_0 \frac{N^2}{l} A \;\text{ or }\; \boxed{L = \mu_0 n^2 A l} \;\text{ where } n = \frac{N}{l}.$$

(Note how L is independent of the current I.)

is another material you might consider. None of these materials are made with any consideration about high frequency dielectric losses but because only a small amount of material is used the losses are probably minimal.

Wood is a convenient coil form and has low losses if very dry. Common sizes that have been used are 2x2, 4x4, or a pair of 2x4 combined to make 4x4. Round dowel rods may also be used but their diameters are often much less than optimum. When using a square coil form there is a logical question about how that affects the inductance calculations. A simplistic (but good) answer is to use an effective circle diameter that has the same area as the square form since area is a strong factor in inductance. Losses with a square form will be somewhat higher than for a circular form. Rectangular forms (such as a single 2x4) have even higher losses in comparison –it takes more wire to encompass a given area.

Winding the coil

Counting turns is a tedious and error prone task. It is much simpler to cut the length of wire needed and then wind that until finished. The resulting turns count will be very close if not exact. Wire is springy and will jump off the form in a tangled mess if not restrained. Start by securing the wire at one end of the form and have a means for easily (preferably with one hand) securing the opposite end when you finish. It is tempting to use some kind of adhesive tape and that will work if you are careful and understand what you are doing. The forces will build and the tape may give way which will result in a frustrating mess of tangled wire. Make sure the tape can not slip. A good way to secure the ends is to first drill a hole in the tube at the starting and end points. Then feed the starting end through the starting hole and bend the wire such that it naturally resists tension and secure the wire with tape. Stuff the loose end length inside the tube so it is out of the way while winding.

It is best to wind the coil by hand as the set up for using a lathe is not worth the trouble for a single coil. There are a number of "poor man's" lathes such as a power drill that have been used but I do not recommend that as you are more likely to make a mess or cause injury than you are to wind a coil. It only takes a couple of minutes to wind a coil by hand so take the time to think what you are doing. It is important to keep the winding tight at all times. The wire will spring off if it ever gets loose. You will very likely have some fractional turn as a result of your calculations. I recommend that you round that to the nearest integer as it is not worth the trouble of making measurements to stop at a specific fractional turn.

References:

1. Electronic and Radio Engineering, fourth edition, Frederick Emmons Terman,
McGraw-Hill Book Company, 1955, pages 30 – 33.
2. The Radio Amateur's Handbook, 44th edition, 1967, The American Radio
Relay League, Newington, Conn., page 26

Let the coil have N turns. Assume that the same amount of magnetic flux Φ links each turn of the coil. The net flux linking the coil is then $N\Phi$. This net flux is proportional to the magnetic field, which, in turn, is proportional to the current I in the coil. Thus we can write $N\Phi \propto I$. This proportionality can be turned into an equation by introducing a constant. Call this constant L, the ***self-inductance*** (or simply ***inductance***) of the coil:

$$N\Phi = LI \quad \text{or} \quad L = \frac{N\Phi}{I}$$

As with mutual inductance, the unit of self-inductance is the henry.

The self-induced emf can now be calculated using Faraday's law:

$$\mathsf{E} = -N\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta(N\Phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

$$\boxed{\mathsf{E} = -L\frac{\Delta I}{\Delta t}}$$

The above formula is the ***emf due to self-induction***.

Example

Find the formula for the self-inductance of a solenoid of N turns, length l, and cross-sectional area A.

The above formula is the ***emf due to mutual induction***.

Example

The apparatus used in Experiment EM-11B consists of two coaxial solenoids. A solenoid is essentially just a coil of wire. For a long, tightly-wound solenoid of n turns per unit length carrying current I the magnetic field over its cross-section is nearly constant and given by $B = \mu_0 n I$. Assume that the two solenoids have the same cross-sectional area A. Find a formula for the mutual inductance of the solenoids.

The magnetic flux in the primary coil is

$\Phi_1 = \mu_0 \frac{N_1 I_1}{l_1} A$ where l_1 is the length of the primary coil.

$$M = \frac{N_2 \Phi_2}{I_1} \quad \text{But } \Phi_2 = \Phi_1: \quad M = \frac{N_2 \Phi_2}{I_1} = \frac{N_2}{\cancel{I_1}} \mu_0 \frac{N_1 \cancel{I_1}}{l_1} A; \quad \boxed{M = \mu_0 \frac{N_1 N_2}{l_1} A}$$

(Note how M is independent of the current I_1.)

Self-Inductance

A current-carrying coil produces a magnetic field that links its own turns. If the current in the coil changes the amount of magnetic flux linking the coil changes and, by Faraday's law, an emf is produced in the coil. This emf is called a *self-induced* emf.

"Professor Coyle" Coil Calculators
By Dan Petersen
http://www.crystalradio.net/professorcoyle/index.shtml

Instructions for Professor Coyle:

Professor Coyle is intended to be a mathematical aid in determining the inductance of a coil and the resonant frequency of a coil-capacitor tuned circuit.

To Use The Coil Calculator:

The coil calculator will calculate the inductance of both closewound or non-closewound or "spaced" coils. To do a closewound coil, enter the coil diameter and the desired number of turns into the appropriate places. Left-click on any of the red numbers to change their value, then press <enter>. Look at the list below the entered numbers ans you will note a series of inductances and lengths paired with the gauge of wire to be used. These are the inductances and coil lengths of a closewound coil as a function of the wire guage. For example, enter a form diameter of 1.75" and 78 turns into the calculator. Now if you wanted to use #28 enamelled wire, you would need a space of 1.11" on the coil and the inductance would be 244.9 uH.

A "spaced" coil is one that has a gap between turns. An example is a coil that is 1 inch long and has 8 turns of #24 wire "spaced" along the 1 inch length.There is a significant gap between turns. To calculate the inductance, enter the coil diameter, the number of turns and the coil length into the calculator. The calculated inductance will appear in blue just under the entered parameters.

To Use the Resonance Calculator:

Enter the known inductance and capacitance into the calculator section at the top of the page. The resonant frequency and the reactance of the L-C circuit will be given in blue immediately below. Example: You have a 40-400pF variable capacitor and a 240uH inductor and you want to find the tuning range. Enter the inductor value, press <enter>, and the highest variable capacitor value into the calculator and press <enter>. You should see a resonance of .514 MHz. Left-click on the capacitance value and now enter 40. You should now see a resonance of 1.624 MHz. In this example would be perfect for tuning the AM broadcast band.The result is a theoretical value. The actual frequency will differ slightly due to distributed capacitance and measurement tolerances

Professor Coyle can be copied for non-commercial purposes only and is meant to be an aid in calculation. The author is not responsible for the results calculated nor any damage to equipment affected by these calculations. This program is not to be bought or sold. Please credit the author, Dan Petersen, when copying this program. Close cover before striking. Place all trays in the upright position before landing. Put the toilet seat down. Exact change only. Don't kiss the dog on the nose.

needle). Hence the secondary coil encloses a *changing* magnetic field. By Faraday's law of induction this changing magnetic flux induces an emf in the secondary coil. This effect in which changing current in one circuit induces an emf in another circuit is called ***mutual induction***.

Let the primary coil have N_1 turns and the secondary coil have N_2 turns. Assume that the same amount of magnetic flux Φ_2 from the primary coil links each turn of the secondary coil. The net flux linking the secondary coil is then $N_2\Phi_2$. This net flux is proportional to the magnetic field, which, in turn, is proportional to the current I_1 in the primary coil. Thus we can write $N_2\Phi_2 \propto I_1$. This proportionality can be turned into an equation by introducing a constant. Call this constant M, the ***mutual inductance*** of the two coils:

$$N_2\Phi_2 = MI_1 \text{ or } M = \frac{N_2\Phi_2}{I_1}$$

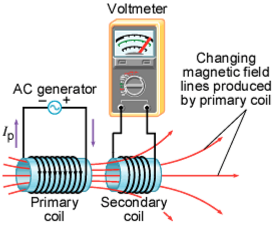
the unit of inductance is $\frac{\text{wb}}{\text{A}} = henry\ (\text{H})$ named after Joseph Henry.

The emf induced in the secondary coil may now be calculated using Faraday's law:

$$\mathcal{E}_2 = -N_2\frac{\Delta\Phi_2}{\Delta t} = -\frac{\Delta\left(N_2\Phi_2\right)}{\Delta t} = -\frac{\Delta\left(MI_1\right)}{\Delta t} = -M\frac{\Delta I_1}{\Delta t}$$

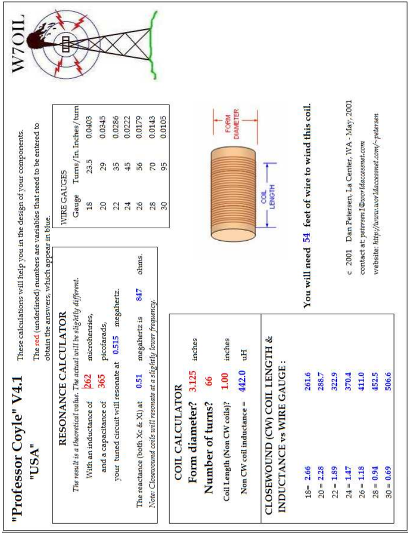
$$\boxed{\mathcal{E}_2 = -M\frac{\Delta I_1}{\Delta t}}$$

Mutual Inductance
Dr. David F. Cattell
faculty.ccp.edu/faculty/dcattell/Sp12/.../Mutual%20Inductance.doc



Suppose we hook up an AC generator to a solenoid so that the wire in the solenoid carries AC. Call this solenoid the *primary coil*. Next place a second solenoid connected to an AC voltmeter near the primary coil so that it is coaxial with the primary coil. Call this second solenoid the *secondary coil*. See the figure at the right.

The alternating current in the primary coil produces an alternating magnetic field whose lines of flux *link* the secondary coil (like thread passing through the eye of a



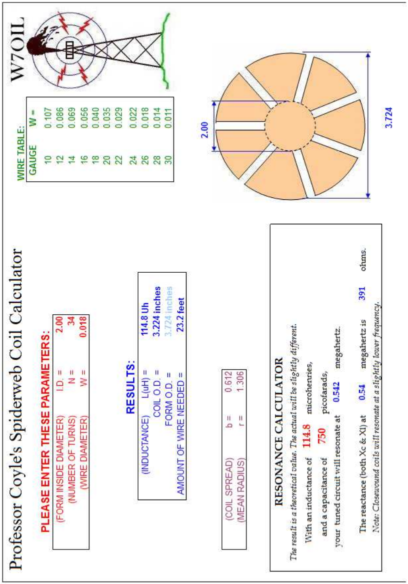


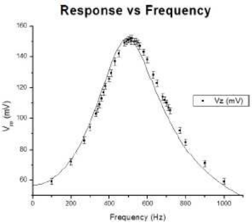
Figure 10: Peak to peak voltage across capacitor as a function of frequency with theoretical voltage fit to data with R = 202 Ω.

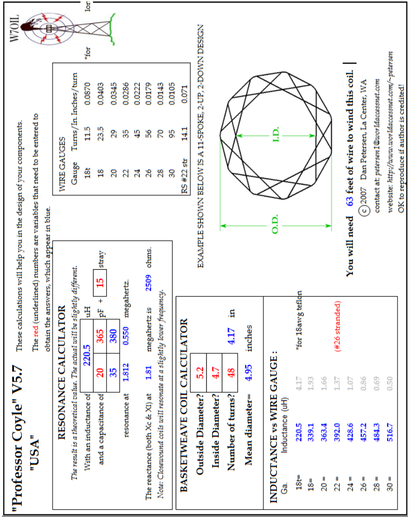
the critical resistance as a function of capacitance there is a problem. But the issue is most likely due to the nature of the measurement as it is difficult to tell exactly when the circuit transitions from under-damped to over-damped. Finally, the model accurately predicts the resonant frequency of a RLC circuit, however, other details of the behavior such as the Q factor are not described correctly in this experiment.

This fit is shown in Figure 10. It can be seen that this does shift the resonance from where measured (to 485 Hz) but the Q factor is the same as in the experiment. So while not entirely responsible for the difference between measurement and theory, as evidenced by the resonant frequency difference, uncounted resistances play a part in the disparity.

4 Conclusions

RLC circuit behavior is described well by oscillating system theory which describes the charge on the capacitor as a 2nd order differential equation. It is demonstrated in this experiment that the theory can be used to reliably find the inductance of a RLC loop by measuring the period of oscillation as a function of capacitance. The predicted exponential damping dependence on resistance is also verified and it is shown that that total resistance of the circuit can be found accurately using a resistance versus log decrement plot. However, when applying the theory to measure the inductance using





Ed Note: I include these screen shots because I believe this is the premiere coil calculator available. I highly encourage any/all crystal radio builders to get a copy, modify it at will, and use it, use it a lot!

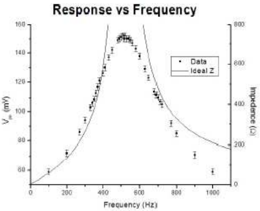


Figure 9: Peak to peak voltage across capacitor as a function of frequency. The data points go to the left axis and the Ideal Z goes with the right axis.

bandwidth of 360 − 15 Hz5. Then, using $Q = f_0/\Delta f$ it was found that the quality of the circuit was 1.4 − 0.1.

The theoretical Q is 52.8 and it is therefore quite obvious why the theoretical impedance curve is much sharper than the measured curve. The plot shows a poor agreement between the theoretical response and the actual results. The peak is much sharper for the ideal case and the bandwidth is much lower as seen in a comparison of Q values. These discrepancies are due to components not behaving ideally and other loses in the circuit such as resistance in the wires or imperfect connections on the bread board. Additionally, the actual voltage falls off faster than expected for the same reasons. Building on this idea, if we assume that the inductance and capacitance of the circuit are the same as measured and fit the function to the data by changing the resistance in equation 14, we find the best fit around R = 202 Ω.

function of frequency. We observed frequencies between 100 and 1000 Hz and set the oscilloscope to average 16 samples and measure the amplitude automatically. The results are shown in Table 6. These values are then

Table 6: Voltage versus frequency. Errors for all values are − 2 mV.

Frequency (Hz)	Vpp (mV)	Frequency (Hz)	Vpp (mV)	Frequency (Hz)	Vpp (mV)
100	59	450	142	600	138
200	72	480	149	630	129
270	86	490	150	650	123
300	94	500	151	680	114
330	103	510	152	690	112
340	106	515	150	700	110
350	109	520	152	710	107
360	113	530	150	720	105
370	117	540	150	770	92
380	121	550	149	800	85
400	126	560	147	900	71
410	130	580	143	1000	59
430	137				

plotted along with equation 14 in Figure 9. Equation 14 is important because RM is so large that the peak to peak voltage (Vpp) should approximately equal V0Z=RM and thus is directly related Vpp. Using this plot we found the resonant frequency to be 515 − 10 Hz. Based on the measurements of the inductor, resistor, and capacitor the theoretical resonance occurs at 514 Hz which is in good agreement with our data. We also measured the bandwidth by finding the two frequencies at which Vpp fell to 70.7% of its peak value. These two frequencies were 350 − 10 Hz and 710 − 10 Hz giving a

Variometer Design

Claudio Girardi

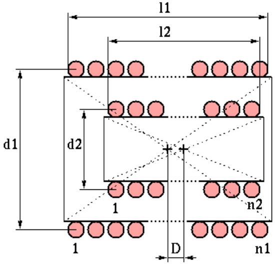
http://www.qsl.net/in3otd/variodes.html

Variometer Design

A variometer consists usually of two coaxial coils, connected in series, where the coils relative position can be varied in some way. If L1 and L2 are the self-inductances of the first and second coil and Lm is the mutual inductance between the two, the total inductance can be written as Ltot=L1+L2±2Lm. The mutual inductance is defined as the flux linked by the turns of an inductance when the other carries a unit current; this of course depends not only on the coil length and diameter but also on their relative position.

For two coaxial coils, like the ones in the picture, the mutual inductance is at maximum when they are also concentrical, i.e. D=0.

If the two concentrical coils are rotated, so that their axes are not parallel any more, the mutual inductance decreases, reaching zero (almost) when the angle is 90°. Continuing the rotation beyond 90° the mutual inductance increases again, but this time with the opposite sign. So, according to the above formula,

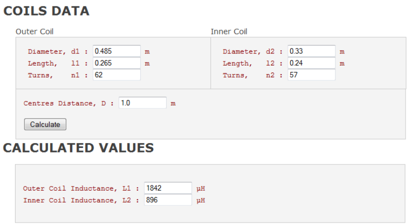


rotating a coil 180° gives a variation of the total inductance of 4Lm.

Two coaxial coils If the two coaxial coils are moved along their axis instead (D<>0), the mutual inductance decreaes, eventually reaching zero at infinity, but does not change sign. This means that there is less variation in the total inductance just moving the coils with respect to rotating them.

The form below computes the main parameters for the two types of variometer described above. The self-inductances are calculated using the formulas in [1], while the mutual inductances use the formulas from [2], with some minor corrections.

These formulas agree very well with the results of some electromagnetic simulations I have done, see the page for details.


COILS DATA

Outer Coil			Inner Coil		
Diameter, d1 :	0.485	m	Diameter, d2 :	0.33	m
Length, l1 :	0.265	m	Length, l2 :	0.24	m
Turns, n1 :	62		Turns, n2 :	57	

Centres Distance, D : 1.0 m

Calculate

CALCULATED VALUES

Outer Coil Inductance, L1 :	1842	µH
Inner Coil Inductance, L2 :	896	µH

In a typical variometer the two coils are concentric (i.e. D=0) and the overall inductance is varied rotating the inner coil; in this case we have:

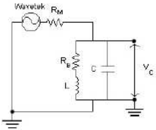


Figure 8: Circuit layout for part D

When analyzing AC circuits it is convenient to use impedances to determine various aspects of the response. For example, with this circuit, the impedance can be used to determine the magnitude of the voltage across the RLC element as

$$V_C = V_0 \frac{R}{R_M + Z} \qquad (13)$$

Impedances are complex numbers in general and for a capacitor Z = 1=j!C, for an inductor Z = j!L, and for a resistor Z = R where j = p □1. As can be seen in Figure 8 the RLC loop has the capacitor in parallel with a series addition of the resistor and inductor. Using the addition rules for circuits the result is that

$$|Z| = \frac{\sqrt{R^2 + (\omega L)^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + (\omega RC)^2}} \qquad (14)$$

where !0 is the resonant frequency.

With this in mind we set out to observe this response by measuring the peak to peak voltage across the capacitor as a difference and based on this it would seem all of our measurements of Rcritical were too low by at least 15%. However, when the resistance is adjusted upwards that much it is readily apparent that the circuit has moved into being over-damped based on the waveform. Therefore, it must be concluded that this is not a reliable way to measure an inductance at least partially due to the subjectivity of the measurement.

3.5 Part D: RLC response to sinusoidal signal

For this section a different circuit was used than in parts A, B, and C. The circuit is shown in Figure 8. RM was 101 k , L was 93 mH, RB was 5.69 , and C was 1.03 ┬ F. Also, the Wavetek was set to output a sine wave with an amplitude of 16 V.

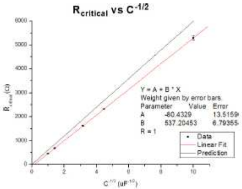
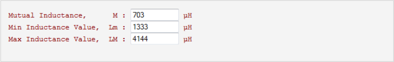


Figure 7: Plot of Rcritical versus 1/p C showing a strong linear relationship. Additionally, a plot of the theoretical behavior given the measured inductance is shown.



When the coils can not rotate but only move along their axis (coaxial coils), assuming a distance D between the two coils centres, we have:

The total inductance value for this case is computed assuming a positive mutual inductance (coils not rotated 180°).

References:

[1] R. Lundin, "A Handbook Formula for the Inductance of a Single-Layer Circular Coil," Proc. IEEE, vol. 73, no. 9, pp. 1428-1429, Sep. 1985.

[2] F.E. Terman, "Radio Engineers' Handbook," London, McGraw-Hill, 1st ed., Sep. 1950.

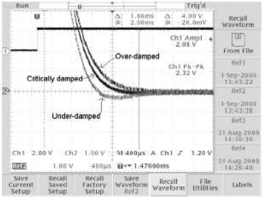


Figure 6: Sample circuit response for this part of the experiment

Table 5: Rcritical versus capacitance.

C (μF)	$R_{critical}$ (Ω)
1	460 ± 10
0.5	678 ± 10
0.1	1620 ± 30
0.05	2320 ± 30
0.01	5300 ± 100

the error values a change in the response was observed that we believe was significant enough to warrant a high degree of confidence (assumed to be 95%) in that range.

A plot of these critical resistances versus 1/p C is shown in Figure 7. Additionally, the ideal behavior is plotted using L = 90 mH. The first result to notice is that the linear fit is very strong and that the error in the slope is only 2.3% with 95% confidence. However, it is also clear that the plot implies a different value for the inductance of the circuit than previously found. Based on the slope of the fit, the inductance of the circuit is 72 − 3 mH with 95% confidence[4]. This gives a 20%

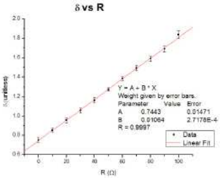


Figure 5: Graph of versus R showing a good fit and match to theory.

3.4 Part C: Dependence of resistance for critical damping on capacitance

For this part of the experiment the same circuit and waveform is used as in part B. The critical resistance is the resistance at which the circuit is critically damped. Therefore b2 = 0 and by solving equation 3 we find

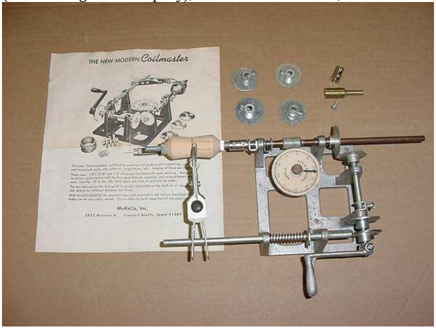
$$R_{critical} = 2\sqrt{\frac{L}{C}} \qquad (12)$$

So a plot of R vs 1/p C should be linear. We determined the critical resistance by adjusting the decade resistance box and observing the resulting waveform. Three example waveforms are shown in Figure 6. To find where the circuit was critically damped we allowed it to be slightly under-damped and then increased the resistance until no overshoot was observed. We did this for 5 different capacitances and our results are summarized in Table 5. Unfortunately we did not have one definitive way to determine the error in our measurement. However, when the resistances were adjusted by 7

Get/Make a "Coilmaster"
http://www.qsl.net/k5bcq/COIL/COIL.html

Get/Make a "Coilmaster"

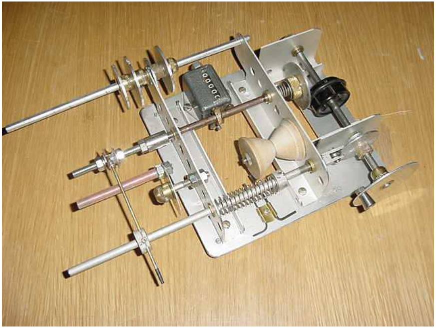
This is "The New Modern Coilmaster" made by MoReCo (MOrris REgister COmpany), Inc. of Council Bluffs, Iowa.



If you want to wind your own coils, and many people did, you need to build one of these. They are really pretty simple as you can see and with a little ingenuity you can make one. The turns counter is driven off a worm gear. The spring in front is to maintain contact between the wire guide assy and the different shaped cams, moving it back and forth. The cam is mounted right behind the crank handle. Each Coilmaster is equipped with four 32 pitch gears, with 39, 40, 42, and 44 teeth. This would give it a spindle to cam ratio of approximately 0.9:1 to

1.1:1. It says enameled wire must first be treated to give it "grip" by passing it through a quick-drying solution, such as resin dissolved in alcohol or other similiar material. Had you guessing too, huh ? Extra cams were $0.75 and gears were $1.00.

A Homebrew "Coilmaster"



This is homebrew "Coilmaster" using "whatever is available" parts. It has a variable spindle to cam ratio from 0.9:1 to 2.1:1, various cams, various wire feed heads, wire spool holder, turns counter, etc......even comes with two allen wrenches. It is shown with a small plastic bobbin coil spool mounted. Additional cams are located on the wire spool holder shaft and also prevent the wire feed spool from coming too close. The wooden spools are mounted on the spindle for large inner diameter coils.

$$\delta = \left(\frac{R}{2L}\right)\frac{2\pi}{\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}} = \pi\frac{R}{L}\sqrt{LC}\frac{1}{\sqrt{1-\frac{R^2C}{4L}}} \approx \pi\sqrt{\frac{C}{L}}R$$

(11)

when R2C=4L 1. In this experiment, the maximum value of R2C=4L is 0.08 and can fairly safely be ignored to test for linearity. Figure 5 shows versus R and a strong linear relationship is observed. Based on the fitting parameters the x intercept of the line is not R = 0
. This is because R is referring only to the resistance of the decade box and ignores the resistance of the coil and the function generator output. This residual resistance is calculated to be 70 − 4
with 95% confidence. The combined resistance of the inductor coil with the stated output resistance of the Wavetek is 69.4
giving a good agreement with the above measurement.

Table 4: versus resistance.

R (Ω)	V1 (V)	V2 (V)	δ
100	-6.28 ± .04	-1.00 ± .04	1.84 ± .04
90	-6.72 ± .04	-1.24 ± .04	1.69 ± .03
80	-7.16 ± .04	-1.46 ± .04	1.59 ± .03
70	-7.56 ± .04	-1.70 ± .04	1.49 ± .02
60	-8.00 ± .04	-2.00 ± .04	1.39 ± .02
50	-8.44 ± .04	-2.36 ± .04	1.27 ± .02
40	-8.96 ± .08	-2.80 ± .08	1.16 ± .03
30	-9.44 ± .08	-3.28 ± .08	1.06 ± .03
20	-10.1 ± .2	-3.88 ± .08	0.96 ± .03
10	-10.6 ± .2	-4.52 ± .08	0.85 ± .03
0	-11.1 ± .2	-5.24 ± .08	0.75 ± .02

Table 3 with the errors in voltage determined by multiplying the minimum change by 2.3 The error becomes worse as the peak number increases

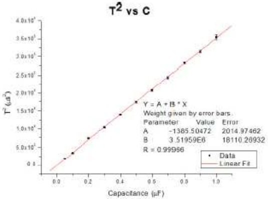


Figure 4: Plot of T2 versus C showing a straight line as predicted by theory.

Table 3: Verification of constant

Peak #	Voltage (V)	δ
1	$7.48 \pm .04$	N/A
2	$3.44 \pm .04$	0.78 ± 0.01
3	$1.56 \pm .04$	0.79 ± 0.03
4	$0.76 \pm .04$	0.72 ± 0.06

because the data was only acquired once with a set vertical resolution on the scope, and thus the uncertainty as a percentage of the value increased as well.

With the consistency of established we proceeded to measure as a function of R. The first and second peaks were measured and used to determine . This data appears in Table 4. Again the error was determined by multiplying the smallest change in V on the scope by 2. Plugging equations 2 and 4 to equation 5 gives

The whole thing is made from aluminum scrap. Since aluminum is soft (and easy to work), the wear points are strengthened with brass bushings from old potentiometers. Be sure to grease all the bearing surfaces or they will jam. All shafts are 1/4" (steel or aluminum) cut to size. The collars are made from old knob inserts after the plastic/bakelite is removed via an application of cold chisel. The cams are made from knobs with thick aluminum skirts (plastic/bakelite removed). Some required a little hammer tweaking to tighten the skirt. Since the cam is soft it rides on a small ball bearing (from an old PC disk drive) cam follower.

The drive "gear" is a knob with a groove cut into it. A small "O" ring is installed into the groove and contacts/drives the aluminum skirt of another knob. A short spring is located behind the skirted knob to insure pressure on the "O" ring. Ran into one problem on my coil winder. The "O" ring kept "walking" off the knob on the right angle drive. After a little study, I decided that the "O" ring footprint is not "zero", so the inside of the track is actually running at a different ratio than the outside of the track. This tends to cause an outward force on the "O" ring in the plane of the "O" ring drive shaft. If the centerline of the "O" ring drive shaft is below that of the driven plate shaft centerline, you also have the additional outward force due to the direction of rotation of the driven plate. The way I solved the problem is to equalize the forces by locating the "O" ring drive shaft centerline ABOVE the center line of the driven plate (for this size plate, "O" ring, etc it turns out that 3/32"-1/8" works best). No more problem, everything runs true at any ratio. This was the first Engineering Change to my coil winder .....the driving shaft bushings are now mounted in vertical slots.

The crank is yet another aluminum skirted knob. The mechanical counter was something from a swapmeet and lends itself to being worked with a beveled collar. I could have used a cam to work the counter, but the counter reset knob would have been difficult to get to. The wire feed is a small hobby shop brass tube run through a drilled knob insert with one set screw used to hold the tube in place and the other used to tighten it to the shaft. This tube easily feeds #24 AWG and smaller wire but a larger tube could be used and you can make several wire feed heads. Most coils I need will be #28 AWG or smaller. It has a bail made from a safety pin on the end. The wire is fed into the back of the tube and wire tension is controlled with your fingers. A small piece of shrink tubing was put on the end to further protect the wire. The wire feed head rests on an adjustable bar to control height (yet another knob with aluminum skirt). Fun project.

The additional holes in the braces are for better cooling and to decrease weight ...Hi, Hi.

......and we have the "Cheapo-Winder"

95% confidence1. Based on this value for the inductance it is determined that for C = 1 uF and R = 70 Ω.

$$\left(\frac{1}{LC}\right) = 1.11x10^7 \gg 1.51x10^5 = \left(\frac{R}{2L}\right)^2 \quad (9)$$

and therefore, according to equation 4

$$T^2 \approx (2\pi)^2 LC \quad (10)$$

This relationship is plotted in Figure 42. Again, the R value shows a strong linear relationship between the values as predicted and the error in the slope is only 0.9% with a 95% confidence interval.

Finally, the inductance of the coil was measured using a Z-meter and found to be 92− 1 mH. This is in good agreement with the value determined from the 1/T2 versus 1/C relationship.

3.3 Part B: Log decrement dependence on resistance

For this section of the experiment the same circuit and waveform was used as in part A. The capacitance was set to 1 ┬ F. First we observed the voltage transient at R = 0, 50, 100, 150, 200, 250, and 300 . Above 100 it became difficult to see more than 2 oscillations. Next we verified that the log decrement ( ) did not change going from one set of peaks to the next. We did this by setting the resistance to 0
and measuring the height of the first 4 peaks allowing 3 successive s to be calculated. The results are summarized in

value. Then the error of the single period was just the error of the total time divided by the numer of periods for that measurement. According to equation 4 a plot of 1/T2 versus 1/C should give a straight line with a

$$m = \frac{1}{4\pi^2 L} \qquad (7)$$

Figure 3 shows this relationship along with the best fit line. The error bars were calculated using the propagation of errors formula:

$$Error = \sqrt{\frac{4}{T^6} T_{error}^2} \qquad (8)$$

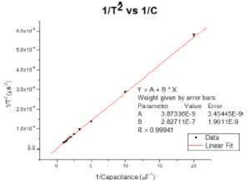
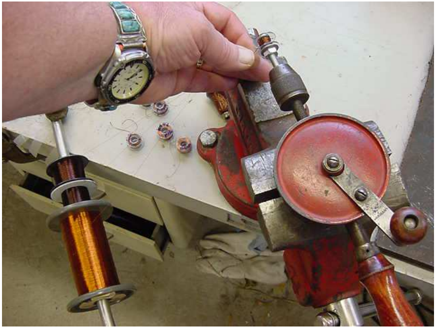


Figure 3: Plot of 1/T2 versus 1/C showing a straight line as predicted by theory.

In this plot the ’Error’ column refers to the standard error and it can be seen that the R value indicates a strong linear relationship. Using the relationship in equation 7 it was determined that the inductance of the circuit was 90− 1 mH to



This is "The Old Cheap Econo-Winder" used by many, including myself. It offers the advantages of: no cams to change, no gears to change, 4x chuck to crank ratio, infinitely adjustable, full control, able to wind multiple strands (above photo shows three #38 AWG strands being fed onto a spool). You are the turns counter....multiply the drill crank turns by 4.3 (this drill gear ratio). Turn the crank with one hand, tension and feed the wire in a pattern with the other hand. If you can "pat your head with one hand and rub your belly with the other", you can wind coils by hand. I you can't ....see above.

A few winding hints/ideas/observations

Your results will probably vary some from mine, but the general ideas will hold.

* It was pointed out to me that the main advantage of a winding machine is to allow a "basket weave" pattern which reduces adjacent turn/layer capacitance and increases coil "Q". Basket weave also tends to stay in place better than level wound coils because it's a diagonal criss-cross ....and it looks cool. You can manually wind a very acceptable "basket weave" pattern by hand with a hand drill or with a coil winder. It doesn't have to be "exactly correct" to see the advantage of higher coil "Q". Hand winding will generally require a spool or something to keep the end turns from collapsing. Coil winder made coils can be wound "free standing" if they are not too tall and you use a 2:1 spindle to cam ratio (my opinion).

* Using Litz wire or multi strand (individually insulated) wire increases the surface area which reduces resistance at RF frequencies due to "skin effect" (concentrates the RF current on the outer surface of the wire) and increases coil "Q". Litz wire is useable at frequencies of 2-3Mhz or LOWER. Above 2-3Mhz, solid wire performs as well as the Litz (from Therman's book). The manufacturers of Litz wire twist and alternately place the conductors relative to each other to reduce eddy currents, capacitance, keep impedance constant, etc. The inductance of one strand is about that of two strands, is about that of the whole bundle for Litz wire ....even though the "individual" coil inductances are, in effect, "paralleled". This is because of the close coupling of the wires.

* When winding with served Litz wire, you will find that it lays down easily and makes nice coils. Solid wire (single or multistrand) will also lay down easily if you feed it across a block of beeswax to give it "stick". You will find that the Litz wire does have higher "Q" but it's "bulkier" than the equivalent solid wire (will take up more space). Litz wire will have a tendency to kink as it comes off the spool due to the wire twist.

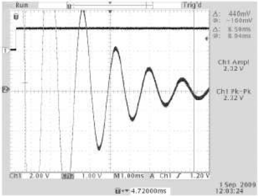


Figure 2: Representative transient signal of voltage across capacitor in this part of the experiment.

used. This was done for 11 different capacitances over the range of 0.05-1 ┬ F. The results are shown in Table 2. The error for the total time was taken to be the resolution

Table 2: Measurement of period of oscillation as a function of capacitance.

C (μF)	# of periods	Total time (ms)	Single period (ms)
1	5	9.40±.08	1.88±.02
0.9	5	8.86±.04	1.77±.01
0.8	4	6.72±.04	1.68±.01
0.7	5	7.82±.04	1.56±.01
0.6	6	8.64±.04	1.44±.01
0.5	5	6.62±.04	1.32±.01
0.4	6	7.08±.04	1.18±.01
0.3	7	7.14±.04	1.02±.01
0.2	9	7.70±.04	0.856±.004
0.1	13	7.64±.04	0.588±.004
0.05	9	3.75±.02	0.417±.002

of horizontal axis of the Tektronix multiplied by 2. We assumed that this large range gives a 95% confidence in the with serial number Phys-943034 was used to measure the resistance of the inductor coil #7. A Wavetek function generator was used in all parts and had the serial number Phys-943026. Additionally a Tektronix TDS3012B oscilloscope with serial number F28819 was used to measure the transient response in all parts.

3.2 Part A: Frequency dependence on capacitance

In this section, along with parts B and C, the Wavetek and circuit were connected as shown in Figure 1. VC was measured using the Tektronix scope and Rout was assumed to be 50 . R in the figure was a decade resistance box and C was a decade capacitance box. L was a large coil inductor (#7) which we measured the resistance of using the DMM before turning on the Wavetek. The resistance was found to be 19.36 . The Wavetek was set to output a 8 V unipolar square waveform with a frequency of 10 Hz and a duty cycle of 50%. The oscilloscope was set to trigger on the leading edge of the unipolar output of the Wavetek.

After all the elements had been setup, the resistance box was set to 0 and it was verified that the period was constant for each oscillation as shown in Table 1. Additionally a representative scope output is shown in Figure 2.

Table 1: Verification of constant period.

Cycle #	Period (ms)
1	$1.86 \pm .02$
2	$1.86 \pm .02$
3	$1.86 \pm .02$

Then we measured the period of oscillation for different capacitances by determining the time to complete multiple cycles and dividing the result by the number of cycles

* To measure the capacitance between conductors of bifilar or Litz wire, measure only a foot or so of wire and extrapolate to the total length (via resistance measurements). If you measure the capacitance on the spool, you will see a very small (in error) reading. The reason is that you are trying to measure capacitance across two really good RF chokes with a tester which uses an RF frequency to determine capacitance.
* Litz wire is sometimes a little confusing. For example: 6/44 unserved Litz wire means 6 strands of #44 AWG wire without an overall wrap (unserved). The AWG "equivalent circular mills" for 6/44 Litz is #36 AWG. This means that 6/44 Litz has the same circular mills and DC current handling capability as one #36 AWG wire. However, the RF current resistance of 6/44 Litz is less than half that of the #36 AWG wire and approaches that of #28 AWG wire. What all this means for the coil is that 6/44 Litz will have a higher "Q" than the equivalent solid wire of equal DC current capability (#36 AWG), but it will take up more space. Unserved 6/44 takes up the space of #34 AWG and served 6/44 takes up the space of #32 AWG.
* From what I've seen, manufacturers of small IF transformers in the 100Khz to 455Khz range use single #38 AWG wire (good), bifilar #40-42 AWG wire (better) or trifilar #44 AWG wire (best) for the windings (allows 18-28ma DC). The smaller JW Miller 100Khz IFs also have a ruberized "powdered iron?" layer inside the aluminum can. I believe the reasons for this layer is to further increase permeability and to isolate the coil from the can. The fact that 100Khz IFs require 5-6mH of inductance dictates a physically large coil, the large coil results in increased capacitive coupling to the can, which reduces "Q" and overall inductance. The higher permeability and resultant higher "AL" value (uH/100turns) is needed to keep the overall coil size manageable in the smaller 3/4" square IF shields.

* Spindle to cam shaft ratios for small IF type coils seem the be 2:1. This means the wire feed head traverses from one side to the other in one spindle revolution and traverses back during the second spindle revolution. I don't think a Morris "Coilmaster" can do that. A 1:1 spindle to cam shaft ratio means the wire feed head traverses to one side and back during one spindle revolution.
* Tuning slug/cup material permeability can vary substantially (just like toroids) and Amidon lists permeabilities up to 35 for their slug tuned coil forms. The 0.01 to 0.50Mhz "3" material has a permeability of 35. As an example: one 455Khz IF (which you would expect to have a high permeability slug) measures 0.7mH without a slug, 1.5mH (max) with it's powdered iron slug and 2.5mH (max) with another powdered iron slug from a JW Miller 100Khz IF (obviously with a higher permeability). Some of the physically small coils, like those in 3/8" cans use very high permeability ferrite cups to reduce coil size requirements.
* When swapping tuning slugs for one you just had to drill out because it broke, make sure the permeabilities are about the same or it won't tune correctly. Could never get that Heathkit HF osc to peak....huh ?.
* One source of low frequency, high permeability slugs is old TV horizontal osc coils. If you need to shorten a powdered iron slug, scribe it with a hacksaw and snap it by hand. A simple method for determining relative permeability is to swap slugs in a given coil form and measure the inductance.
* Coil spools solve the problem of the end turns "collapsing" on small coils, especially if you are not using a coil winder. The coil spools used are plastic "Singer" Class 15 (11/32" tall winding) or "PFAFF" (7/32" tall winding) sewing machine bobbins available at any sewing store (or the wife's sewing room ....but be careful, they don't share your enthusiasm). Coil winding using the wife's sewing machine "bobbin winder"

$$\delta = \ln \frac{q(t_{max})}{q(t_{max} + T_1)} = aT_1 \qquad (5)$$

The last equality in equation 5 can be made based on the form of q(t) where e^{-at} is the attenuating factor. Lastly, the quality factor Q of the circuit is defined as

$$Q = 2\pi \frac{total\ stored\ energy}{decrease\ in\ energy\ per\ period} = \frac{\pi}{\delta} = \frac{\omega_1 L}{R} \qquad (6)$$

with $\omega_1 = 2\pi f_1$ being the damped angular frequency. As can be seen, a lower resistance leads to a higher quality while a higher inductance increases Q.

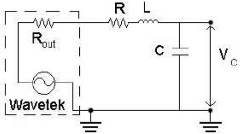


Figure 1: Circuit layout for parts A, B, and C.

3 Experiment

3.1 Equipment

A decade capacitor and decade resistor were used in parts A-C. The serial number for the capacitor was A-1678 and for the resistor was 48288. A HP 34401A Digital Multimeter (DMM)

$$b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} \qquad (3)$$

There are three distinct types of solutions depending on whether b2 is positive, negative or zero.

When b2 = 0 the circuit is said to be critically damped. In this case the two roots of the characteristic equation are real and the same value. Therefore, the charge in the capacitor falls to zero exponentially and quicker than for any other value of b.

When b2 > 0 the two roots of the characteristic equation are real and again the charge drops to zero in an exponential fashion. However, the falloff of charge is slower than for b2 = 0. This can be seen by observing that at later times the decay constant is (a − b) < a. With this type of response the circuit is said to be over-damped.

Finally, when b2 < 0 the two roots are imaginary and thus the charge oscillates about 0 before finally decaying to 0 (assuming a 6= 0). Under these conditions the circuit is said to be under-damped. The frequency of the oscillation is

$$f_1 = \frac{1}{T_1} = \left(\frac{1}{2\pi}\right)\sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2} \qquad (4)$$

where T1 is the period of the oscillation. Because the charge is still decaying the logarithmic decrement δ can be defined as the natural log of the ratio between two successive peaks of charge

does not work and can shorten your life. A "Singer" Class 15 or "PFAFF" bobbin has a thicker core and lends itself to being drilled out to 19/64", 9/32", or 17/64" which allows them to be put onto standard 1/4" slug tuned cores. Trim the spool with wire nippers and since it's an outside curved edge, a finger nail clipper works well. Trim the spools after you have wound them to keep the newly created rough edge from cutting small gauge wire or smooth the spool edge before winding. If the spools are not the correct height, cut them in half or make some "washers". It's important to secure these solidly to the coil form or the winding will tend to "walk" the washer ends outward. When done, carefully remove the washers and add some beeswax/Q dope to seal the outer turns.

* To keep the initial turns from slipping add a layer of double sided tape to the core or drip some beeswax onto the core (preferred).

* Many types of magnet wire allow direct tinning of the wire which burns off the insulation in the process. This is MUCH easier than scraping the insulation off with a razor blade, especially for the smaller gauges. Burning off the insulation over a flame is not advised.

* Aspect ratio is important. A fixed length of wire was wound on a given form and slug. The coil winding height on one was 3/8" and 3/16" on the other. The 3/16" winding (larger outer diameter) measured 40% (max) more inductance. Therman says optimum inductance is achieved if the winding cross section is square. Slug tuned coils may be different.

* To give the wire more "stick" as you are winding, feed it across and into a beeswax block (available at art/craft and sewing stores).

Factors Affecting Coil Q ...some experiments

EXPERIMENT #1 ....Skin Effect

Four coils were wound by hand. Coil Q will increase if the turn-to-turn capacitance is reduced. This can be accomplished by using a "basket weave" pattern. The "basket weave" coils have roughly 1-2 spool length traverses of the wire per revolution of the spool (coil Q will be even higher on fixed pattern wound coils made using a coil winder). Q can also be increased by using multi strands of smaller gauge wire due to increased surface area (skin effect) at frequencies below about 3Mhz. It's a substantial improvement in Q to use two strands of #39 AWG vs one strand of #36 AWG, or better yet .....six strands of #42 AWG vs one strand of #36 AWG. All three of those examples have the same current handling capability. Many small, higher quality, IF transformers are wound with two strands of smaller gauge wire. Larger, high quality IF transformers are wound with Litz wire, dipped, etc.

All coils were trimmed to 2.0 mH with no slug, and 3.0 mH with a slug. Q Data is "unloaded Q" as measured on a Boonton 260-A.

At 200Khz, all coils trimmed to 2mH, no tuning slug

Coil #1 #34 AWG, level wound, 325pf, Q of 52
Coil #2 #34 AWG, basket weave wound, 320pf, Q of 55
Coil #3 two #38 AWG, basket weave, 310pf, Q of 78
Coil #4 three #38 AWG, basket weave, 320pf, Q of 83

At 170Khz, all coils trimmed to 2mH, no tuning slug

Coil #1 #34 AWG, level wound, 440pf, Q of 55
Coil #2 #34 AWG, basket weave wound, 447pf, Q of 57
Coil #3 two #38 AWG, basket weave, 433pf, Q of 78

RLC loop when an external voltage was applied. The capacitance was varied and the periods of the oscillations were used to determine the inductance in the circuit. Next we measured the log decrement as a function of resistance to verify the response is approximately linear and to estimate the total resistance of the circuit including the inductor and the function generator. Following that we determined the resistance required for critical damping as a function of capacitance. Using this we verified the theoretical result that the critical resistance is proportional to 1/ p C. Finally, we measured the voltage across the capacitor in a different RLC circuit driven by a sinusoidally varying voltage. The peak-to-peak voltage was measured as a function 1 of frequency to determine the resonant frequency, the bandwidth, and the quality factor Q. We also compared the resonant frequency with the theoretical value.

2 Theory

The governing equation for a resistor, inductor and capacitor in series with a voltage source is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V(t) \qquad (1)$$

This is the equation for an oscillator with damping and a driving function. Solving the characteristic equation gives two roots s1; s2 = a − b with

$$a = \frac{R}{2L} \qquad (2)$$

Transients and Oscillations in RLC Circuits

Will Chemelewski with Brian Enders

http://courses.physics.illinois.edu/phys401/Files/Misc/Sample%20report%20RLC.pdf

Abstract

Transient responses of RLC circuits are examined when subjected to both long time scale (relative to the decay time) square wave voltages and sinusoidally varying voltages over a range of frequencies about the resonant frequency. In general, a good correspondence is found between theory - describing the charge in the system in terms of a 2nd order differential equation with a harmonic oscillator form - and experiment. It is demonstrated that the inductance can be accurately measured using period of oscillation versus capacitance measurements. Furthermore, the exponential decay of the response is described well by the model and the resonant frequency of a sinusoidal external voltage is accurately predicted. However, some discrepancies were found though not necessarily a result of theoretical failures. One problem is the failure to predict the inductance of a circuit based on the critical resistance variation with capacitance, although the problem could lie in how the measurement is conducted. Additionally, the quality and bandwidth of a RLC element is poorly predicted but this could also be a result of experimental problems.

1 Purpose

The purpose of this experiment was to observe and measure the transient response of RLC circuits to external voltages. We measured the time varying voltage across the capacitor in a

Coil #4 three #38 AWG, basket weave, 446pf, Q of 85

At 150Khz, all coils adjusted to 3mH with the tuning slug

Coil #1 #34 AWG, level wound, 400pf, Q of 82
Coil #2 #34 AWG, basket weave wound, 380pf, Q of 85
Coil #3 two #38 AWG, basket weave, 380pf, Q of 109
Coil #4 three #38 AWG, basket weave, 385pf, Q of 114

EXPERIMENT #2 ....Coil Form Dielectrics

Other factors which effect coil Q are the coil form material and the adhesive used to hold the windings. Any coil form which can absorb moisture, like the old cardboard oatmeal boxes and toilet paper tubes used for crystal sets, is a problem unless the cardboard is treated with varnish, shellac, etc ....but it's cheap.

Some have suggested using old plastic pill bottles which is OK if they are sturdy so the coil turns don't move. Recently some testing has shown that coils wound on those amber pill bottles have just has high a "Q" as those wound on the Amphenol 5-pin forms. Some PVC is the same way. What does affect "Q" is the material you use to coat the windings (materials to be tested later ...beeswax, refined paraffin, varnish, laquer, other "glues"). Here is an example of coils wound (adjacent turns) with #28 AWG magnet wire:

"Q" at 6 Mhz with all coils measuring 20.5uH				
Reference	Note 1	Note 2	Note 3	note 4
1-1/4" Amphenol 5-pin	175	--	175	--
1-1/4" amber pill bottle	180	--	177	172
1-1/2" amber pill bottle	161	147	--	--
1-1/4" untreated cardboard	152	130	--	--
1-1/4" PVC	173	--	175	--

Notes:

Plain wiring, no adhesives, no tape to hold windings
Use a light coat of clear fingernail polish to hold windings (strips across windings didn't work), Q reading 15-20 points lower when first applied, above readings are after 48 hours.
Use Elmer's Stix-All silicone to hold windings (4 strips across coil windings)
Use Scotch black electrical tape to hold coil windings (1-1/2 turn)

EXPERIMENT #3 ...Turn Spacing

The highest Q coil for a given inductance is "air wound" with space between the turns (less turn-to-turn capacitance), the next is space winding on a good coil form (ceramic, phenolic, etc). Space winding on a coil form by winding string or another wire along with the coil wire and removing the "spacer" later is good. Ceramic and some phenolic coil forms came with wire groves to space the turns. You do have the problem of having to stabilize the separated turns with Q-Dope, paraffin, Duco cement, etc.

Here is some data on two coils:

stripped. You will be left with nice clean varnish-free wires, which can then be tinned as usual.

As a test of this method I measured the AC impedance of each of the wires of the same length at 10kHz using my LCR meter, and found that they all were very consistent, indicating that I was effectively and consistently removing the varnish and using all the wires in the bundle.

By the way, the speakers sound terrific! I'm not sure how much of the improvement was from the litz wire, but it certainly didn't hurt the sound!

found it works, but you end up oxidizing the wires and even incinerating some of the smaller strands),

5) Individually sanding each strand (I'd be doing this job for years as each wire has dozens of strands and I have 6 tweeters, 6 midranges and 4 woofers to wire up to 18 capacitors and 20 inductors!).

I tried several of these methods, and none were satisfactory in producing a nice clean tinned wire end, without breaking or damaging the strands. However, I did eventually come up with a simple, fast and very effective method, so I thought I would share it with the other DIY'ers here, in case they are crazy enough to build something with litz wire.

A Dremel tool with a stainless steel wire wheel attachment works very well. I found that the stainless wheel works faster and lasts longer than the carbon steel wheel, with the added advantage that the small pieces of the wheel brush that wear off during the process are non-magnetic, so they won't attach to nearby driver magnets (don't need little pieces of wire inside my EMIMs and EMITs!). Simply remove ~ 1/2" of the insulation, then untwist the various strands from each other (the Cardas Litz has 3 counterwound layers of different diameter wire to untwist, do them one at a time) and flatten them into a flat fan shape. Set the Dremel to about 1/2 of maximum speed and apply to the coated wire, ensuring that the wheel is ALWAYS turning toward the ends of the wire (otherwise they will entangle with the spinning wire wheel and snap off!). 10 seconds on each side of the fanned wire is very effective at removing the varnish coating without destroying the wires. Then re-fan the wire in a different direction and repeat the process 2 or 3 times to ensure that all the wires get

1-1/4" coil form (amber pill bottle)
both coils 13.8uH +/- 0.1uH
both coils #28 AWG wire
coil "A" 16 turns are adjacent
coil "B" 20 turns are on .025" centers +/- (spacer was removed)
both coils, no adhesives/coatings to hold the turns in place

At 3.4MHz ..... coil "A" Q=107, coil "B" Q=175

Granted, these are unloaded Q measurements on a Boonton 260-A and loaded Q will be less, however the improvement will be carried over and the "trend" is there.

What does this mean for the signal (loaded Q will be less dramatic):

Sensitivity ...the Boonton uses a 20mVAC "e" signal and measures "E" with a very sensitive voltmeter, so a Q of "107" means "E" is 2.14VAC ....a Q of "175" Q means "E" is 3.5VAC ....higher sensitivity.

Selectivity ....Q is also the -3db voltage BW divided into the frequency (Q = Fo/-3dbBW ...or -3dbBW = Fo/Q). For a Q of "107" the -3dbBW is 31.8Khz. For a Q of "175" the -3dbBW is 19.4Khz .....better selectivity.

EXPERIMENT #4 ....Coil Dope

"E6000" Craft Glue was used on a coil with a Q of "175" ...absolutely no change in Q as you put the material on, as it dries, or 24 hrs later, even for a very heavy application of glue. I believe it's some kind of silicone based product .....waterproof, clear, and remains flexible. My wife didn't even tell me she had some "really good coil dope" material in her crafts pile.

"CG Clear Ice" fingernail polish was used on a coil with a Q of "107" and the Q dropped to "97".

Considering the amplitudes of successive cycles when $\cos(\omega_d t - \phi) = 1$

$$x_n = X_0 e^{-\zeta\omega_n t_n} \quad \text{and} \quad x_{n+1} = X_0 e^{-\zeta\omega_n (t_n + T_d)}$$

$$\text{Therefore} \quad \frac{x_n}{x_{n+1}} = e^{\zeta\omega_n T_d}$$

The logarithmic decrement δ is ln(xn / xn+1).. Normally n =1 and n+1 is therefore 2.

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \zeta\omega_n T_d = \zeta\omega_n \frac{2\pi}{\omega_d} = \zeta\omega_n \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

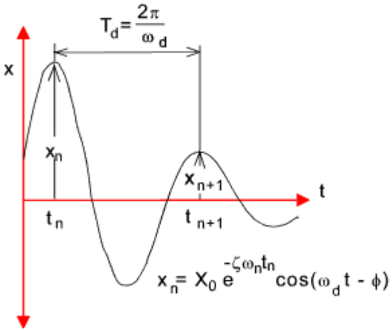
The damping ratio ζ can be expressed in terms of the logarithmic decrement δ as follows

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

It is clearly possible to determine the damping ratio experimentally for a mechanical system be initiating vibrations and measuring the amplitude of the vibrations.

Logarithmic Decrement....
Roy Beardmore 17/01/2013
http://www.roymech.co.uk/Useful_Tables/Vibrations/Free_Vibrations.html

The rate at which the amplitude of vibrations decays over time provides a very useful method of identifying the degree of damping. See the figure below for the plot of a typical underdamped vibration.



The equation for of motion for underdamped oscillations arrived at above can be used to establish the amplitude of any of the cycles. i.e.

$$x_n = X_0\, e^{-\zeta \omega_n t_n} \cos(\omega_d t - \phi)$$

Understanding LC Tank Q, Impedance, and Losses
David Wagner
http://wiki.waggy.org/dokuwiki/crystal_radio/tank

A resonant inductor-capacitor (LC) tank is the beating heart of a crystal radio, filtering the signal of interest from the sea of radiofrequency (RF) energy received by an antenna. Properly loaded, a high-quality (high-Q) tank will give a crystal radio greater sensitivity and selectivity.

For a parallel tank, a smaller inductor and larger capacitor can result in higher Q.

$$Q = \omega R_p C = \frac{R_p}{X} = \frac{R_p}{\omega L}$$ 1)

Tank Impedance

When tuned to the station of interest, the crystal radio's LC tuning tank between the antenna and detector presents a very high resistive impedance *to ground*. Estimating the magnitude of this effective shunt resistance is not difficult, but it is important to keep in mind how the tank impedance is in parallel with both the source and the detector/load impedance.

If the source and load impedances are low in comparison to the tank's shunt resistance, this resistance may be neglected, the tank loss will be fairly low, and the antenna can be matched directly to the serial combination of the detector and the audio load. However, passing a low impedance signal significantly loads the tank and results in broader tuning. To minimize tank loading, the signal impedance will approach the tank's shunt resistance, and this resistance must be considered as part of the driven load. Estimating the parallel LC tank's effective shunt resistance can be done by measuring or assuming the tank's unloaded Q.

$$R_{tank}=2\pi f\ L_{tank}Q_{unloaded}$$

Tank Loading

For efficient energy transfer, the antenna impedance should be matched to the parallel combination of the load and shunt impedances. In effect, the tank must be kept fully 'powered up' to maintain its narrow frequency response while reflecting the rest of the signal into the load.

$$R_{src}=\frac{1}{\frac{1}{R_{load}}+\frac{1}{R_{tank}}} \Rightarrow \frac{1}{R_{load}}+\frac{1}{R_{tank}}=\frac{1}{R_{src}}$$

The LC tank shunt resistance can then be used to estimate upper limits on the tank's loaded Q and the matched source and load impedances.

$$R_P=\frac{1}{\frac{1}{R_{src}}+\frac{1}{R_{load}}+\frac{1}{R_{tank}}}=\frac{1}{\frac{1}{R_{src}}+\frac{1}{R_{src}}}=\frac{1}{2}R_{src}$$

$$Q_{loaded}=\frac{R_P}{2\pi f\ L_{tank}}=\frac{R_{src}}{4\pi f\ L_{tank}}$$

$$R_{src}=2R_P$$

This is for very small signals.

oscillation of IL relative to the oscillation of I changes by about 180 deg. Increasing the value of the Q-factor makes this variation of phase more abrupt. However, the most important feature of the steady state solution described by Equations (19-21) is that one can reduces the frequency width of the peak in the amplitude of IL, increases its height, and chooses its frequency position, by an appropriate choice of L, r and C. That the enhancement can be made to occur in a selected, narrow frequency band by tuning the parameters of a resonant circuit is particularly useful in applications, as this makes it possible to isolate a signal occurring at a particular frequency (e.g., the emission from a radio station) from an irrelevant background.

6. To conclude

More details on LR and series LRC circuits can be found in Young and Freedman [1], chapters 30 and 31. The Physics and Maths of LRC circuits will also be studied in the Level 2 Electronics course, amongst many other things.

References

[1] H. D. Young and R. A. Freedman, University Physics, 13th Ed., Pearson Addison-Wesley, San Francisco (2012).

$$R_{load} = R_{det} + R_{audio} = 2R_{det}$$

This may not be correct since R_load(RF) may be only about half of R_load(audio) and R_load(DC).

Assume all loss is through the LC tank shunt resistance (R_tank); losses due to circulating current resistance are small in comparison to primarily capacitive losses to ground.

$$R_{src} = \frac{1}{\frac{1}{2R_{det}} + \frac{1}{R_{tank}}}$$

$$\Rightarrow R_P = \frac{1}{2} R_{src} = \frac{1}{\frac{1}{R_{det}} + \frac{2}{R_{tank}}}$$

$$\Rightarrow Q_{loaded} = \frac{1}{\left(\frac{2\pi f L_{tank}}{R_{det}} + \frac{2}{Q_{unloaded}} \right)} =$$

$$\frac{Q_{unloaded} R_{det}}{2\pi f L_{tank} Q_{unloaded} + 2R_{det}}$$ 2)

$$\Rightarrow Q_{loaded} = \frac{1}{\left(\frac{4\pi f L_{tank}}{R_{load}} + \frac{2}{Q_{unloaded}} \right)} =$$

$$\frac{Q_{unloaded} R_{load}}{4\pi f\ L_{tank} Q_{unloaded} + 2R_{load}}$$

The tank loss can now be estimated.

Efficiency (%): $\frac{R_{tank}}{R_{tank} + 2R_{det}} = \frac{\pi f\ L_{tank} Q_{unloaded}}{\pi f\ L_{tank} Q_{unloaded} + R_{det}} =$

$$\frac{1}{\frac{R_{det}}{\pi f\ L_{tank} Q_{unloaded}} + 1}$$

Tank Loss (dB): $10\log\frac{R_{tank}}{R_{tank} + 2R_{det}}$ dB

Tank Loss (%): $\frac{R_{det}}{R_{tank} + 2R_{det}} = \frac{R_{det}}{2\pi f\ L_{tank} Q_{unloaded} + 2R_{det}}$

Circulating Current Losses

The preceding should be valid so long as the tank's Q is limited primarily by its shunt resistance. To check this assumption, verify the tank's circulating current series resistance (R_circ) is

at the natural frequency of the LRC circuit, $f_0 = \omega_0/2\pi$. (This last feature might not be completely obvious from the form of the equation if you are not much used to analysing the behaviour of functions.) At its maximum, the amplitude of oscillation of IL is about a factor Q larger than the amplitude of oscillation of I. (Note that $I_{L0} = QI_0$ when $\omega = \omega_0$. The amplitude of IL can exceed that of the total current, I0, because the current passing through the inductor includes a current circulating round the LCR circuit formed by the inductor and the capacitor; this circulating current may be large when $f \approx f_0$.)

This enhancement is an example of a resonance, a common phenomenon in many different fields of Physics and Engineering: When an oscillating system is excited by a periodic perturbation whose frequency (almost) coincides with one of its natural frequencies, the response of the system to the perturbation is greatly magnified when damping is light. (See Young and Freedman [1], Section 14.8, for a short discussion of this matter.)

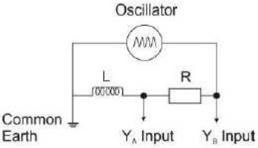


Figure 3: The circuit considered in tasks 1 and 2.

One also sees from Equation (20) that when the frequency of the generator sweeps through the resonance, the phase of the common feature of the evolution of dissipative systems kicked away from their equilibrium position by a sudden perturbation.

The steady state solution is particularly interesting when the total current generated by the signal generator varies sinusoidally. Setting $I = I_0\cos(\omega t)$, as given by Equation (2), Equation (15) now reads

$$LC\frac{d^2I_L}{dt^2}+rC\frac{dI_L}{dt}+I_L=I_0\cos(\omega t) \qquad (18)$$

This last equation can also be solved exactly using general methods (which you will study in your Maths course). Doing so results in the following steady state solution:

$$I_L = I_{L0}\sin(\omega t-\phi) \qquad (19)$$

where the phase-angle f is defined by the equation

$$\tan\phi = Q\frac{\omega^2-\omega_0^2}{\omega\omega_0} \qquad (20)$$

and

$$I_{L0}=\frac{I_0}{\sqrt{\frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}+\left(\frac{\omega_0^2-\omega^2}{\omega_0^2}\right)^2}} \qquad (21)$$

Equation (19) shows that in the steady state the current through the inductor oscillates at the frequency of the signal generator, $f=\omega/2\pi$. Equation (21) tells us that when the Q-factor is much larger than 1, then, as a function of f, IL strongly peaks

much lower than what it would need to be to be solely responsible for the LC tank's unloaded Q.

$$R_\circ \ll \frac{2\pi f\, L_{tank}}{Q_{unloaded}} = \frac{1}{2\pi f\, C Q_{unloaded}}$$

For the 'worst case' broadcast band values, this series resistance is small, but not impossible to beat.

$R_\circ \ll \frac{2\times\pi\times f\ \times 90\times 10^{-6}}{1000} \approx$ 0.3 Ω at 520 kHz, and 1 Ω at 1720 kHz.

For comparison, the 30' (10 m) of 12 AWG (2 mm) solid wire sufficient to make a stout cylindrical air-core inductor of this value is about 0.05 Ω.
One thing to note is how much circulating current resistance by itself is equivalent to the tank loading resistance (again by itself) resulting in the same tank Q.

$$Q=\frac{R_{tank}}{2\pi f\, L_{tank}}=\frac{2\pi f\, L_{tank}}{R_\circ}$$

$$\Rightarrow \quad R_{tank}R_\circ=\left(2\pi f\, L_{tank}\right)^2$$

$$R_{tank}=\frac{\left(2\pi f\, L_{tank}\right)^2}{R_\circ};$$

$$R_{\circ}=\frac{\left(2\pi f\ L_{tank}\right)^2}{R_{tank}}$$ [3]

Thus, at 1 MHz, one ohm of circulating current resistance is equivalent to loading a 100 µH tank with about 400 kΩ, while it would load a 200 µH tank only one-quarter as heavily, about 1.6 MΩ. Since parallel loading dominates most crystal radio tank designs, it seems worthwhile to use tanks with smaller inductors. However, parallel loading decreases Q with increasing frequency, and BCB reception requires an increasing Q to maintain constant bandwidth, so some trade-offs may be worth considering.

Now, if it were true that the unloaded tank Q can be modeled accurately from only the inductance (L_tnk) and parallel tank resistance (R_tnk), then the Q should always decrease with frequency, but this is not the case. Perhaps a combination of parallel and serial (circulating) resistance can be used.

$$Q_{unloaded}=\frac{1}{\frac{2\pi f\ L_{tank}}{R_{tank}}+\frac{R_{\circ}}{2\pi f\ L_{tank}}}$$

Tank Design

Typical ferrite toroid inductor Q is 300.[4]
Extremely good tanks have R_tank increasing from 1 MOhm to 2 MOhm across the broadcast band.[5]

[1] Q-Factor

$$LC\frac{d^2I_L}{dt^2}+rC\frac{dI_L}{dt}+I_L=I \tag{15}$$

The solutions of this equation will depend on how the total current I varies in time. For a square wave signal, I can be taken to jump rather abruptly, but periodically, from one value to another, staying constant between the jumps. Between any two consecutive jumps, the time evolution of IL is thus governed by the equation

$$LC\frac{d^2I_L}{dt^2}+rC\frac{dI_L}{dt}+I_L=I_0 \tag{16}$$

where I0 is a constant. The relevant solution of this equation is

$$I_L = A\exp(-\alpha t)\cos(\omega_0' t+\chi)+I_0 \tag{17}$$

with the values of the constants A and depending on the precise variation of I during the jumps. (What these values are is not important for us.)

It is clear from Equation (17) that immediately after the jump in I, the current across the inductor oscillates at the natural frequency of the circuit, about a value equal to I0, with an amplitude of oscillation decreasing in time due to the exponential factor. Thus IL tends towards I0, and becomes indistinguishable from I0 once the initial oscillation has sufficiently decrease in amplitude. The time-evolution of IL can thus be divided into two parts: At first, immediately after the jump in I, IL displays transients (the free oscillations), which disappear gradually and leave place to a steady state solution (IL = I0). The occurrence of transients is a very is large (which will be the case for the circuit built in this practical).

As can be easily checked from Equation (10), the amplitude of the oscillation decreases by a factor 2 over any interval of time of duration $\Delta t_{1/2} = \ln 2 / \alpha$, that is, over a number of periods of oscillation equal to $n_{1/2} = \Delta t_{1/2} / T_0$. (We neglect the difference between ω_0 and ω_0'.) Given that $\alpha = r/(2L)$ as noted above, and that $T_0 = 1/f_0$, we have, in good approximation when Q is large,

$$Q = \frac{\pi n_{1/2}}{\ln 2} \qquad (14)$$

This last result will be used in Task 5. An important conclusion one can derive from it is that a large Q-factor means that the oscillations last for a long time, i.e., that the damping is light.

However, the case of interest here is not that of an LRC circuit in isolation but that of Figure 4 below, in which an LRC circuit is connected to other components. In the experiment, the function generator, or "oscillator", may generate a signal which is either a square wave or a sinusoidal wave, and, due to the large resistance of the resistor R, the current flowing through the whole circuit is practically the same as if the capacitor and inductor were not present. We denote by I the total current flowing through the whole circuit. Therefore we now have $I_C + I_L = I$ instead of $I_C + I_L = 0$.

The result is that Equation (9) acquires a non-zero right-hand side and becomes

[2]
$$Q_{loaded} = \frac{1}{\left(\frac{1}{R_{det}} + \frac{2}{R_{tank}}\right) 2\pi f L_{tank}} =$$

$$\frac{1}{\left(\frac{1}{R_{det}} + \frac{2}{2\pi f L_{tank} Q_{unloaded}}\right) 2\pi f L_{tank}} =$$

$$\frac{1}{\left(\frac{2\pi f L_{tank}}{R_{det}} + \frac{2}{Q_{unloaded}}\right)} = \frac{Q_{unloaded} R_{det}}{2\pi f L_{tank} Q_{unloaded} + 2 R_{det}}$$

[3] This gives the same results as Series - Parallel Impedance Conversion Calculator//.
[4] Experiments with Coils and Q-Measurement, Wes Hayward, w7zoi, October, 2007 (Update 01Dec07.)
[5] Experiments with LC circuits part 10

$$\omega_0' = \sqrt{\frac{1}{LC} - \frac{r^2}{4L^2}} \tag{11}$$

[Equations (10) and (11) are not correct when the argument of the square root function is zero or negative; however, we do not need to worry about this as in the practical r, L and C are such that the argument is positive.]

The time evolution described by Equation (10) is a damped oscillation: the current oscillates, now at a natural angular frequency ω_0' rather than ω_0, and the amplitude of the oscillation decreases exponentially in time. The oscillations disappear because the energy they contain is dissipated – here it is dissipated as heat due to the resistance of the coil. The "quality" of an oscillator, i.e., the length of time the oscillations last once started, is usually quantified by a dimensionless parameter denoted Q, the Q-factor of theoscillator. Here Q is given by the equation

$$Q = \frac{2\pi f_0 L}{r} \tag{12}$$

where f0 is the natural frequency of the circuit as defined above, $f_0 = 1/2\pi\sqrt{LC}$.Thus $Q = \omega_0 L / r$, and, from Equation (11),

$$\omega_0' = \omega_0\sqrt{1 - \frac{1}{4Q^2}} \tag{13}$$

The displacement of the natural frequency from its zeroresistance value is therefore small when the quality factor

T_0. $f_0 = 1/T_0$, is called the natural frequency of the circuit. (Recall the relations between frequency, angular frequency and period: $T_0 = 2\pi/\omega_0$ and $\omega_0 = 2\pi f_0$.)

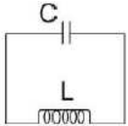


Figure 2: An LC circuit

It is a simple matter to correct Equations (6) and (7) for the internal resistance of the inductor. (With this resistance taken into account, the circuit shown in Figure 3, is a "parallel LRC circuit.) If the resistance of the inductor is r, then $V_L = L\, dI_L/dt + rI_L$, so that Equation (6) now gives

$$LC\frac{d^2 I_L}{dt^2} + rC\frac{dI_L}{dt} + I_L = 0 \qquad (9)$$

This last equation has exactly the same form as the equation of motion of a classical harmonic oscillator for the case where the oscillating mass is submitted to a frictional force proportional to its velocity – see, e.g., Section 14.7 of Young and Freedman [1]. The equation can be solved analytically using general methods. The result is that

$$I_L = A\exp(-\alpha t)\cos(\omega_0' t + \chi) \qquad (10)$$

where A and are two arbitrary constants (i.e., two constants which can have any value), a = r / (2L) , and

MEASURING THE Q OF LC CIRCUITS

Dick Kleijer crystal-radio.eu
http://www.crystal-radio.eu/enqmeting.htm

In theory we can determine the Q of a circuit as follows:

Step 1

Couple a RF signal generator to the LC circuit.The coupling between generator and LC circuit must be loose, otherwise the output resistance of the generator will load the circuit and reduces the Q.

Step 2:

Set the generator to the frequency at which you want to measure the Q.Adjust the LC circuit (turn the tuner capacitor) so you have maximum voltage over the circuit, the circuit is now in resonance, this frequency is the resonance frequency of the circuit (f.res).

Step 3:

Measure the voltage over the LC circuit at resonance frequency (f.res).

Step 4:

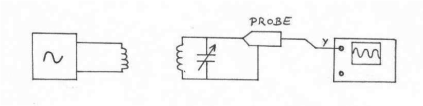
Vary the generator frequency a little above and below f.res. and determine the two frequenties were the voltage over the circuit is 0.707 times the value at f.res.The voltage reduction to 0.707 times, is the -3 dB point.One -3 dB point, is lower in frequency then f.res, this frequency we call: fl.The other -3 dB point is higher in frequency then f.res, this frequency we call: fh.

Step 5:

Calculate the bandwidth BW: **BW= fh - fl.**Calculate the Q: **Q= f.res / BW**

For performing these 5 steps, we can use the following test setup

Test setup 1 Measuring the Q with a signal generator and a probe.



In the schematic above you see from left to right the following components.
A signal generator
A coupling coil
The LC circuit
A 1:100 oscilloscope probe
A oscilloscope

Connect the output of the signal generator to the coupling coil having e.g. 50 turns.
Place the coupling coil at about 20 cm from the coil of the LC circuit.
The coupling coil don't have to be high Q.

These currents and voltages and the charge of the capacitor may vary in time. The instantaneous voltage across the capacitor is given by $V_C = q/C$. Since $I_C = dq/dt$ and $I_C = -I_L$, we can write

$$C\frac{d}{dt}(V_C) = -I_L \tag{6}$$

From Equation (1), we also have VL= L dIL/dt. (As above, we assume that there is no significant potential drop across the coil originating from its internal resistance.) Since C L V =V , we arrive at the equation

$$LC\frac{d^2 I_L}{dt^2} = -I_L \tag{7}$$

You may recognize from the Foundations course that this equation has exactly the same form as the equation of motion for a mass attached to a spring and sliding on a frictionless table (see Chapter 14 of Young and Freedman [1]). Its solutions have thus the same mathematical properties as the solutions found in the mechanical problem, although the Physics context is completely different. Therefore IL (and thus q, IC, VC and VL) oscillate harmonically in time, i.e., like a sin or a cos function. Repeating the calculation done for the mechanical harmonic oscillator, one finds that the angular frequency of these oscillations is

$$\omega_0 = \sqrt{\frac{1}{LC}} \tag{8}$$

and that their period, T_0. is $2\pi\sqrt{LC}$. The inverse of

$$V_L = -\omega L I_0 \sin(\omega t) = \omega L I_0 \cos(\omega t + \pi/2) \tag{4}$$

Comparing equations (2) and (4), we see that the maxima of VL occur a time $\pi/(2\omega)$ before the maxima I, i.e. a quarter of a cycle earlier: one says that the voltage leads the current by 90 degrees.

From the last two equations we also see that the maximum of the voltage across the inductor, V_L^{MAX}, is $\omega L I_0$, while the maximum of the voltage across the resistor, V_R^{MAX}, is RI_0. We thus have

$$V_L^{MAX} = \left(\frac{\omega L}{R}\right) V_R^{MAX} = \left(\frac{2\pi f L}{R}\right) V_R^{MAX} \tag{5}$$

This relation will be used in Task 2 to obtain the inductance of the inductor provided. Let us now consider the circuit shown in Figure 2. To begin, we'll assume that the resistance of the wires connecting the capacitor and the inductor and the resistance of the wire forming the inductor (its internal resistance) are both negligible. The voltage across the capacitor and the voltage across the inductor will be denoted by VC and VL, respectively, the capacitance and inductance of these two components by C and L, the current flowing through the capacitor by IC, that flowing through the inductor by IL, and the charge of the capacitor by q. For convenience, we measure VC , VL , IC and IL from left to right. Hence VC =VL and C L I = -I . (To see why there is a minus sign in the last equation, note that the current must flow in opposite directions in the upper and lower parts of the circuit.)

Because of the 20 cm distance, there is a loose coupling between the coils.

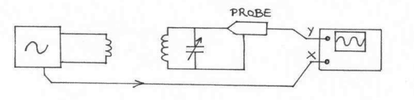
Connect the probe to the LC circuit.
The earth connection of the probe must be connected to the housing of the tuner capacitor.
The probe is connected to the oscilloscope.
The probe provides a small loading of the circuit, so the Q will not reduce so much.
There are also 1:1 and 1:10 probe's, but these will load the LC circuit too much.
The 1:100 probe I use has a input resistance of 100 M.Ohm, and a input capacity of 4 pF.

The output voltage of the generator must be set so high, that the oscilloscoop gives a clear picture of the RF signal.
Because the 100 times attenuation in the probe, the signal generator output must be set fairly high.
When measuring low Q circuits, I must set the generator output to it's maximum of 20 Volt peak-peak.

For measuring the Q: perform the 5 steps described on the top of this page.

The frequency adjustment is done by hand, by turning the frequency knob of the generator.

Test setup 2 Measuring the Q with a sweep generator and a probe.



In this schematic you see from left to right the following components:

A sweep signal generator
A coupling coil
The LC circuit
A 1:100 oscilloscope probe
A oscilloscope

This method uses a sweep generator, this is a signal generator where the frequency is constant varying between two set values.
I use a sweep function generator of brand "Hung Chang" with model number G305, it can produce signals up to 10 MHz.
It has a "sweep output" which gives a voltage going up and down with the frequency sweep.

The "sweep output" is connected with the X input of the oscilloscope, the oscilloscope is placed in the X-Y mode.
Now the lightspot on the scope runs from left to right and back over the screen, this makes a frequency scale with on the leftside the startfrequency and on the rightside the stopfrequency of the sweep generator.
The sweepfrequency must be set at about 10 Hertz, this means the frequency is running 10 times per second from startfrequency to stopfrequency and back.

The Y input of the oscilloscope is connected via the 1:100 probe with the LC circuit.

The RF output of the sweep generator is connected to the coupling coil, which is placed about 20 cm from the coil of the LC circuit.

L is always positive. Hence, when the current increases, so does the potential drop across the coil, which means that the coil effectively opposes the increase in current. Likewise, when the current decreases, the potential difference across the coil is negative, which tends to increase the current and opposes its change. These effects are a manifestation of Lenz’s law, which states, in the words of Young and Freedman [1], that ‘the direction of any magnetic induction effect is such as to oppose the cause of the effect’.

The coil does not act like a resistor following Ohm’s law, though, since VL is proportional to the rate of change in current, not to the current itself. For instance, consider the LR circuit of Figure 1. We’ll denote the inductance of the inductor by L, the resistance of the resistor by R, and the frequency of oscillation of the emf produced by the function generator (or oscillator) by f. The values of L, R and f relevant in this practical are such that the total potential drop across the circuit is dominated by the drop across the resistor, which means that the current flowing through the circuit, I, is almost the same as if the inductor was not present. Let us thus assume that

$$I = I_0 \cos(\omega t) \tag{2}$$

with I0 a constant and Then, if VR is the voltage across the resistor, we have, by Ohm’s law

$$V_R = RI = RI_0 \cos(\omega t) \tag{3}$$

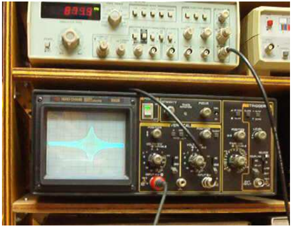
If the resistance of the wire forming the inductor is negligible, then the potential difference between its two ends, VL, is identical to the voltage induced by the change of current, Vind. In this case, from Equation (1), happens when the circuit enters resonance. By the end of the practical you will have seen how resonant circuits can be used to tune radios.

While the work will focus on measurements on a relatively simple circuit, it actually addresses the general issue of how “under-damped” oscillating systems behave when subjected to a periodic perturbation, which is an issue of wide significance in Physics and Engineering. The main features of the currents and voltages measured during the session – their free damped harmonic oscillation, the excitation of transients by a sudden perturbation, the occurrence of resonances under a periodic forcing, and the dependence of the height and width of the resonance peaks and of the accompanying phase changes on the Q-factor of the system – are in fact quite general. They are found in any physical system governed by the same type of differential equations as the LRC circuit considered here.

2. Preparation

A current passing through a coil produces a magnetic field – this is the principle of electromagnets and transformers. If this current changes as a function of time, then so does the magnetic field. As you will see in the Foundations course, the change in the magnetic field present inside the coil induces a potential difference Vind between the two ends of the wire forming the coil, and this potential difference is proportional to the rate of change in the current [1]. By definition, the inductance of the coil (or more generally, of an inductor) is the quantity L such that

$$V_{\text{ind}} = L\frac{dI}{dt} \tag{1}$$



At the top: the sweep signal generator.

Under: oscilloscope with the curve of the LC circuit on the screen.

We can turn the tuner capacitor and get the curve of the LC circuit on the oscilloscope screen.
Adjust with the amplitudeknob of the sweep generator the hight of the peak of the curve to 2.83 cm.
(The peak-peak distance is then: 2x2.83=5.66 cm).

Determine the width of the curve at 2 cm high, this is the -3 dB point (because 2.83x0.707= 2).

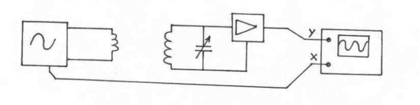
Calculate the bandwidth:
BW= (stopfrequency-startfrequency) x curve width at -3 dB / total screenwidth.

And the Q:
Q= f.res / BW

The great advantage of this method is that changes in resonance frequency of the LC circuit, can direct be seen on the screen.
Also changes in Q can direct be seen, because the hight of the peak will change then.

At high Q circuits, we can see the hight of the peak halve for instance, when we touch the (insulated) litzwire with our fingers.

Test setup 3 measuring the Q with a sweep generator and a amplifier.



In this schematic we see from left to right the following components:

A sweep signal generator
A coupling coil
The LC circuit
A amplifier
A oscilloscope

When using a 1:100 probe between LC circuit and oscilloscope there are two problems:

a- Because the 100 times attenuation of the signal, the amplitude on the oscilloscope will often have a very low level.
b- The probe can give dielectric losses, which reduces the Q.

To solve these two problems, I replaced the probe by a selfmade amplifier with a gain of 1x.
The amplitude on the oscilloscope will now be 100 times higher then with the 1:100 probe.

Question: Compare this last equation to Equation (18) of Section 2, and show, by identifying the constants between these two equations, (i) that, within a change of notation and of physical dimensions, the amplitude IL0 defined by Equation (21) is nothing else than the amplitude A defined by Equation (14.46) of Young and Freedman; and (ii) that the ratio b / km is the inverse of the Q-factor of the harmonic oscillator. (Hence, in Figure 14.28 of Young and Freedman, the amplitude A is more and more peaked at the resonance for higher and higher values of the Q-factor.)

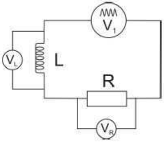


Figure 1: The LR circuit considered in Section 2.

1.What it's about

This session explores the behaviour of "LRC circuits", also known as "LCR circuits" and "RLC circuits", namely circuits containing resistors (R), inductors (L) and capacitors (C). It builds on the second Circuits Skills experiments, in which you looked at circuits containing resistors and capacitors and measured voltages using an oscilloscope. Here the objective of your measurements will be to find the inductance and internal resistance of an inductor and the Q-factor and frequency of oscillation of an LRC circuit. In the process, you will investigate the response of a driven LRC circuit to different frequencies, look at the effect of damping, and see what

Oscillations and Resonances in LRC Circuits
Durham University, UK
http://level1.physics.dur.ac.uk/projects/script/lcr.pdf

Preparatory Task:

Read the entire script and familiarise yourself with the objectives of the session. In particular, read through Section 2 in advance of the session, as the theory developed in this section is essential for understanding the Physics behind the results you'll obtain. You also need to write in your lab book a brief plan of what you will be doing, task by task, and answer the question below. This practical will involve measuring timedependent voltages with an oscilloscope, error analysis, plotting diagrams (with error bars), and least square fitting using the LINEST function of Excel. If you are not confident in having already acquired the necessary skills in these matters, make use of the online resources available at the url http://level1.physics.dur.ac.uk/general/index.php.

In Sections 14.7 and 14.8 of Young and Freedman [1], you'll find a discussion of the dynamics of a damped harmonic oscillator consisting of a mass m sliding on a table under the effect of a spring of force constant k, a frictional force proportional to the velocity of the mass, and an additional force varying with time sinusoidally. In the notation of Young and Freedman, the equation of motion for the position of the mass is

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx + F_{\max}\cos\omega_d t$$

This equation can also be written in the equivalent form

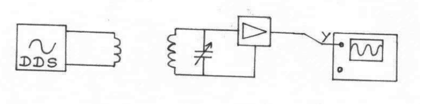
$$\frac{m}{k}\frac{d^2x}{dt^2} + \frac{b}{k}\frac{dx}{dt} + x = \frac{1}{k}F_{\max}\cos\omega_d t$$

The input of the amplifier uses a FET (Field Effect Transistor) and a capacitive voltage divider, which will load the circuit only very little.

A complete schematic of the amplifier you will find here

For the rest, this test setup is the same as test setup 2.

test setup 4 Measuring the Q with a DDS signal generator and a amplifier.



In this schematic you see from left to right:

A DDS signal generator
A coupling coil
The LC circuit
A amplifier
A oscilloscope

DDS means "Direct Digital Synthesis".

The output signal is in a DDS generator made in a digital way.
The great advantage of this kind of generator is the accuracy of the frequency setting.
The output has also a very low distortion.
The DDS generator I use, is a build yourself electronic kit from the company ELV .
You can also buy a complete build and tested module.

The specification are:

Output frequency: 0.1Hz -- 20 MHz.
Output voltage: 0 -- 4 Volt peak peak (not loaded).
Output impedance: 50 Ohm.
Minimum stepsize of frequency setting is 0.1 Hz. (up to 10 MHz output frequency).
Minimum stepsize of frequency setting is 1 Hz. (10 to 20 MHz output frequency).

As stepsize you can also select, e.g. 10 Hz, 100 Hz, 1 KHz, etc.

I now use the DDS generator because the used sweep generator could not be set accurate enough on frequency.
And also the frequency changes slightly during the measurement.

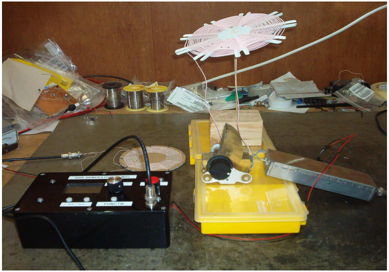


Photo of the test setup.
The coil laying on the table is the coupling coil.

You can get an accurate measure of the width by finding the frequencies just above and just below the resonance at which vC is reduced by $1/\sqrt{2}$ from the peak value. The width $\Delta\omega$ is then $2\pi\Delta f$, where the 2π converts from frequency in Hertz to angular frequency. Do your measured values of $\Delta\omega$ and τ satisfy Eq. 12?

The FET amplifier is connected via short wires to the tuner capacitor.
The coil of the LC circuit is placed at the top of a wooden stick, so it has not much influence from surrounding obstacles.

The windings are laying in a horizontal plane, so the coil picks up less signal from radiostations which can influence the measurement.

During measurements on high Q circuits, I tune the DDS generator in 10 Hz steps.
This test setup is in my opinion very reliable for determining the circuit Q.

Tips for measuring the Q:

During measurements don't come with your hands too close to the LC circuit, because this has influence on frequency and Q. Keep a minimum distance of 20 cm.

Don't lay the coil of the LC circuit during measurement on the table, but keep a minimum distance of 20 cm from wooden or metal objects.

Driven oscillations

To study the driven solution we need to provide a sine-wave signal source, as shown in Fig. 5. Our source is equivalent to an ideal sine-wave generator in series with a 50 resistor, labeled Rg in the diagram. Since Rg is large enough to affect the damping of the circuit, we add the 3.3 ! resistor in parallel to reduce the effective resistance of the generator. The other change from Fig. 4 is to replace the LabPro with a DMM, set to read AC voltage. (The DMM can read AC voltages between about 30 Hz and 1000 Hz. It is unreliable outside of that range.)

Connect the circuit as shown in Fig 5, and then vary the driving frequency to find the frequency at which vC is maximum. This identifies the resonance frequency, in Hertz. Next, you should plot vC as a function of frequency, taking care to get enough data around the resonance frequency to clearly define the curve. This goes very quickly if you enter f and vC directly into

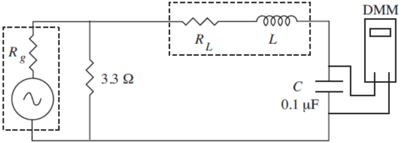


Fig. 5 Circuit for measuring driven oscillations in an RLC circuit.

Graph.cmbl and then pick data points to trace the regions of interest. Does your plot look like Fig.3?

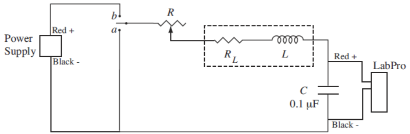


Fig. 4 Circuit for measuring free oscillations in an RLC circuit.

a will start the discharge, and should result in a plot resembling Fig. 2. There may be some irregularity at the beginning, due to bouncing when the switch first makes contact, but you can ignore that in your analysis.

When you have a reasonable looking plot, select the portion of the data after any effects of switch bounce, and fit it to a damped sine wave. You should be able to obtain estimates of the time constant and the angular frequency for your circuit. (Delete the fit or restart RLC.cmbl before taking more data. Otherwise you will have to wait while the program tries to fit the new data.)

Now increase the variable resistor R slightly and obtain another voltage vs time plot. Describe what happens to the time constant and to the shape of the curve as R becomes progressively larger.

Critical damping occurs when R is just big enough that the voltage no longer crosses zero during the decay. Try to find this setting, and measure the value of the variable resistor using a DMM. The exact value is not very clear, but make a reasonable attempt. Reminder: You must disconnect R from the circuit to get an accurate measurement with the DMM.

Q Factor Measurements on L-C Circuits
Jacques Audet, VE2AZX
http://www.arrl.org/files/file/QEX_Next_Issue/Jan-Feb_2012/QEX_1_12_Audet.pdf

The author reviews existing measurement techniques and offers insight into loaded and unloaded Q factors as applied to LC circuits and antennas. A simpler method is proposed that uses an SWR analyzer along with a spreadsheet that easily computes the unloaded Q.

Introduction

The Q factor gives a figure of merit for inductors and capacitors. It is the ratio of reactance to resistance. For filters, it relates directly to the circuit selectivity: The higher the Q, the better the selectivity and the lower the insertion loss of the filter. For oscillators, higher Q also means that lower phase noise is produced. In the case of antennas, a lower Q is generally preferred, giving a larger SWR bandwidth.

Transmission methods are traditionally used for making quality factor (Q) measurements on L-C circuits. This implies that a signal source and an RF voltmeter or spectrum analyzer must be available for such measurements. These are not always available however. Since SWR analyzers are becoming commonplace in many amateur radio rooms, it then becomes tempting to use this instrument for Q measurements on L-C circuits.

Let’s review the existing methods that are currently used for Q measurements that only require scalar measurements — that is, no phase measurements are required. The last method details how the SWR analyzer can be used to measure the unloaded Q of L-C circuits.

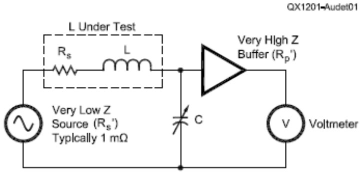


Figure 1 — Block diagram of the classical Q meter as used in the HP 4342A.

1 – The Classical Q Meter Method

This is the technique you would use if you had access to a Q Meter, such as an HP / Agilent Q Meter model HP 4342A. See Figure 1. A very low impedance source is required, typically 1 milliohm, and a very high impedance detector is connected across the L-C circuit. The unloaded Q (QU) of a single reactance component is given by Equation 1.

$$Q_U = \frac{X_S}{R_S} = \frac{R_P}{X_P} \quad \text{[Eq 1]}$$

where XS and RS are the series reactance and loss resistance, and RP and XP are the corresponding parallel loss resistance and reactance components.

In the test set-up we need to make the source resistance RS′ as small as possible, since it adds to the coil or capacitor resistance. Similarly, the detector resistance, RP′ shunting

demonstrating the intimate relation between time and frequency response parameters. In obtaining this result, we assumed , so that at the peak.

EXPERIMENTAL PROCEDURE

The RLC circuit is assembled from a large solenoid, a capacitor on the circuit board, and an additional variable resistance to change the damping. The circuit can be charged up with a DC power supply to study the free oscillations, or driven with a sine wave source for forced oscillations.

Free oscillations

To study the homogeneous solution we will use LoggerPro to record the voltage across the capacitor as a function of time. The required circuit is shown in Fig. 4. Since the coil is not made with superconducting wire, we account for the wire resistance with RL. The variable resistor, shown by a resistor symbol with an arrow in the middle, should be set to minimum resistance initially. (You can check with the DMM in ohmmeter mode, before connecting R into the circuit.) The power supply can be set to maximum output.

Start LoggerPro with the file RLC.cmbl. This will configure the program to collect data at 10,000 samples per second, triggered when the voltage decreases across 9.5 V. Set the switch to position b to charge the capacitor, and then start data acquisition. Flipping the switch to position

At the maximum, the oscillation amplitude is considerably greater than the driving amplitude. In fact, if τ were infinite (no damping), the response would be infinite at ω_0 .

Since the shape of the peak in vC characterizes the resonance, it is convenient to have some parameter to specify the sharpness of the peak. Traditionally, this is taken to be the full width $\Delta\omega$, shown in Fig. 3, at which the voltage or current have fallen to $1/\sqrt{2}$ of their peak value.

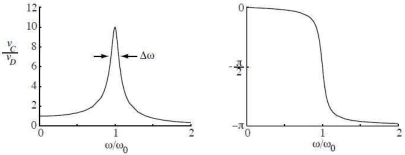


Fig. 3 Amplitude and phase of capacitor voltage as a function of frequency.

The reason for this choice is that the power dissipation is proportional to i^2, so these frequencies correspond to the points at which the power dissipation is half of the maximum. Using Eq. 9 we find that the width is related to τ by

$$\Delta\omega = \frac{2}{\tau} \quad (12)$$

the L-C should be much higher than the RP of the component under test. Note that the L-C circuit is commonly represented as having a resistor in series (RS) with L and C or a shunt resistor (RP) in the parallel model to represent the losses.

Measurement consists of setting the source frequency and adjusting the tuning capacitor, C, for resonance to maximize the voltage across the resonating capacitor C. The Q is calculated using Equation 2.

$$Q = \frac{\text{Voltage Across } C}{\text{Source Voltage}} \quad \text{[Eq 2]}$$

Note that the source voltage is in the millivolt range, since it will be multiplied by the Q factor. A 10 mV source and a Q of 500 will give 5 V across the L-C circuit. In order to preserve the high impedance of the detector even at the higher frequencies, a capacitive voltage divider is used.

This circuit measures the unloaded Q called QU provided that the series resistance RS of the inductor under test is much higher than the source resistance RS′ and the detector parallel resistance RP′ is much larger than the L-C circuit RP.1

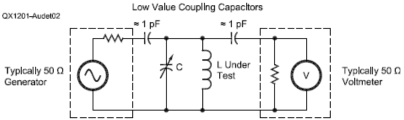


Figure 2 — The signal generator and the source are weakly coupled to the L-C circuit under test, allowing measurement of the 3 dB bandwidth.

2 – Transmission Method Using Coupling Capacitors

A signal is coupled into an L-C parallel circuit using a low value capacitor and extracts the output signal using the same value of capacitor.2 See Figure 2. Note that inductive coupling is also possible, as used in transmission line (cavity) resonators. The – 3 dB bandwidth (BW) is measured at the resonant frequency f0, and the loaded Q (QL) is calculated with Equation 3.

$$Q_L = \frac{f_0}{BW} \quad \text{[Eq 3]}$$

The obtained bandwidth, BW, is a function of the coupling, and another calculation is required to get QU, the unloaded Q. When the input to output coupling is identical, we use Equation 4.

$$Loss = 20\log\left(\frac{Q_U}{Q_U - Q_L}\right) \quad \text{[Eq 4]}$$

where Loss is in positive dB. The ratio QU / QL may now be calculated using Equation 5.

$$\frac{Q_U}{Q_L} = \left(\frac{10^{\frac{Loss}{20}}}{10^{\frac{Loss}{20}} - 1}\right) \quad \text{[Eq 5]}$$

1/# is referred to as "critical damping" because the charge reaches zero in the shortest time without changing polarity.

The solution for sine-wave driving describes a steady oscillation at the frequency of the driving voltage:

$$q_C = A\sin(\omega t + \phi) \quad (8)$$

We can find A and ϕ by substituting into the differential equation and solving:

$$A = \frac{v_D / L}{\left[\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega / \tau\right)^2\right]^{1/2}} \quad (9)$$

$$\tan\phi = \frac{2\omega / \tau}{\left(\omega^2 - \omega_0^2\right)} \quad (10)$$

These two equations are plotted in Fig. 3, where we use the fact that $v_C = q_C/C$ to plot the voltage across the capacitor relative to the driving voltage. (Our apparatus does not allow us to observe the phase of the response, so we won't consider that further.) The angular frequency for maximum amplitude is given by

$$\omega_{peak} = \omega_0\left[1 - 2/(\omega_0\tau)^2\right]^{1/2} \quad (11)$$

Where qC0 and ! are determined by initial conditions, and

$$\omega_1 = \omega_0\left[1 - (\omega_0 \tau)^{-2}\right]^{1/2} \quad (5)$$

This solution is plotted in Fig. 2 for a case where the capacitor is initially charged and no current is flowing. (For a mass on a spring the equivalent situation would be to pull the mass aside and release it from rest.) Evidently there are oscillations at ω_1, approximately equal to ω_0, within an exponential envelope. Note that the amplitude falls to 1/e of the initial value when $t = \tau$.

As $\omega_0 < 1/\tau$ gets smaller (larger resistance R), ω_1 becomes smaller and finally imaginary. The corresponding solutions do not oscillate at all. For $\omega_0 < 1/\tau$, there are two exponentials

$$q_C = A_1 e^{-t/\tau_1} + A_2 e^{-t/\tau_2} \quad (6)$$

where τ_1 and τ_2 differ somewhat from τ. When $\omega_0 = 1/\tau$, the solution is slightly simpler:

$$q_C = \left(A_1 + A_2 t\right) e^{-t/\tau} \quad (7)$$

If the capacitor is initially charged, these results tell us we will get a monotonic decay for sufficiently large R. The case "0 =

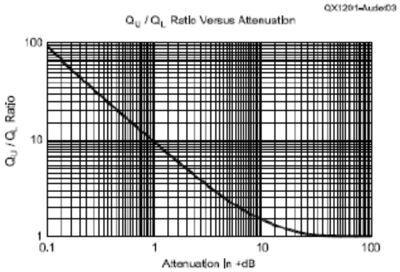


Figure 3 — Graph showing the ratio of unloaded Q (QU) to loaded Q (QL) as a function of the L-C circuit attenuation at resonance. Applies to a single tuned circuit with identical in / out coupling. When the attenuation is large (> 20 dB) the coupling is minimal and the loaded Q closely approaches the unloaded Q.

This equation is plotted in Figure 3. Note that this method applies well to transmission line resonators with equal impedance inductive coupling loops at the input / output. Note also that these two methods do not require knowledge of the L or C values to compute the Q factors.

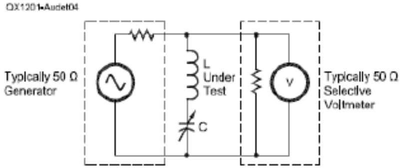


Figure 4 — Here the L-C circuit is connected in series and across the generator - detector. At resonance the L-C circuit has minimum impedance and by measuring the attenuation, the effective series resistance ESR of the L-C series may be computed at the resonant frequency f0.

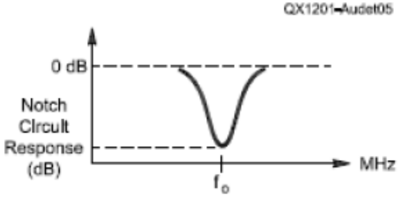


Figure 5 — Frequency response of the notch circuit of Figure 4, where f0 is the frequency of maximum attenuation.

3 – Shunt Mode Transmission Method With L-C in Series

with $\tau = 2L/R$ and $\omega_0^2 = 1/LC$. Two situations will be of interest. We will examine the free oscillations, when vD is exactly zero, meaning that the sine-wave generator has been replaced with a short circuit. We will then connect the sine wave generator and calculate the response as a function of frequency.

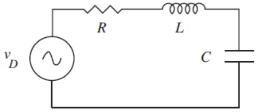


Fig. 1 Idealized series RLC circuit driven by a sine-wave voltage source.

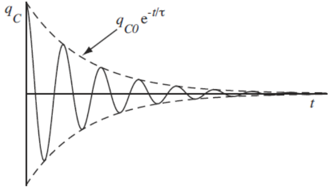


Fig. 2 Damped oscillation, showing decay envelope

If the resistance in the circuit is small, the free oscillations are of the form

$$q_C = q_{C0} e^{-t/\tau} \cos(\omega_1 t + \phi) \quad (4)$$

RLC Circuits
Rice University
http://www.owlnet.rice.edu/~phys102/Lab/RLC_circuits.pdf

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.
Richard Feynman (1918-1988)

OBJECTIVES
To observe free and driven oscillations of an RLC circuit.

THEORY
The circuit of interest is shown in Fig. 1, including sine-wave sources. We start with the series connection, writing Kirchoff's law for the loop in terms of the charge qC on the capacitor and the current $i = dq_C/dt$ in the loop. The sum of the voltages around the loop must be zero, so we obtain

$$v_L + v_R + v_C = v_D \sin(\omega t) \quad (1)$$

$$L\frac{d^2q_C}{dt^2} + R\frac{dq_C}{dt} + \frac{q_C}{C} = v_D \sin(\omega t) \quad (2)$$

For reasons that will become clear shortly, we rewrite this as

$$\frac{d^2q_C}{dt^2} + \frac{2}{\tau}\frac{dq_C}{dt} + \omega_0^2 q_C = \frac{v_D}{L}\sin(\omega t) \quad (3)$$

Here the L and C are series connected and placed in shunt across the transmission circuit. See Figure 4. (This method is also detailed in the reference of Note 2.) It basically computes the effective series resistance (ESR) of the combined L and C, based on the attenuation in dB at resonance, using a selective voltmeter to prevent source harmonics from affecting the measurement. The ESR may be calculated using Equation 6.

$$ESR = \frac{Z}{2\left(10^{\frac{dB}{20}} - 1\right)} \quad \text{[Eq 6]}$$

where dB is a positive value of attenuation and Z is the source and detector impedance. Once the ESR is known, it is only necessary to compute:

$$Q_U = \frac{X_L}{ESR} \quad \text{[Eq 7]}$$

where XL = 2π f0L, and L has already been measured separately.

Note 3 gives a reference to a spreadsheet that I have developed to eliminate the need to measure L and C. Two more attenuation points are required to compute these values. There is also a version for crystals, which computes the equivalent R L C values and other related parameters.

4 – Reflection Measurement Using an SWR Analyzer

This method has been recently developed to make use of an SWR analyzer, thus eliminating the need for the source detector combination. Adjust the SWR analyzer to the resonant frequency of the circuit. The L-C circuit may be coupled to a link coil on the SWR analyzer, which provides variable coupling. As shown in Figure 7, the amount of coupling is adjusted until the SWR drops to 1:1. The frequency is recorded as f0. Then the frequency is offset above or below f0 to obtain an SWR reading between 2 and 5. Now plug the new frequency and SWR values in the spreadsheet that I provide to calculate the unloaded Q factor. Note that the L or C values are not required to compute the Q factor.

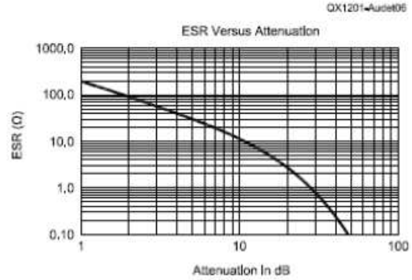


Figure 6 — Effective series resistance as computed from the attenuation in dB, in a 50 Ω system.

When $R = 0 \Rightarrow R_{total} = R_p \Rightarrow R_p = \beta_{exp} \cdot 2L$

3. Calculate experimental angular frequency of oscillations

$$\omega_R = \frac{2\pi}{T}$$

4. For the resonance conditions in RLC circuit, calculate capacitance C for each inductance L using the equations:

5. Calculate the uncertainty of the capacitance _C

6. For the case of critical damping, compare the experimental value of critical resistance RC with its theoretical value calculated on the basis of the equation:

$$R_{Ctheory} = 2\sqrt{\frac{L}{C}}$$

7. Write the conclusions.

Literature:

1. Halliday, Resnick “Fundamentals of Physics - 8th edition”, John Wiley 2007

2. Zięba “Pracownia Fizyczna Wydziału Fizyki I Techniki Jądrowej AGH”, Uczelniane Wydawnictwo Naukowo-Dydaktyczne 1999.

Updated: 11.02.2011 by Barbara Dziurdzia resistor and repeat the observations. Investigate how the increase in resistance R affects the oscillation parameters.

6. Increase R until the critical aperiodic oscillations occur and write down the Rcritical in the measurement table.

7. Change the inductance to L2 at the decade coil and for resistances R0=0, R1, R2 repeat all measurements according to 2-6. Investigate how the increase in inductance L affects the oscillation parameters.

8. Write down the results in Table 1.

6. Data Handling

1. Calculate the log decrements $\delta = \ln\left(\frac{U_i}{U_{i+2}}\right)$ and the experimental damping coefficient

β exp of oscillations: $\beta_{exp} = \frac{\ln \frac{U_i}{U_{i+2}}}{T}$ for all combinations (L1, R0), (L1, R1), (L1, R2), (L2, R0), (L2, R1), (L2, R2)

2. Calculate the parasitic resistance of a coil Rp taking advantage from the fact that at the beginning there is always set R=0 at the resistance decade. the total resistance Rtotal in the RLC circuit consists of the resistance of a decade R and the parasitic resistance of a coil R_p: $R_{total} = R + R_p$.

$\beta_{exp} = \frac{R_{total}}{2L}, \quad R_{total} = \beta_{exp} \cdot 2L$

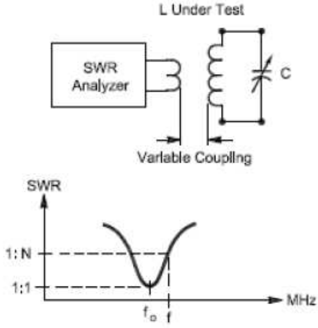


Figure 7 — Set-up for measuring the unloaded Q of L-C resonant circuit. The coupling is adjusted for 1:1 SWR at resonance. The link should have approximately 1 turn per 5 or 10 turns of the inductor under test.

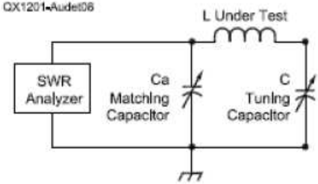


Figure 8 — Here the variable coupling to the SWR analyzer is provided by the capacitive divider formed by Ca and C. Ca is typically 10 to 50 times larger than C.

As shown in Figure 8 the coupling may also be realized with a second variable capacitor, Ca, which serves as an impedance divider. Both capacitors have their frame grounded, along with the SWR analyzer. This makes the construction and operation of the SWR - Q meter much easier.

Both circuits transform the L-C circuit effective parallel resistance RP to 50 + j 0 ohm as required by the analyzer to obtain a 1:1 SWR at resonance. The capacitance value of Ca will be in the order of 10 to 50 times the value of C, the main tuning capacitor. A slight interaction will be present between these two adjustments when adjusting for 1:1 SWR.

Note that a variable inductive divider can be used, in series with the inductor under test, instead of the capacitive divider of Figure 8. In this case the SWR analyzer is connected across this variable inductor, which allows matching to the SWR analyzer 50 ohm input. I do not recommend this as the Q of the variable inductor is probably not going to be as high as the matching capacitor, Ca. The Q of these matching components have a second order effect on the measured QU.

At a Q factor of 300, a 1% error in SWR gives a 1% error in Q. Assume that the measured SWR is 4 and the SWR meter has a resolution of 0.1. If the error in SWR is 0.1, then the error in Q is 2.5%. Other errors include the Q of the tuning capacitor, and the Q of the matching component, Ca, or the link coil to a lesser degree.

2. Set the inductance at the decade coil to the certain value L1 . Set the resistance at the decade resistor to R0=0. Observe the damping oscillations on the oscilloscope selecting the proper time base setting and vertical gain.

3. Measure on the oscilloscope display the period of oscillations T (Fig. 8). To increase accuracy, measure periods of a few successive oscillations and calculate the average value which will be further used to get the experimental angular frequency ω exp of oscillations:

$$\omega_{exp} = \frac{2\pi}{T}$$

4. Measure on the oscilloscope display the amplitude of maxima U2, U4, (Fig. 8)

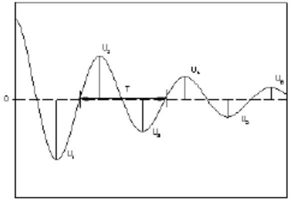


Fig. 8.. Damping oscillations

5. Keep the inductance L1 at the decade coil unchanged and set the another resistance R1 at the decade resistor. Observe the damping oscillations for the settings (L1, R1) and repeat the measurements 3-4. Set the resistance R2 at the decade

connected in series . The oscilloscope monitors the potential difference uC(t) across the capacitor as a function of time.

5. Measurements

1. Set the RLC circuit as shown in Fig. 6 . Fig. 7 shows the decade coil, the decade resistor and the capacitor with its charging system.

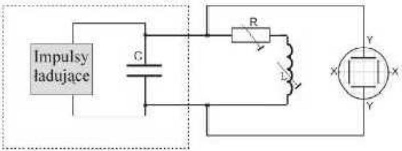


Fig. 6. The measurement system


Fig. 7. The decade coil, the decade resistor and the capacitor with its charging system.

Here are some simulation results:

At Q = 200 the average error on the calculated Q is – 0.5% when the test frequency is below resonance. Above resonance, the average error drops to – 0.05%.

At Q = 50 the average error on the calculated Q is –1.0% when the test frequency is below resonance. Above resonance, the average error drops to –0.7%.

These small errors come from the reactance of the coupling element (link coil or Ca in the capacitive divider).

The following errors were obtained by simulation with an inductor Q of 200.

The matching capacitor Ca should have a Q of 500 or larger. A Q value of 500 for Ca gives – 6% error and a Q value of 1000 for Ca gives – 0.6% error. Interestingly the link coil Q may be as low as 50 and the error is only – 0.4%. The losses of the transmission line between the SWR analyzer and the Q measuring circuit should be kept to a minimum. An attenuation of 0.05 dB gives –1.6% error on the calculated Q.

So far the Q factor measurement included both the combined Qs of the L and C, since it is difficult to separate their individual Q values. When the capacitor Q (Qcap) is known the inductor Q (Qind) may be calculated like parallel resistors:

$$\frac{1}{Q_{ind}} = \frac{1}{Q} - \frac{1}{Q_{cap}} \quad \text{[Eq 8]}$$

Note that the combined Q of the L-C circuit is 17% lower with Qind = 200 and Qcap = 1000, compared to having a capacitor with an infinite Q.

Measuring the Q of Low Value Capacitors (< 100 pF) Using the Delta Q Method

Two Q measurements are required:

First measure the Q of a test inductor (preferably having a stable high Q) and record it as Q1 and record the amount of capacitance C1 used to resonate it.

Second, connect the low value capacitor to be tested across the tuning capacitor, decrease its capacitance to obtain resonance again. Note the measured Q as Q2 and the amount of capacitance C2 used to resonate. (Q2 should be less than Q1). Compute the Q of this capacitor as follows:

$$Q = \frac{(C_1 - C_2) \times Q_1 \times Q_2}{C_1 \times (Q_1 - Q_2)} \quad \text{[Eq 9]}$$

Note that (C1 – C2) is the capacitance of interest. It may be measured separately using a C meter. If Q2 = Q1 the Q of the capacitor becomes infinite.

Unloaded and Loaded Q

Looking back at method 1, the measured Q is the loaded Q and it approaches the unloaded Q as the source impedance goes toward zero and the detector impedance is infinite.

With method 2, the measured Q is the loaded Q and a correction is required based on insertion loss to obtain the

For damped oscillations, the log decrement is equal to:

$$\delta = \ln\left(\frac{V_1}{V_2}\right) = \frac{R}{2L}T$$

Critical damping in an RLC circuit is achieved when:

$$\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 = 0,$$

from which we obtain:

$$R_{critical} = 2\sqrt{\frac{L}{C}}$$

We vary the resistance and search for a signal that has no undershoot and has a maximum decay rate. Typical oscilloscope traces for the overdamped, underdamped and critical damping are shown in Fig. 5.

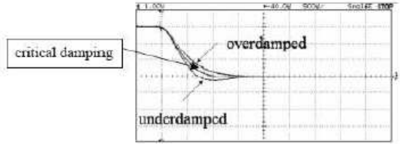


Fig. 5. Typical oscilloscope traces for critical damping critical damping

4. Equipment

RLC circuit consists of a capacitor C combined with its charging system, a decade resistor and a decade coil – all

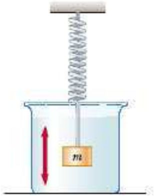


Fig. 3. A mechanical analogous to the RLC circuit – a block–spring system moving in a viscous medium.

The log decrement is determined from the ratio of the voltages of successive maxima of the damped oscillatory signal. This measurement is taken from the oscilloscope display, as shown in the Fig. 4. The figure shows the damped signal for two different values of resistance for fixed value of capacitance. The arrows indicate the first and second maxima.

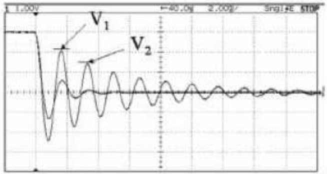


Fig. 4. Typical oscilloscope trace showing log decrement measurement

unloaded Q. In this case the L-C circuit under test "sees" the same Q as we have measured, (if it could look at its environment by looking towards the source and detector).

Such is not the case in methods 3 and 4. In method 3 the L-C circuit will typically "see" a 50 ohm source and a 50 ohm detector that are effectively in parallel. This adds 25 ohm in series with the ESR of the L-C circuit, and reduces the Q that it sees. This loaded Q sets the selectivity of the notch filter created by the circuit.

In method 4, the L-C "sees" the SWR analyzer internal impedance, say 50 ohm. Remember that the Rp value is transformed to 50 ohm at the analyzer, and effectively is in parallel with the analyzer's internal impedance. This means that the actual Q factor seen by the L-C circuit is reduced by half. This value is the loaded Q of the L-C circuit. In this case the coupling factor equals 1.5

Now consider a 50 ohm dipole antenna. The series R-L-C model may be applied to a resonant dipole antenna, which presents approximately 50 ohm at the feed point. When the dipole is fed by a low loss transmission line the SWR meter connected at the transmitter will measure the antenna unloaded Q at an SWR = 2.62 as calculated from equation 3: Q = f0 / BW. Note that the spreadsheet given in Note 4 will calculate the dipole Q factor under these conditions.

When the antenna is fed by a 50 ohm source, its effective bandwidth will double, since the total resistance seen by the dipole is now the radiation resistance plus the transmitter output resistance.

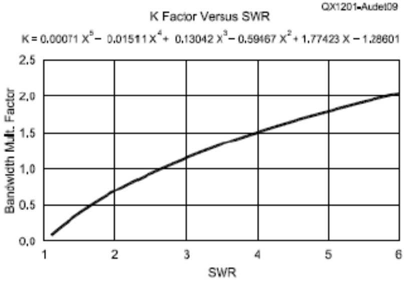


Figure 9 — This graph shows the K factor versus SWR. At SWR = 2.62 the correction factor K equals 1 while at SWR = 5.83 the K factor is 2.

Here we still have the loaded Q = unloaded Q / 2. This assumes that the transmitter output impedance and the feed line are 50 ohm. The Q derived from the bandwidth at SWR points of 2.62 gives the unloaded Q value, independently of the transmitter output impedance.

In the general case, the complex impedance of the transmitter reflected at the antenna will modify its effective bandwidth and its actual resonant frequency. Note that the SWR meter connected at the transmitter will not show this effect since it can only measure the unloaded Q of the dipole. Also, the bandwidth of the matching circuits at the transmitter will affect the effective impedance seen by the antenna and modify its loaded Q and bandwidth.

is a circuit damping constant

The equation **** describes a sinusoidal oscillation with an exponentially decaying amplitude $Qe^{-\frac{Rt}{2L}}$.

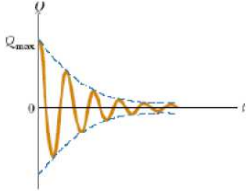


Fig. 2 Charge versus time for the damped RLC circuit.

The RLC circuit is analogous to the damped harmonic oscillator illustrated in Fig. 3 and described by the equation:

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

Q corresponds to the position x of the block at any instant, L to the mass m of the block, R to the damping coefficient b, and C to 1/k, where k is the force constant of the spring.

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2 R$$

$$i = \frac{dq}{dt} \text{ and } \frac{di}{dt} = \frac{d^2 q}{dt^2}$$

$$L\frac{d^2 q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

$$\frac{d^2 q}{dt^2} + 2\frac{R}{2L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

$$\frac{R}{2L} = \beta, \qquad \frac{1}{LC} = \omega_0^2$$

$$\frac{d^2 q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = 0$$

***

The solution to this differential equation for damped oscillations in an RLC circuit is:

$$q = Qe^{-\frac{Rt}{2L}}\cos(\omega t + \varphi)$$

****

where:

$$\omega = \sqrt{\omega_0{}^2 - \left(\frac{R}{2L}\right)^2}$$

is the angular frequency of the damped oscillations,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

is the angular frequency of the undamped oscillations.

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\beta = \frac{R}{2L}$$

One way to measure the effective bandwidth of the dipole might be to insert an RF ammeter in series with one dipole leg. Find the frequencies where the current is reduced by ~ 30% and compute the effective bandwidth this way. In the receive mode the dipole antenna will also exhibit the same loaded Q = unloaded Q / 2 if the receiver impedance is 50 ohm. Your antenna effective bandwidth may be twice as large as you really thought!

Summary of the Four Methods Presented in this Article

1 – Classical Q Meter. See Figure 1. This shows a set-up as used by the HP / Agilent model HP 4342A.

This technique uses a transmission method that requires a very low impedance source and very high impedance detector, which are not easy to realize.

Set the frequency and adjust the reference capacitor for resonance.

The measurement approximates the unloaded Q. Corrections are difficult to apply.

You don't have to know the L-C values.

2 – Transmission Method Using Coupling. See Figure 2. This technique requires a source and a voltmeter.

Find the –3 dB transmission bandwidth points. The measurement requires two low value, high Q coupling capacitors.

It measures the loaded Q as used in the test circuit.

You need to correct the loaded Q value for attenuation to find the unloaded Q, if the attenuation is less than 30 dB.

The accuracy of the Q measurement remains somewhat dependant upon the Q factor of the coupling capacitors. Refer to Figure 2. With 1 pF coupling capacitors having a Q of 1000, the error on Q was – 1.4%.

The L or C values are not required.

3 – Shunt mode transmission method with LC connected in series. See Figure 4.

This method requires a source and a selective voltmeter to prevent source harmonics from affecting the measurement. Compute the ESR from the minimum attenuation measured at resonance.

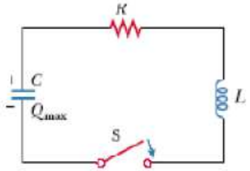
The L or C values are required to compute the Q value.

The author’s spreadsheet computes the series/parallel R-L-C values from 3 attenuation measurements. This technique is also useful for crystal measurements.

This is potentially the most accurate method, if the attenuation is measured with high accuracy using a vector network analyzer (VNA).

4 – Reflection measurement using an SWR analyzer. See Figures 7 and 8.

oscillations of charge, current and potential difference continuously decrease in amplitude, and the oscillations are said to be damped.



Let's write the equation for the total electromagnetic energy U in the circuit at any instant. As we know the resistance doesn't store electromagnetic energy, so we have:

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C} \quad *$$

This total energy decreases because energy is transferred to thermal energy. The rate of this
transfer is:

$$\frac{dU}{dt} = -i^2 R \quad **$$

where the minus sign indicates that U decreases.
By differentiating * with respect to time and then substituting the results in ** we obtain:

Damped oscillations in RLC circuits
by Barbara Dziurdzia
AGH University of Science and Technology in Cracow
http://home.agh.edu.pl/~lyson/downloads/Manual_8.pdf

1. Goal

To observe damped oscillations in the RLC circuit and measure the amplitude, period, angular frequency, damping constant and log decrement of damped oscillatory signals. To find the critical resistance for which the critical damping occurs.

2. What to learn?

Transfer of energy in LC circuit. The electrical-mechanical analogy. Differential equation describing damped simple harmonic motion in the RLC circuit.. Solution of this equation. Angular frequency of the damped oscillator. Damping constant. Angular frequency of the undamped oscillator. Forced oscillations and resonance. Kirchhoff's rules. Log decrement of damped oscillatory signals. Critical damping. How does the oscilloscope work?

3. Background

A circuit consisting of an inductor, a capacitor, and a resistor connected in series (Fig. 1) is called an RLC circuit. The resistance of the resistor R represents all of the resistance in the circuit. .With the resistance R present, the total electromagnetic energy U of the circuit (the sum of the electrical energy and magnetic energy) decreases with time because some portion of this energy is transferred to thermal energy in the resistance. Because of this loss of energy, the

This technique requires a matching capacitor or variable link coupling. There will be a slight interaction between the matching components and the main tuning capacitor.

Adjust two variable capacitors or vary link coupling to obtain an SWR of approximately 1:1

Toroidal inductors are difficult to test with variable link coupling. Use the capacitive divider method instead. With link coupling, keep the link close to the grounded end of the coil, to minimize capacitive coupling. In general air wound coils are more delicate to test since they are sensitive to their environment.

The L and C values are not required.

Offset the frequency to have an SWR increase. Note the frequency and SWR.

The author’s spreadsheet computes the unloaded Q factor directly.

This technique requires the least equipment of any of the methods.

Best accuracy is obtained with a link coil or a variable capacitive divider.

While all four methods do not require phase or complex impedance measurements, method 4 only requires an SWR analyzer. The accuracy of the SWR analyzer method has been validated from RF Simulations and by actual measurements.

Since this method relies on SWR measurements, the accuracy of the measuring instrument should be verified. The SWR = 2 point may easily verified by placing two accurate (≤ 1%) 50 ohm resistors in parallel at the analyzer output using a tee connector. Note the reading obtained at the frequency of interest and compute the SWR offset from the ideal value of 2.00. Then perform the Q measurements around this same SWR value, possibly above and below the resonant frequency and average the Q results.

To compensate for inaccuracies in determining the frequency (f0) of 1:1 SWR, it is recommended that you perform the Q measurement at two frequencies above and below the f0, as done in the spreadsheet provided in Note 4.

Note 5 covers other methods of measuring Q using a vector network analyzer, mostly suitable for use at microwave frequencies.

Jacques Audet, VE2AZX, became interested in radio at the age of 14, after playing with crystal radio sets and repairing old receivers. At 17, he obtained his first ham license, and in 1967 he obtained his B Sc degree in electrical engineering from Laval University. He then worked in engineering functions at Nortel Networks, where he retired in 2000. He worked mostly in test engineering on a number of products and components operating from dc to light-wave frequencies.

His areas of interest are in RF simulations, filters, duplexers, antennas and using computers to develop new test techniques in measurement and data processing.

Notes

Let us summarize what we have learned about parallel and series RLC circuits:

PROPERTY	SERIES RLC	PARALLEL RLC
Resonant Frequency	$f_R = \frac{1}{2\pi\sqrt{LC}}$	$f_R = \frac{1}{2\pi\sqrt{LC}}$
Voltage Across R	maximum at f_R	constant = V_0
Current Through R	constant = V_0/R	minimum at f_R
Q	$Q = \frac{2\pi f_R L}{R}$	$Q = \frac{R}{2\pi f_R L}$
Bandwidth	$BW = \frac{f_R}{Q}$	$BW = \frac{f_R}{Q}$
Impedance below Resonance	Capacitive	Inductive
Impedance above Resonance	Inductive	Capacitive
Effect of changing R	increasing R increases BW	increasing R decreases BW
Effect of changing L/C	increasing L/C decreases BW	increasing L/C increases BW

frequency at which the current increases to 141.4% of the minimum. The 141.4% current points were chosen because they correspond to a doubling of the power.

BW = f_U - f_L

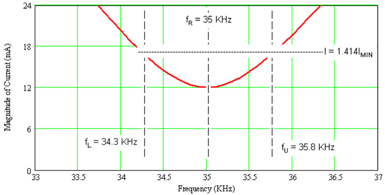
Q is defined as:

$$Q = \frac{R}{2\pi f_R L}$$

Bandwidth and Q are also related as follows:

$$BW = \frac{f_R}{Q}$$

The graph below illustrates the definition of BW.



The bandwidth is 35.8 - 34.3 = 1.5 KHz. Now we can determine Q:

$$Q = \frac{f_R}{BW} = \frac{35KHz}{1.5KHz} = 23.3$$

As a check, we can use our other definition to determine Q:

$$Q = \frac{R}{2\pi f_R L} = \frac{1000}{42.89} = 23.3$$

As one would expect, both results agree.

1Hewlett Packard Journal, September 1970, www.hpl.hp.com/hpjournal/pdfs/IssuePDFs/1970-09.pdf

2Wes Hayward W7ZOI “Two Faces of Q,” w7zoi.net/2faces/twofaces.html

3There is an Excel spreadsheet to perform the various Q calculations on my website: ve2azx.net/technical/Calc_Series-Par_RLC.xls for RLC circuits.

4The MathCad™ and the corresponding Adobe PDF file as well as the Excel file may be downloaded from the ARRL website at www.arrl.org/qexfiles/. Look for the file 1x12_Audet.zip.

5 Darko Kajfez “Q Factor Measurements, Analog and Digital,” www.ee.olemiss.edu/darko/rfqmeas2b.pdf

Appendix 1

In the SWR analyzer method, the equivalent parallel resistance RP of the L-C circuit is transformed to 50 ohm to give 1:1 SWR at resonance by using an adjustable link coupling or a variable capacitive divider. As the frequency is varied around resonance, it may be set at two frequencies where the absolute value of the reactance, X, is equal to RP.

Abs(X) = RP

Normalizing these impedances gives impedances of 1 for RP and j for X. Since these are in parallel, the resulting impedance, Z, will be:

$$Z = \frac{1 \times j}{1 + j} = 0.5 + 0.5j$$ [Eq A-1]

Taking the absolute value of Z we get Z = 0.707. This is the –3 dB point, since this value is 3 dB below the initial value of 1. The complex reflection coefficient is:

$$\rho = \frac{(0.5+0.5j)-1}{(0.5+0.5j)+1} = -0.2+0.4j$$ [Eq A-2]

The absolute value of the reflection coefficient $|\rho|$ is 0.447. The return loss RL in dB is given by:

RL = –20 log $|\rho|$ = 6.99 [Eq A-3]

The corresponding SWR is given as:

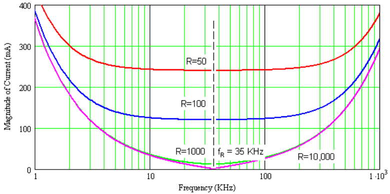
$$SWR = \frac{1+\rho}{1-\rho} = 2.62$$ [Eq A-4]

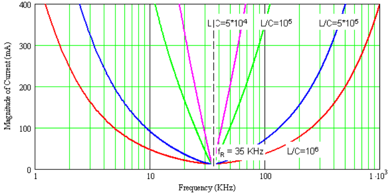
At the – 3 dB points, the bandwidth (BW) is related to the unloaded Q factor as follows:

$$Q_U = \frac{f_0}{BW}$$

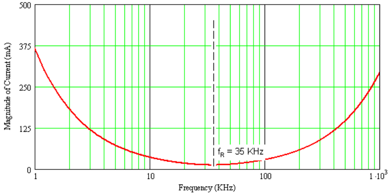
Therefore, measuring the bandwidth, BW, at SWR = 2.62 allows us to compute the unloaded Q (QU) of the circuit. So far, we need three measurement points. One point at frequency f0 that gives 1:1 SWR and two points at fL and fH below and

The shape of the band-stop response of a parallel RLC circuit depends on the value of R and L/C as shown in the next graphs.

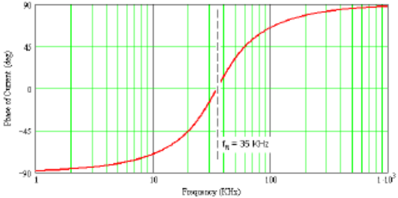




Note that the band-stop characteristic becomes narrower as the value of R increases. This the opposite of what we saw in the series RLC circuit. The response also becomes narrower as the L/C ratio decreases, which again is the opposite of what we saw in the series RLC circuit. We can summarize this behavior by using two additional parameters, Q and the bandwidth, BW. We define bandwidth for parallel RLC circuits, as the difference in frequency between the upper frequency at which the current increases to 141.4% of the maximum and the lower frequencies near the resonant frequency, while allowing others to pass.



The graph below shows the variation in phase shift of the current through the parallel RLC circuit.



Notice that the current has no phase shift at the resonant frequency. At frequencies below the resonant frequency the current lags the applied voltage and the circuit acts like an inductor. At frequencies above resonance, the current leads the applied voltage and the circuit acts like a capacitor.

above resonance that give an SWR of 2.62. The last two points are symmetrical around f0, so only one is measured as f = fH.

Since fL × fH = f0 [Eq A-5]

The other frequency fX is assumed to be at:

$$f_X = \frac{f_0^2}{f_H} = \frac{f_0^2}{f}$$ [Eq A-6]

The bandwidth BW1 may now be calculated:

$$BW_1 = f - f_X = f - \frac{f_0^2}{f}$$ [Eq A-7]

Since we want to be able to measure at any SWR value, we need to add a correction factor, K, resulting from the use of bandwidth BW1.

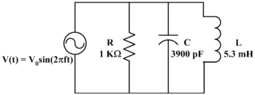
$$Q_U = \frac{K f_0}{BW_1}$$ [Eq A-8]

The correction factor, K, is the bandwidth multiplication factor. It is a function of the SWR. I derived K using Mathcad™ numerical calculations and then normalized the value. I used a fifth order polynomial to fit the computed data. See Figure 9. Note 4 gives the Mathcad™ and Adobe PDF files that I used, as well as more details on the derivations. The complete equation as used in my spreadsheet for the unloaded Q factor becomes:

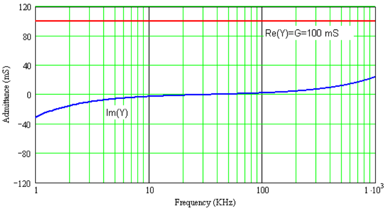
$$Q_c = \frac{f_0 \cdot (1.77423 \cdot SWR - 0.59467 \cdot SWR^2 + 0.13042 \cdot SWR^3 - 0.0151 \cdot SWR^4 + 0.00071 \cdot SWR^5 - 1.28601)}{\left| f - \frac{f_0^2}{f} \right|}$$

[Eq A-9]

Let us look at a specific parallel RLC circuit and explore its behavior.



The graph below shows the variation of the admittance of this parallel RLC circuit with frequency.



We can compute the resonant frequency from the component values:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{0.159}{\sqrt{(5.3 * 10^{-3})(3900 * 10^{-12})}} = \frac{0.159}{\sqrt{2.067 * 10^{-11}}} = \frac{0.159}{4.546 * 10^{-6}} = 35.0\ KHz$$

Note that this parallel RLC circuit has the same resonant frequency as the series RLC circuit examined earlier. In general, the resonant frequency depends only on L and C and not whether they are connected in parallel or series.

The next graph shows the variation in the current through the parallel RLC circuit. Notice that the current is a minimum at the resonant frequency. This is an example of a "band-stop" circuit response. A parallel RLC circuit can be used to block

Inductive susceptance:

$$B_L = \frac{-j}{2\pi fL}$$

Capacitive susceptance:

$$B_C = j2\pi fC$$

And of course, conductance is defined as follows:

$$G = \frac{1}{R}$$

By analyzing the parallel RLC circuit using admittance, we will be able to use nearly all of what we have already learned about series RLC circuits. Admittances in parallel add, just like impedances in series. We can immediately write the equation for the admittance of the parallel RLC circuit by adding the admittances of the three components:

$$Y = G + B_C + B_L = \frac{1}{R} + j2\pi fC - \frac{j}{2\pi fL}$$

This can be simplified to the following expression:

$$Y = G - j2\pi fC\left(1 - \frac{1}{4\pi^2 f^2 LC}\right)$$

The imaginary part of the admittance vanishes when:

$$4\pi^2 f^2 LC = 1$$

We define the frequency at which this occurs to be the resonant frequency, given by the following formula:

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

The equation for the admittance of the parallel RLC circuit can be simplified further:

$$Y = G + j2\pi fC\left(1 - \left(\frac{f_R}{f}\right)^2\right)$$

Experiments with Coils and Q-Measurement

Wes Hayward, w7zoi,
October, 2007 (Updates 01Dec07, 08Dec08.)
http://web.archive.org/web/20101226143919/http://w7zoi.net/coilq.pdf

I recently became interested in building a “Zero Power Receiver (ZPR)”, a circuit that would receive the AM broadcast band while using no external power. There would be no batteries or other sources of energy. This is, of course, the classic crystal set that most of us built in our youth. But building a contemporary ZPR is a different exercise than it was for those youthful interludes. The main difference is the presence of considerably more science in the process than we used in the construction of that first crystal set. There are some really wonderful web sites out there that present much of this work. Another difference is that the modern ZPR may not even use a “crystal,” or diode detector. Instead, it may use a field effect transistor. The most recent published design that I know about was presented by Bob Culter, N7FKI, in QST for January, 2007. Bob used a new zero threshold MOSFET in his ZPR.

The w7zoi amateur radio set up and listening post, September 30, 2007. A ZPR resides on top of the Icom R75 communications receiver.

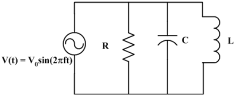
Those interested in finding out more about the modern crystal set should go to the world wide web with a good search engine such as Google. The first place to go is the Birmingham, Alabama Crystal Radio Group. They have a collection of links listed on their web page that will get you to many of the other really good sites. Birmingham really is the center for much of this activity, probably the result of some local activity that grew into something greater. These guys are to be commended for work that gets a lot of us thinking and trying something different. Be sure to look at their photo collection and at the results from their contests.

I'll not spend too much more space with further discussion of crystal sets here. This note is related to some of the measurement methods that I have used for Q measurements. But there is one piece of data that I do want to emphasize, for this was the thing that got my attention: The gang from Birmingham hold an annual listening contest that goes on for

The voltage response of the series RLC circuit can be completely characterized by its resonant frequency, f_R, and Q or the bandwidth, BW.

Parallel RLC Circuits

A parallel RLC circuit, also called a tank circuit, is one in which R, L, and C are in parallel, as shown below.



Much of what we just learned about series RLC circuits will carry over into parallel RLC circuits. Before we begin we need to define a new concept known as admittance. Admittance is the reciprocal of impedance and its symbol is Y:

$$Z = \frac{1}{Y}$$

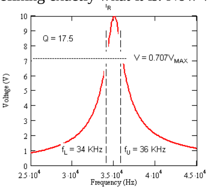
The unit of admittance is the Siemens (S). 1 Siemens = 1 ohm^{-1}. Like impedance, admittance is a complex number, consisting of a real part and an imaginary part. The real part is called conductance, G, and the imaginary part is called susceptance, B:

Y = G+jB

Admittance is very handy when analyzing parallel circuits because the total admittance of a group of components in parallel is simply the sum of their individual admittances. Our formulas for inductive and capacitive reactance can be converted to formulas for susceptance:

Another way to look at this is to say that the Q is the ratio of the reactance of the inductor to the resistance, at resonance. High Q RLC circuits have a very sharp response. Low Q RLC circuits have a broad response.

We have talked about the bandwidth of a circuit without defining exactly what it is. Now we will define it.



Referring to the figure above, bandwidth (BW) is defined as the difference in frequency between the upper frequency at which the voltage drops to 70.7% of the maximum and the lower frequency at which the voltage drops to 70.7% of the maximum. The 70.7% voltage points were chosen because they correspond to a power decrease of 50%. Thus:

$BW = f_U - f_L$

Bandwidth and Q are also related as follows:

$$BW = \frac{f_R}{Q}$$

In the graph above, Q = 17.5, and f_R = 35 KHz. The bandwidth is:

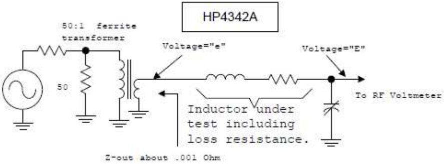
$$BW = \frac{35000}{17.5} = 2000\,Hz = 2\,KHz$$

over a week in the dead of winter. They then collect the logs to see who is hearing the good stuff. The folks who win are not just listening to the local stations in their area. Rather, they log stations from hundreds, and even thousands of miles away! A winning log may well contain over a hundred stations. One recent winner was from Hawaii.

Q-Measurement using a Q-Meter

The basic ZPR has at least one tuned circuit that serves to tune the receiver to a desired station. Most of the receivers that are built by those who listen to far distant broadcasts will have several. While there is considerable lore on the web that present an algorithm for coil construction, this does not provide the numbers we need for design. If we are to really be able to analyze the circuits that we build, even in a circuit as primitive as a crystal receiver, we must do measurements on key components to characterize, and hence, model them. We must have resonator Q.

Shown below is the basic topology of a Q-Meter, the HP-4342A.



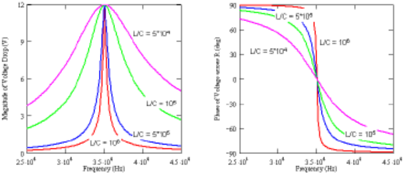
A relatively modern Q Meter. This particular model is perhaps the last of its kind.

This instrument has a built in signal generator (22 kHz up to 70 MHz) that supplies energy to a ferrite transformer with an extremely low output impedance, around one milliohm. The output has a magnitude "e". This level is maintained with a detector and feedback loop. The signal is then applied to the inductor under test, which is attached to terminals on top of the instrument. The inductor is tuned to resonance with a high quality, calibrated, built-in variable capacitor. The voltage across the capacitor is measured with a very high impedance RF voltmeter. The value of this voltage, E, is then directly related to Q. The lower the net series resistance, the higher E will be.

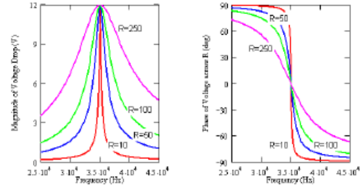
Some amateur experimenters have built homebrew Q meters using the HP scheme. Calibration may be a challenge, but the basic concepts are fundamental and would form a sound basis for experimental efforts. I might be tempted to try some of the very wideband op-amps that are available today as a way to generate the ultra low output impedance drive.

A modern measurement performed in industry or at an institution of higher learning will probably be done with a network analyzer. We will not go into any detail here, for that will take us far from our immediate goals. However, we should state that it is now quite possible for the amateur experimenter to build his or her own network analyzer. Some of the "antenna analyzers" now on the market edge in this direction. A network analyzer was described a few years ago in QEX. In my opinion though, the finest example of a homebrew analyzer is that presented by Paul Kiciak, N2PK. Paul's vector (meaning that it gives impedance magnitude and angle, or real + imaginary) is found on the web at http://n2pk.com/ and offers measurements to 60 MHz with a 100 dB dynamic range.

Although there are many combinations of L and C that will give the same resonant frequency, the shape of the curve representing VR depends on the L/C ratio as shown below.



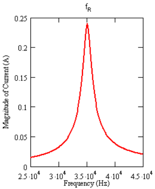
As the L/C ratio increases, the response becomes sharper.
The value of series resistance, R, also affects the shape of the V_R curve as shown below:



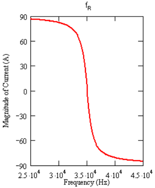
As the series resistance gets larger, the voltage curve gets broader.

In order to make sense of all these possibilities, it is necessary to introduce some new parameters. This first is Q, the quality factor of the series RLC circuit. Q is defined as:

$$Q = \frac{2\pi f_R L}{R}$$



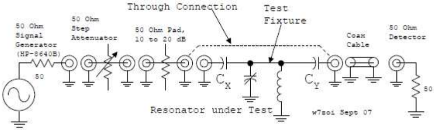
The magnitude of the current is maximum at the resonant frequency and $|I_{max}| = V_0/R = 12/50 = 240$ mA. This type of circuit behavior is called a band-pass response. If a complex signal containing many different frequencies was applied to the series RLC circuit and the output taken across R, the output would contain mostly frequencies near the resonant frequency. The next graph shows the phase of the current through the series resonant circuit.



At frequencies below the resonant frequency, the current leads the voltage, which is characteristic of an RC circuit. At frequencies above resonance, the current lags the voltage, and the series RLC circuit looks like a series RL circuit. At resonance, the current and voltage are in phase.

A Still Viable Classic Q Measurement

In earlier times when a network analyzer was not as available to the experimenter as it is today, resonator (another term for tuned circuit) Q was measured by examining the bandwidth of the circuit. The Q is then just the ratio of the center frequency to the bandwidth. A system for this is shown below.



A classic scheme for measurement of Q. CX and CY are adjusted for high loss. See text.

The use of this method is very general and is not restricted in frequency. I've used it to evaluate low frequency inductors used in audio filters as well as components for microwave filters. Like most RF measurements, a 50 Ohm system is used for both a driving RF source and for the load. It is vital to have digital frequency readout. A counter can be attached to an analog calibrated generator. The generator is followed by a step attenuator. This can be homebrew and is trivial for measurements up through the HF spectrum. All we will need for this measurement is a 3 dB step. However, it should be fairly accurate. (See EMRFD chapter 7 for info on attenuators.) The step attenuator is followed by a fixed pad to establish a Z0 environment. This pad should ideally be right next to the following test fixture that contains the tuned circuit we are measuring. The test fixture output is routed to a 50

Ohm detector. This can be a power meter, a spectrum analyzer (my usual choice), or a 50 Ohm terminated oscilloscope. If the ‘scope option is picked, the termination should be at the input to the oscilloscope and not out at the test fixture end of the connecting cable. With a 50 Ohm terminator at the ‘scope, the test fixture will see 50 Ohms at all frequencies, no matter what the cable length might be.

The first step in use is to put a jumper (usually a BNC barrel) from the fixed pad to the cable leading to the detector, shown as a dotted line in the figure above. The level in the detector is then noted. If you are using a ‘scope, calculate the power in dBm. Power in dBm is usually read directly if you are using a power meter or spectrum analyzer. Then connect the test fixture with its tuned circuit and tune the signal generator and/or variable capacitor to obtain a detector peak. The peak response will always be less than was seen with the "through" connection. That direct connection represented a matched impedance case where all of the available power from the signal generator is transferred to the detector.

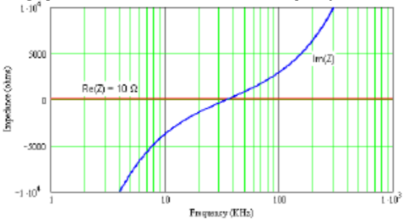
It’s now time to extract some numbers. Set the step attenuator to 3 dB and then tune the generator for a peak in output response. Peak the resonator capacitor if necessary. Note the frequency for this peak response. Now comes an important part of the procedure that is easy to bypass. Note the power seen in the detector and record it in your notebook. It should be at least 30 dB below the maximum available power that we measured with the through connection. Let’s assume, for the moment, that it is.

We now carefully note the detector level. If we are using a spectrum analyzer or an oscilloscope, we adjust the signal generator amplitude just a bit so the trace is right on a cursor

$$Z = R + j2\pi fL\left(1-\left(\frac{f_r}{f}\right)^2\right) = 50 + j6.2832\, f * 5.3*10^{-3}\left(1-\left(\frac{3.5*10^4}{f}\right)^2\right)$$

$$= 50 + j\left(0.0330\, f - \frac{40.79*10^6}{f}\right)$$

The graph below shows how Z varies with frequency:



We can find the current through the series RLC circuit by using Ohm's Law:

$$I = \frac{V}{Z} = \frac{12}{50 + j\left(0.0330 f - \frac{40.79*10^6}{f}\right)}$$

rather than simplify this ugly expression further, we will graph the current as a function of frequency:

called the resonant frequency. It is possible to derive an equation for the resonant frequency in terms of L and C from the condition $4p^2f^2LC = 1$:

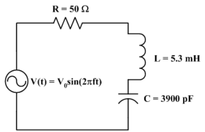
$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

where:
f_R = resonant frequency (Hz)
L = inductance (H)
C = capacitance (F)

The resonant frequency depends on the product of L and C, not the individual values, and is independent of R. We can use the formula for the resonant frequency to simplify our equation for the impedance of the series RLC circuit as follows:

$$Z = R + j2\pi fL\left(1 - \left(\frac{f_R}{f}\right)^2\right)$$

Let us examine the behavior of a series RLC circuit in more detail by studying the circuit shown below:



.
We can immediately calculate the resonant frequency:

$$f_R = \frac{1}{2\pi\sqrt{LC}} = \frac{0.159}{\sqrt{(5.3*10^{-3})(3900*10^{-12})}} = \frac{0.159}{\sqrt{2.067*10^{-11}}} = \frac{0.159}{4.546*10^{-6}} = 35.0\,KHz$$

We can substitute in values for fR, R, and L to get the total impedance:

line on the display. An adjustment of oscilloscope vertical position may also be used for this.

Next, remove the previously added 3 dB attenuation. This will cause the response to increase, doubling the detector power. (An oscilloscope voltage response will go up by a factor of 1.41.) Now carefully tune the signal generator to a higher frequency until the response has dropped to produce exactly the same level that we had at the peak. Record this upper frequency in your notebook. Next, the generator is tuned back toward and through the peak until the previously noted amplitude is again obtained. This occurs at a lower bandwidth edge. The difference between the upper and the lower frequencies is the resonator bandwidth. The Q is then the ratio.

$$BW = F_{upper} - F_{lower}$$

$$Q = \frac{F_{center}}{BW}$$

Consider now the earlier assumption that there was at least 30 dB loss through the test fixture containing the resonator and connecting capacitors. This loss relates to the choice of CX and CY shown in the schematic diagram for the classic measurement. First, it is important that the values be approximately the same value. This guarantees that any loading by the Z0 source will equal the loading by the Z0 detector. If CY was much larger that CX, for example, we could have a situation where the detector would severely load the resonator, but we could still have 30 dB or more attenuation. So make the two loads about the same. A loss of 30 dB or more means that the dominant loss mechanism is the loss within the tuned circuit and not the loss related to loading by the source or detector.

The way we adjust the loading at the ends of this single resonator filter is by picking CX and CY. The values we use will depend upon the capacitor in the resonator. If, for example, we have a resonator capacitance of 300 pF, we can probably get our 30 dB loss with values of perhaps 3 pF for CX and CY. I would probably grab 1 pF for each part from the junk box. Exact details will change as we go to much higher frequency. The concepts are the same though. For example, when evaluating a VHF helical resonator, or similar LC-like structure, we might have nothing more than a pair of coaxial connectors mounted in the resonator wall. The normal center pins from the connectors may well be enough coupling. Indeed, I have encountered situations where it was necessary to recess the connectors in the wall so that the center wires are partially hidden. Good measurement results are guaranteed if the 30 dB rule is maintained and symmetrical loading is achieved. If the Q exceeds 500 or 1000, it may be useful to go to loss greater than 30 dB. Alternatively, a correction can be applied. See page 58 of Introduction to RF Design (ARRL, 1994.)

Some of these details are not intuitive. A good way to build some intuition is to do simulations in LT Spice, which is free from Linear Technology on the web. (Bravo for LT—many thanks!) You can then "build" inductors in software, with appropriate series resistance. Then sweep the filter and see if you get the right Q by observing the 3 dB points.

The procedure outlined is general and applies to any frequency. The use of low impedances allows extreme flexibility. Some folks have measured Q with a similar approach where a small probe coil is driven by the generator and is placed near the resonator being studied. An oscilloscope

This expression in more complex than those derived in earlier sections for the series RL and RC circuits and as a result, the behavior of a series RLC circuit is much more interesting. Let us consider several interesting cases:

Case 1: f <<1.

If the frequency is very close to 0 Hz, the j2pfL term in the original expression for Z is approximately 0. In this case the impedance of the parallel RLC circuit can be approximated by the following equation

$$Z \approx R - \frac{j}{2\pi f C}$$

At very low frequencies, the series RLC circuit behaves like a series RC circuit.

Case 2: f >> 1

When the frequency is very large, the -j/(2pfC) term in the original expression for Z is approximately 0. In this case the impedance of the series RLC circuit can be approximated by the following equation

$$Z \approx R + j2\pi f L$$

At very high frequencies, the series RLC circuit behaves like a series RL circuit.

Case 3: $4p^{2}f^{2}LC = 1$

When this condition occurs, the imaginary part of Z is zero. The impedance of the series RLC circuit is real and is equal to the resistance:

Z = R

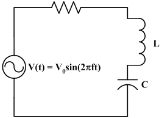
The circuit behaves as if the inductor and capacitor were not present. The negative reactance of the capacitor and positive reactance of the inductor add up to 0, creating a condition is known as resonance, which occurs at a specific frequency

RLC Circuits in: Electronics for Radio Amateurs
York County ARS K4YTZ
http://www.ycars.org/EFRA/Module%20A/AC%20Circuit%20Analysis%205.htm

RLC Circuits

An RLC circuit contains resistors, inductors and capacitors. In this section we shall look at the two simplest RLC circuits, the series RLC circuit and the parallel RLC circuit.

Series RLC Circuits



Since the resistor inductor and capacitor are in series, the total impedance of the series RLC circuit is the sum of the individual impedances:

$Z = R + X_L + X_C$

After substitution of the formulas for X_L and X_C one gets the following expression for Z in terms of f, R, L and C:

$$Z = R + j2\pi fL - \left(\frac{-j}{2\pi fC}\right)$$

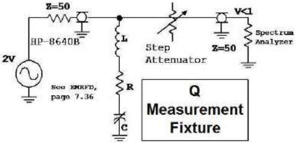
This can be simplified by combining the imaginary terms:

$$Z = R + j2\pi fL\left(1 - \frac{1}{4\pi^2 f^2 LC}\right)$$

is then used with a 10X probe and a small value series capacitor as a detector. The series capacitor is important, for even with a 10X probe, the impedance at RF might still excessively load the resonator. It is important to keep the source probe inductor well away from the coil. The concepts are the same and can all be modeled with appropriate calculations.

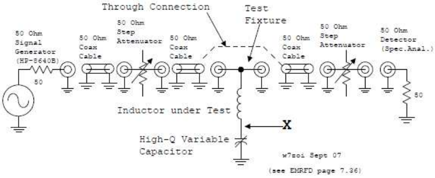
The EMRFD Q-Measurement Scheme

An alternative scheme is presented in Experimental Methods in RF Design (ARRL, 2003) in Chapter 7, page 7.36. In this method, a tuned circuit containing the inductor to be measured is configured as a series resonator and is then connected as a shunt element. The basic scheme is shown below.



Alternative Q measurement scheme.

A steady strong output is seen in the detector as the signal generator is tuned. As resonance is reached, the output dips down. The reactance of the inductor is cancelled by that of the capacitor exactly at resonance, leaving nothing but the loss resistance, R, to attenuate the signal reaching the detector. The higher the Q, the greater the dip will become. A more detailed diagram is shown below.



Alternative Q measurement system. See text for explanation.

The procedure for using this method begins with setting the signal generator to the desired frequency. The level is noted in the spectrum analyzer. A power meter or 50 Ohm terminated oscilloscope can also be used if the signal generator is known to be low in harmonic content. (Not all signal generators fulfill this criterion though, so be careful. The HP-8640B that I use has very low harmonic output. If you are concerned, use a low pass filter with the generator.) The capacitor in the test fixture is then tuned to produce a dip. Use the highest possible amplitude resolution you have in your detector. I go to a 2 dB/division mode with my spectrum analyzer. After an initial dip has been set with the capacitor, use the signal generator to select the lowest dip response. Then use an amplitude control on either the detector or generator to move the observed signal to a cursor line.

Record the signal generator frequency. Then take the test fixture out of the system, realized in my setup by substituting a BNC barrel for the test fixture. The signal will increase at the detector. Increase the attenuation in the step attenuator until the same response at the cursor line is observed. This will provide an attenuation or dip value. Interpolate as needed to

EE. Bucher, 1919, "Wireless Experimenter’s Manual", Wireless Press, New York.

Wes Hayward, W7ZOI, Experiments with Coils and Q-Measurement, October, 2007 (Updates 01Dec07, 08Dec08. http://web.archive.org/web/20101226143919/http://w7zoi.net/coilq.pdf

Steve Ratzlaff, AA7U kindly made detail measurements on the Q of the capacitors and coils used in this report.

Dick Kleijer , 2013, Measuring the Q of LC Circuits, http://www.crystal-radio.eu/enqmeting.htm

Barbara Dziurdzia, 2013, Damped oscillations in RLC circuits, AGH University of Science and Technology in Cracow, http://home.agh.edu.pl/~lyson/downloads/Manual_8.pdf

Rice University EE., RLC Circuits, http://www.owlnet.rice.edu/~phys102/Lab/RLC_circuits.pdf

Durham University EE., Oscillations and Resonances in LRC Circuits, http://level1.physics.dur.ac.uk/projects/script/lcr.pdf

This is slightly low to Steve's measured 240Q. I am of the opinion that the original tank measurement may have been a bit on the low side. Still, again the results are within the ballpark and the technique works if you have a good measure on the Q of the capacitor used in your tank. Alternatively, starting with a known coil Q, the technique will work equally well to determine the Q of an unknown capacitor.

Conclusion:

If you have access to, or own a digital oscilloscope, or have been hankering to purchase one, then the time may be now. The analytical method presented above is simple in practice and robust in theory. I would also say that, by working with your wave forms, you learn to recognize your data and soon get a good "feel" for what is good and what is not. The technique also produces, should you wish, an auditable report of the wave and analysis. It is hard to argue against the results with the waveform sitting there staring right at you!

Measuring your coil Q should not be difficult.

A copy of the evaluation spreadsheet in xls format can be found here CoilQbyDecrement_kjs.xls

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Bibliography:

Jacques Audet, VE2AZX., Q Factor Measurements on L-C Circuits., QEX - January/February 2012, pp 7 - 11. http://www.arrl.org/files/file/QEX_Next_Issue/Jan-Feb_2012/QEX_1_12_Audet.pdf

obtain data within 0.1 or 0.2 dB. Record this value in the notebook with the previous center frequency.

Finally, unsolder the inductor from the circuit at the point marked with the large X in the figure. Then attach a capacitance measuring meter to measure the C value. I used one of the AADE L/C meters. (Just Google AADE on the web.) The C value is recorded. The F and C data allows calculation of the effective inductance. The inductor equivalent series resistance can then be calculated from the attenuation, which then leads directly to Q. The equations are included in the EMRFD presentation.

An advantage of this scheme is that the Q value is directly related to the attenuation value, which is relatively easy to determine with an accurate step attenuator. (I use a surplus HP-355C and HP-355D combination.) The detector operates directly on a null point rather than along the slope of an attenuation function where frequency drift can complicate the results. But the method is sensitive to the driving and load impedance at the test fixture. Values other than 50 Ohms will compromise results It may even be important to actually use 50 Ohm cables; the 52 Ohms of RG-58 might not be close enough. A conservative approach would place a 50 Ohm pad (6 to 10 dB) at the edge of the test fixture.

This photo shows the test fixture that we used with the EMRFD scheme. The inductor in this case is a ferrite toroid wound with Litz wire. This variable capacitor is believed to be made by Jackson Brothers, Ltd, from England and is the highest Q variable capacitor we had in our junk box. More on this below.

Comparing the methods.

Most of the measurements we have done in recent times used the EMRFD method. In years past, we
would use a Q meter when available, but most often, the 3 dB BW method. Short of a calibrated Q-Meter, the EMRFD scheme is quicker than the 3 dB resonator bandwidth measurement. In one recent case, I measured the Q of an inductor with both methods. The inductor was a basket weave coil, described below. Using the classic 3 dB bandwidth method, the Q was measured as 579 at 1 MHz. Insertion loss was 48 dB for this measurement. The same coil with the same variable capacitor measured Q=650 at 1 MHz with the EMRFD scheme. That’s about a 10% spread between the two schemes.

The coil used in this experiment was a 5 inch diameter solenoid wound in a basket weave pattern for L=265 uH. The wire was 175/46 Litz, which is 175 strands of #46. The coil sat on a piece of plastic suspended on a stack of wood blocks during measurement in an effort to avoid external loads.

Experiments with various coils.

APC cap with red/black leads (149 pF meter): 937Q at 904kc (Q meter alone and coil 1060Q)
APC cap with my Litz leads (149 pF meter): 995Q at 904kc (Q meter alone and coil 1060Q)

Steve's measurements above provide important datapoints with which to calibrate my technique. Note first of all that the Litz coil is not a modest 506Q but a robust 1140Q at 1MHz. The reason for my low measurement was my assumption of a high-Q capacitor. Steve's measurements of two of my APC's are 995 and 937, hardly very high. There is an obvious need for a correction. Wes Hayward in his excellent notes on Q measurements provides the key in the form of the following formula:

$$Ql = (Qc * Qres) / (Qc - Qres) \qquad (14)$$

where Ql is the desired coil Q, Qres is the measured Q of the tank/resonator, and Qc is the tank capacitor Q.

Substituting the values Qres = 532 and Qc = 995 the equation simplifies thus:
Ql = (937 * 506) / (937 - 506)
Ql = 1100.

The result is distressingly close to Steve's measured value of 1140Q. This shows the possible level of accuracy in the analytical technique used, both reassuringly high and very "Litz-like".

Taking the first solenoid and substituting the values Qres = 167 and Qc = 937 the equation simplifies thus:
Ql = (937 * 167) / (937 - 167)
Ql = 203.

my opinion that the capacitor Q ought to be high enough as to not influence the coil Q measurement in any significant manner. I said that for three reasons: 1) Just about every published article starts with such an assumption and 2) I had not done much research on the subject of capacitor Q and so had no particular expectations other than assumption 1 and finally 3) I had no measurements or specific knowledge of the Q for the capacitor used in my setup.

Reason 1 is a punt and should be rejected, reason 2 has been remedied and I give some analysis on the interesting subject of capacitor ESR (Equivelant Series Resistance) on a separate page so that readers may set their own expectations. For the third reason I have received the kind assistance of Steve Ratzlaff, AA7U who offered to make actual Q measurements on the coils and APC capacitors with his HP4342A Q-meter, results follow:

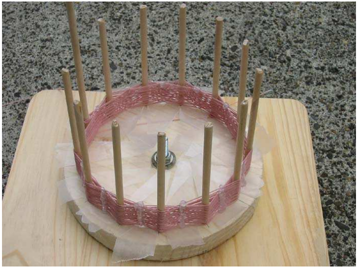
Litz basket coil,
550kc 1127Q;
1000kc 1140Q,
1700kc 674Q (194.2 uH on aade.com LC meter)

Solenoid coil,
550kc 205Q,
1000kc 240Q,
1700kc 222Q (218.2 uH on aade.com LC meter)

Using Steve's Litz wound ferrite rod coil (1000kc 1047Q) as the reference for external caps:

100 pF silver mica (99.2 pF meter):859Q at 1068kc (Q meter alone and coil 1025Q)

Some of the coils investigated in this study. The basket weave coils were wound with the fixture shown below. The two toroids shown are Amidon FT-114A-61 and used #18 enamel wire or 175/46 Litz wire. The rod is similar to an Amidon R61-050-400 and is wound with 50/46 Litz wire.



Fixture used to wind basket weave coils. Strips of waxed paper are placed under the coil cross over lines before application of hot melt glue.

Toroid Experiments

Most of the filters I build at HF use toroid cores. Powdered iron cores seem to offer higher Q at HF (3 to 30 MHz) than I can obtain with other practical, reasonable sized forms. The -61 and -63 Ferrite materials from Amidon offer high Q in the MF region from 300 kHz to 3 MHz. Powdered iron can provide high Q, but have such low permeability that the number of turns becomes excessive. So the first experiments I did used ferrite toroids.

Not all of the data collected will be presented here. However, I'll give a few of the data points of interest. There are some surprises.

qc = 18.55 exp(-t / 2*Rp*123)*cos(6.283*1.044*t + 1)

Where t = 0 to 50 uS
R = 0.627 Mohm in this case for best match.

Q = Rp / 2pifL = 627000 / 6.283 * 1.044 * 189 = 506

The results above for a big litz coil clearly demonstrate its superiority to a more common tapped solenoid. Published data though has let me to expect a big litz coil Q to be in the 1000+ range. I have run this test backwards and forwards, checking all possible causes for this unexpected low Q with no difference in the final value. In fact I am quite impressed with the repeatability of the technique and the correspondence between the results from the decrement and those from modeling the wave form.

To be certain I even tested the coil using the old -3dB method. As stated previously, I am uncomfortable using a signal generator in addition to the scope, and I have already found trouble connecting the 1:100 scope probe directly to the tank, as is necessary in this method. With that said, the results yield a bandwidth of 2.3kHz about a nominal frequency of 1018.8kHz for a tank Q = 450. I suspect the method (with the problems noted) lowballs somewhat the final result. In any case, this confirms the test to have a moderate Q and nothing like the expected values one reads online.

Notes on Capacitor Q and its impact on the measurement value.

This is where we begin to think more seriously about the capacitor used to resonate the tank. The assumption of very high Q needs to be questioned. I stated in the setup discussion

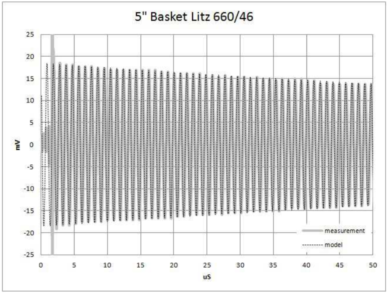


Figure 6. Second oscillogram with modeled damped wave response.

The coil tested values as follows:

n = 45
T = (tn - to) / n = (47.82 - 4.70)/45 = 0.958 uS
f = 1/T = 1/0.958 = 1.044 MHz

Ao = = 18.05 mV
An = = 13.65

d = 1/n ln(Ao/An) = 1/45exp(18.05/13.65) = 0.00621
Q = pi / d = 3.142 / 0.00621 = 506

And:

qc = qo * e^(-t/2RpC) * cos(2pif * t + phi)
C = 123 pF

First, the best toroid I found for BC band use was 38 turns of #18 enamel wire nearly filling a FT-114A-61 core. Inductance was 195 uH on an AADE L/C meter. Q=383 at 1 MHz and peaked up to almost 400 at 811 kHz. Q remained well over 300 at both 500 and 1610 kHz.

A slightly more practical and easier to wind coil used a FT-114A-61 form with 41 turns of #22 enamel wire. At 1 MHz, the inductance was 240 uH with a Q of 384. The Q dropped nearly to 300 at the band extremes.

The receiver I was building was to use a variable capacitor with only 188 pF maximum C. So a higher L was needed. I managed to get 56 turns of #22 enamel wire on a FT-114A-61 form for L(AADE) of 449 uH and Q=344 at 1 MHz. Q was 316 at 549 kHz and 274 at 1.55 MHz. Although compromised, this seemed a practical coil for a first zero power receiver.

A question that immediately came up was what could be done with Litz wire on toroids. The results were disappointing. 39 turns of 175/46 Litz wire on a FT-114A-61 form yielded L=244 uH and Q=318 at 1 MHz. The Q improved to 352 down at 550 kHz. A second FT-114A-61 core was wound with 40 turns of smaller 50/46 Litz wire, resulting in L=248 uH and Q=298 at 1 MHz. Again, low frequency Q improved slightly. My conclusion at this point is that the 175/46 Litz wire is not justified for ferrite toroids, although that conclusion is certainly more general than my data can support. I have not investigated any of the expensive Litz "rope" using over 600 strands of #46 wire. (See later note.)

Variable Capacitor Q (data updated 08Dec08 with more discussion at the end.)

Many of the web sites I read that deal with crystal sets and similar receivers state that the absolute best possible variable capacitors should be used. They then define these as being those capacitors with ceramic insulators, with silver plating being preferred on the metal. Double bearing designs are also recommended. Some sort of special means to minimize resistance related to moving contacts is deemed important. While all of these things are intuitively reasonable, they are not numbers.

The Q measurement schemes I have presented all relate to a determination of resonator Q. Actual inductor Q can only be obtained if one knows the Q of the capacitors in a tuned circuit, or if can be established that capacitor Q is very high. The folks building Q meters go to great extremes to optimize the variable capacitor Q within the instrument. (See Kito and Hasegawa, "Measuring Q" Easier and Faster,(HP Journal, September, 1970.)

Let me again emphasize that all of the numbers presented here, with all measurement methods, relate to resonator Q. We can calculate capacitor Q only if we independently know the inductor Q.

This still leaves us with the question of what we should be using for variable capacitors, both in our receivers and for Q measurement. To begin an evaluation, I took the highest Q toroid that I had built at the time (38 turns of #18 enameled wire on a FT-114A-61 core) and attached it to various variable and fixed capacitors from my junk box. Measurements were done at or near 1 MHz using the EMRFD measurement scheme. The data, although less than conclusive, is interesting. Here are some results:

decrement method. An exact match will not always be the case, but they ought to be fairly close. The match between theory and the decrement method gives confidence that this analytical technique is as robust as it is simple.

qc = qo * e^(-t/2RpC) * cos(2pif * t + phi)
qc = 12.2 exp(-t/2*Rp*106)*cos(6.283*1.055*t + 1)

Where t = 0 to 50 uS
R = 0.237 Mohm in this case for best match.

Q = Rp / 2pifL = 237000 / 6.283 * 1.055 * 215 = 167

Example Two:

To be sure the technique works I have also assessed the quality of a 5inch diameter basket weave coil wound from 660/46 litz wire. Such coils are rightly considered "performance coils" for their low skin-effect and low resistivity.

This leaves the resistance of the coil (R) as the primary variable. Model the above equation in the spreadsheet by adjusting R until the model results match the measured oscillation from the coil. From the above example we see Figure 5 with the modeled damped wave overlaid on the test oscillation.

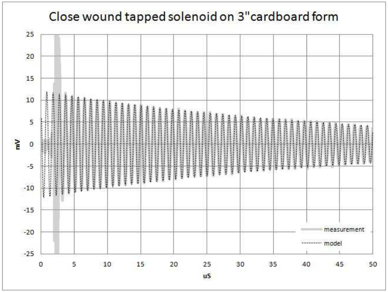


Figure 5. Oscillogram with modeled damped wave response.

Here the background data from the test is in grey and overlaying it is the dashed model in black. The fit is nearly perfect using a resistance of 0.22 Mohms. Relating resistivity to Q we have the following equation (5):

Q = Rp / 2pifL

Substituting f = 1.055 MHz, C = 106 pF, and R = 0.237 Mohms the result is Q = 167, exactly the same as with the

Capacitor comparisons, all operated near 100 pF.

1. 107 pF Silver Mica. This consisted of two series connected 214 pF 1% 500 volt units.
1. Resonator Q=341.
2. Mica compression trimmer, 30-300 pF, marked GMA40400, Resonator Q=314.
3. "Jackson Brothers" (that’s a guess) dual section, 497 pF/section max, built-in gear drive, Resonator Q=383. This was the best variable capacitor in my junk box.
4. TRW four section. This was the most elegant looking capacitor I had in my stash. It came from a Tektronix 191 sine wave generator. Resonator Q=354. An OK cap, but not the best.
5. Collins TCS Transmitter VFO capacitor, built-in gear drive. Resonator Q=357.
6. ARC-5 Receiver variable. (BC-454?, all three sections in parallel yield 47-514 pF) Resonator Q=378.
7. BC-221 capacitor, 13 to 188 pF, Resonator Q=364.
8. Hammarlund single bearing, 150 pF max. Resonator Q=279. (ugh).

Much of the lore seems correct. However, the best measured resonator Q came from the Jackson Brothers dual section, not from antiquity, but from just slightly more recent times. The beautiful TRW capacitor, while good, was not at the top of the list. The aging single bearing capacitor was not expected to be great, and there was no surprise. I was surprised that the Silver Mica fixed capacitors were not at the top. But perhaps these were rejects. Although they were brand new unused capacitors, they were purchased at a surplus emporium and loss may have been the reason they were declared surplus. Additional measurements are called for in this area.

Additional information regarding capacitor measurements is appended at the end of this note.

Litz Wire Connection.

Classic lore suggests that it is vital to solder all strands in a Litz wire bundle at both ends. If one strand is left unsoldered, the argument is that current can no longer flow in that wire, so it will not contribute to the expanded surface area that leads to improved Q with Litz wire.

I had measured less than stellar Q with some 175/46 Litz wire on a toroid form and wondered if perhaps I did not manage to get all of the strands soldered. So I did an experiment while using the mica compression trimmer as the capacitor (poor capacitor choice.) After doing my best to solder all strands by carefully applying rosin flux from a SMT rosin pen, I heated and tinned the ends with an 800 degree tip in my Weller soldering station. The ends looked very good in a visual inspection.

I then started doing Q measurements near 1 MHz. After each one, a few strands of the Litz wire were cut and a 1 inch section was removed near one end. The strands were removed from the capacitor end of the test resonator. Here’s a plot of Q from 0 to 152 clipped strands. N was estimated by counting the number of pieces of wire after a clipping operation.

To test the validity of this technique we need a bit more theory, just to be certain this is working. With the data in spreadsheet form, it becomes possible to model the coil from a theoretical standpoint. The model can then be plotted with and compared to the acquired data to test for correlation. Various web pages, especially university-sponsored lab exercises provide a good source for practical techniques and their theoretical underpinnings. Some good sources I utilized include:
"Damped oscillations in RLC circuits" by Barbara Dziurdzia at AGH University of Science and Technology in Cracow, "RLC Circuits" a lab note from Rice University, and "Oscillations and Resonances in LRC Circuits" from Durham University in the UK. And very many others.

From the above we find the general equation for damped electrical oscillations as:

qc = qo * e^(-a*t) * cos(2pif * t + phi) (13)

Where
qc is the charge in volts
qo is the initial charge
a is the damping factor (=Rs/2L (series) and = 1/2RpC (parallel)
R is the coil resistance in ohms
L is the coil inductance in uH
t is the time in uS
f is the frequency in MHz
and phi is the phase angle

The phase angle and qo charge are initial conditions and adjustments should be minor to fit the calculated oscillation to actual. L, the inductance comes from the calculation we made with equation (12), and frequency comes from equation (9).

An example to show off a bit. This test was made with a close-wound tapped solenoid on a 3" cardboard form. The graphic in Figure 4 shows some initial contact hash followed by a lovely damped oscillation. Measurements on this coil showed a test frequency of 1.055 MHz, L of 215 uH and C = 106 pF. Q was calculated at 167. A decent coil but not great.

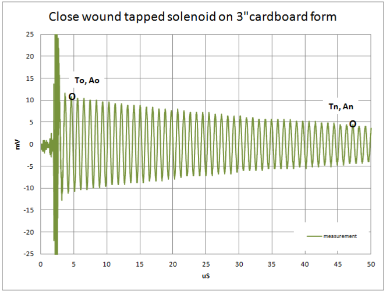
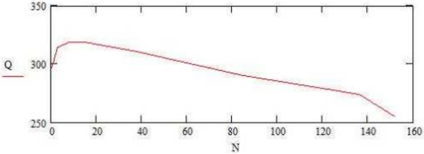


Figure 4. Oscillogram of a coil under test.

n = 45
T = (tn - to) / n = (47.24 - 4.59)/45 = 0.948 uS
f = 1/T = 1/0.948 = 1.055 MHz

Ao = = 11.2 mV
An = = 4.80

d = 1/n ln(Ao/An) = 1/45exp(11.2/4.80) = 0.01883
Q = pi / d = 3.142 / 0.01883 = 167



Q versus number of removed strands in a Litz wire toroid. There are 175 total strands in this wire.

This is certainly not a definitive experiment. It should be repeated with a coil where the Litz wire seemed to actually make a difference, and that was not the case with the toroid. However, the classic lore is not supported. But this makes sense after some thought (and some comments from colleague KK7B.) The weak dependence suggests that even though a cut at the end of a wire means that conduction is eliminated at the end of the wire, this does not mean that it is removed from the interior of the coil. Strand to strand capacitance guarantees that similar current will flow in each. The current in each forces the flow to the surface. If a strand is cut, there will still be some current flow in that strand back from the cut. The capacitance between strands connects the strands to those that are still connected at the end.

I hope to repeat this experiment with a different coil and a better resonator capacitor.

The Basket Case

Solenoid coils tend to have the highest Q if the wires are spaced by about 1 wire diameter. This lore is found in Doug DeMaw’s book on Ferromagnetic Inductors (Prentice Hall,

1981) and is quoted on the Amidon web page. This rule of thumb is reasonable. The highest inductance will occur with tight spacing. If the spacing increases beyond this, there is a greater chance that some of the magnetic flux from one wire in a coil will escape from the side of a coil and not link to all turns.

If we plot the inductance of a solenoid of constant pitch versus the number of turns, L is proportional to N2 when N is small. However, the parabolic shape relaxes toward a linear dependence as N becomes large, especially as the length exceeds the diameter. Linear L versus N is akin to adding inductors in series where each one is isolated from the others.

When one turn is directly next to the next turn with no gap, the current flowing in one turn will force some of the current in the next wire to move away. The overall effect produces non-uniform current in the wire surfaces. This is alleviated with a slight spacing between turns. But too much spacing decreases inductance too much.

A classic coil type is the so called basket weave. A couple of coils of this type are shown, along with a coil winding fixture on page 7 of this report. There are an odd number of rods in the winding jig. This means that as we weave the wires among the rods, one turn will not be directly adjacent to the next turn. This makes the coil behave as if it was a solenoid spaced by 1 turn. But there is no real gap, so the coil length, and hence, inductance is the same as a tightly wound one would be.

I had never built a basket weave coil, yet this is one of many designs that the folks describe on the web sites mentioned. So a couple of coils were built. The first used #22 enamel wire. The coil diameter is 5 inches wound on 13 posts with a total of

5000 data points to the computer. At 1 MHz that is about 100 data points per cycle, more than enough sampling to avoid any question of aliasing.

The decrement measurement is based on measuring two peaks. The data is in digital format so one should eliminate "estimation error" by actually reading the value off the spreadsheet, noting both time (in uS) and amplitude. Before taking readings, care must be taken to center the oscillations around zero. Amplitude measurements assume that the waveform is symmetrical around zero yet the scope may or, more likely, may not be set perfectly for this. In the spreadsheet the solution is simply to take the average of all the readings and subtract that from each.

Type the values for “to” (initial peak time), “tn” (final peak time), Ao (initial peak amplitude) and An (final peak amplifude), and the number of periods analyzed “n”.

T = tn - to in uS (8)
f = 1/T in MHz (9)

d = 1/n ln(Ao/An) (10)
Q = pi / d (7)

Next, disconnect the battery and coil and measure carefully the capacitance, C, that was used in the test. With this you can calculate the actual coil inductance from the frequency and capacitance. Why buy an expensive L meter?

L (uH) = 1E6 * ((1/ 2pif)^2) / C (pF) (12)

Example One:

10uS / division for a longer recording. Set the scope trigger appropriately to capture the data.

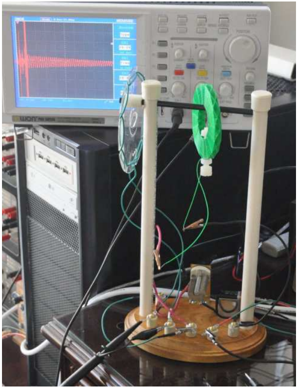


Figure 3. Photo of the test setup.

Tap, or "key" the battery to pulse the tank and review the waveform on the screen. Setting the scope on its measurement mode allows reading the wave frequency with each pulse. Adjust the capacitance after each pulse to bring the tank to the desired frequency, near 1 MHz in my case. Once at the desired frequency you are collecting data. When you get a good clean oscillation free of contact hash and noise, send the data to the computer for analysis. For my scope set at 5uS/div it returns

36 turns. The 1 MHz Q was a poor 244 with Q slowly increasing with frequency. Inductance was 265 uH A similar coil was then wound with 175/46 Litz wire. Again, inductance was 265 uH. The 1 MHz Q was a spectacular 650. The Q dropped off at both 530 and 1630 kHz, but was OK over most of the range. This is the highest 1 MHz Q I had ever seen at that time. I remember frequent Q values well over 1000 with some helical resonators at 500 MHz, but that is common at UHF.

Ferrite Rod Experiments

The next resonator type to be investigated was the ferrite rod. The only new part that I had around was an Amidon R33-050-750, meaning that it was built from mix 33 type ferrite with a diameter of 0.5 inch and length 7.5 inch. This material has an initial permeability of 800, which is high enough to make me wonder about it as a high Q antenna material. Owing to the high permeability, the number of turns needed should be much less than the 70 turns suggested in the N7FKI paper. I put a layer of paper on the core and then would 36 turns of #22 enamel wire over the central 3 inches. The inductance was 158 uH. Additional turns would be needed to reach the desired 240 uH.

Then Q was measured. It was a miserable 58.6 at 1 MHz. On the other hand, I did see several local radio stations in the spectrum analyzer display during testing. It may be a viable antenna, but it is not much as a resonator. I didn’t even bother changing to Litz wire.

The next experiment was with a junk box ferrite rod. This was a piece that was 3.85 inches long with a diameter of 0.475 inch. After putting a layer of paper on the core, I wound 37 turns of #22 enamel over the inner 2.8 inches. The inductance was 80 uH. A 1 MHz Q measurement was a more respectable 293.

The dimensions of this rod were very close to an Amidon R61-050-400. The inductance constant in the Amidon literature, after conversion, was 43 nH/turn2, so the 37 turn winding should have an inductance of 59 uH if it was wound evenly over the total core. My winding was bunched near the center which would increase the inductance a bit, so 80 uH would be reasonable. I concluded that the material is probably -61 mix, or a similar mix with initial permeability of 125, perhaps from a different vendor. I calculated that 65 turns over the same part of the core would yield 250 uH.

Based upon the measurements above, I wound 62 turns of 50/46 Litz wire on a single layer of paper over the junk box rod. The result was a spectacular Q of 610 at 1 MHz. The Q dropped off to 577 at 550 kHz and 500 at 1551 kHz, both still quite respectable. This high Q was a major surprise.

N7FKI has a larger R61-050-750 ferrite rod in his parts box and has offered it for some measurements. That is on the list of "To Do" experiments.

Classic Solenoid Coil

Among the goodies in the N7FKI junk box were some large cylinders that looked like they would work well as coil forms and yield reasonable Q. The outside diameter was 4.2 inches. After earlier coil construction experiments, about 55 feet of Litz wire remained on a spool (purchased on EBay from Paul Cianciolo, W1VLF), so I put solder lugs on each end and wound a coil. The first one had the wires spaced approximately

pF. These are silver or nickel plated (depending on the source of information) and ceramic (steatite) insulated and should have a high Q with little impact on the test. At least this is the general assumption used in most Q-measurement articles I have run across. In actual fact the capacitor Q is not infinite and not even that high so resistive losses associated with the capacitor can be expected to lower the measured Q. One needs to keep this in mind and know that this measurement is on the resonator, not on the coil itself. More on this topic in a later section. The APC capacitance range is limited but sufficient for my intended test frequencies of 1.0 to 1.1 MHz, this is a test, not a radio.

Keying, or pulsing the circuit is somewhat tricky. In order to obtain a clean wave trace the pulse contact needs to be fast and sharp. Contact "bounce" and arcing, especially at radio frequency are difficult to avoid and pulsing the circuit several times may be needed prior to finding a clean trace. I initially used a transmitter key but found the contactor at RF to be messy with visible arcing. The best method I have found for pulsing the circuit is to connect a pointed probe to one side of the circuit and tap it sharply but lightly against the hot side of the battery. After a few tries one gets a "feel" for the contact. Pulse the circuit and get a wave, things are this simple.

Procedure:

Hang the coil under test on a stand where it is clear of other components or metal objects. Connect the coil as per the schematic. Connect the probe and scope to the coupling coil leads and set the scope for a good scale, (Figure 3). I generally use 5uS / division horizontal scale which gives me 40 to 50 oscillations at about 1 MHz. For higher Q coils I may go to

Setup:

The following schematic shows the setup, (Figure 2). In this configuration, I have used a simple 9V battery to stimulate the coil and have the scope connected to the tank via a coupling coil. An earlier configuration with a power supply resulted in too much noise and ripple to obtain a good reading. The tank is coupled lightly to the scope with a coupling coil. This coupling coil is simply two turns of hookup wire about 4 inches in diameter placed an inch or two from the coil to be tested and I have found no reason to space it further. Between the coupling coil and oscilloscope I utilize the 1:100 probe although testing with my 1:10 probe showed little impact. In this way, the coil under test is clean and well isolated from the measurement circuit.

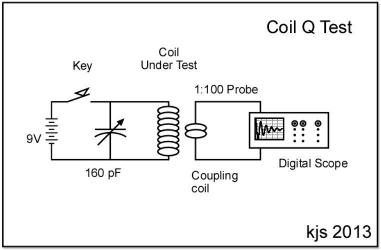
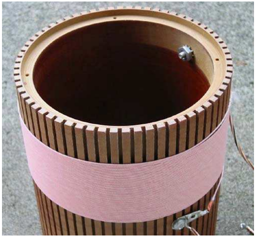


Figure 2. Schematic of test circuit.

For the tank I have used a small CT1C150 military spec cap ("APC"-type) with a capacitance range from about 20 to 160

with a 1 wire diameter gap. Measured inductance with the AADE L/C meter was 275 uH. The Q at 1 MHz was 353; OK but disappointing. The winding length was 2.9 inches.

The coil was then rewound with close spacing, resulting in 53.25 turns over a length of 1.52 inches. Inductance with the AADE meter was 378 uH. A 1 MHz Q measurement showed a higher effective L of 419, but with Q=219.

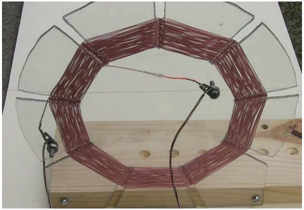

Close spaced solenoid coil.

I have not discussed the self resonant frequency of any of the inductors described in this note. However, it is easily measured by merely inserting the inductor in a 50 Ohm line. The fixture on page 6 of this note can be used. The "through" wire between the BNC connectors is removed and the coil is inserted. The signal generator is then tuned to find the first null frequency. This null is the result of stray parallel C resonating with the inductance to form a trap. The parallel resonant frequency was 4.0 MHz for the close spaced inductor shown in the above photo. This inductor had an inductance of 378 uH, which was measured at about 500 kHz with the AADE L/C meter. This yields an equivalent parallel capacitance of 4.2 pF. Using this and the 378 uH inductance measurement we calculate an effective 1 MHz L of 403 uH. This is still below the value we measured. The AADE number may well be in error because of the coil's parallel capacitance. Additional number crunching should yield a better model.

Update: 01Dec07

A Spider Coil.

Many of the crystal radio related web sites present information on Spider coils. I picked up a piece of Lexan Polycarbonate at Home Depot and cut it to form a nine segment coil form. The diameter ranged from 4.4 to 6.1 inches. This form was then wound with 39 turns of the 175/46 Litz wire. A photo follows.



The inductance was 373 uH, but the Q results were disappointing at best: 550 kHz, Q=499; 1.002 MHz; Q=474; 1.504 MHz, Q=382. The EMRFD Q measuring scheme was used.

be related to the circuit components as follows, (Bucher, 1919):

d = pi (R / 2pifL) (3)

The angular frequency is often expressed as w= 2pif so this can be substituted into equation (3) above to simplify as

d = pi R / w L (4)

We turn now to looking at Q and its relation to the circuit components. The general expression for Q is:

Q = 2pifL / R (5)

Simplifying to

Q = w L / R (6)

With the term w L in both expressions, we can solve for w L in each and set them equal to each other:

QR = piR / d so:

Q = pi / d (7)

Dreadful, wasn't it? The inspiration here is that with a simple determination of the logarithmic decrement, deriving the circuit Q is a trivial exercise even a geologist can manage. No signal generators, no attenuators, no multiple measurements. All this should serve to reduce sources of error and give a faster and easier way to determine the quality of that most central component of your crystal radio, the coil.

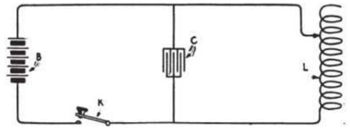


FIG. 8. Simple oscillation circuit illustrating the production of radio frequency currents.

Figure 1. Simple oscillation circuit, Bucher 1919.

As a professional Geologist, I must admit that most math involving more than ten fingers and ten toes (for advanced computations) is not my specialty. As such I will keep this aspect to an absolute minimum. Still, a bit of algebra may be useful to explain damping and logarithmic decrement. RF oscillations in a tank circuit are damped due to losses (primarily resistive) associated with that circuit. The amount of damping thus is related to the quality, or Q of the tank. In old texts the damping is generally determined by measuring the "Logarithmic Decrement" or the amplitude of successive oscillation peaks and taking the log of the ratio.

$$d = \ln(A1/A2) \qquad (1)$$

A more generalized version of this formula taking into account measurements over many periods can be expressed as:

$$d = 1/n \ln(Ao/An) \qquad (2)$$

Where n is the number of periods analyzed, Ao is the amplitude of the first peak and An is the amplitude of the peak n periods away. This equation allows a very simple determination of the log decrement with high accuracy. Because damping is due to resistive losses in the circuit, it can

Using Litz Rope.

The initial experiments were encouraging, although I was frustrated in not having achieved higher Q values. A review of many sites on the web quoted "Q over 1000" as if it was to be expected. One of the most informative web sites that I found was from Europe, http://www.crystal-radio.eu/enlctest.htm . He built numerous coils and then did careful Q measurements. Many of his measurements were consistent with mine. There was one central theme that emerged: If Q in excess of 1000 is desired, it is probably achievable only with Litz wire using many more strands than the 175 that I had used. Specifically, the wire that really made the difference in the measurements from Europe was 660/46 Litz. This is 660 strands of #46 wire. I've seen this stuff referred to as "Litz Rope," although that term is often reserved for the bundles that are up to two inches in diameter. The 660/46 wire is manufactured by Kerrigan-Lewis. They will sell to individuals, but have a 2 pound minimum. That's a lot of rope. You can find smaller pieces of this wire for sale on the web, but with a high price tag.

Then my experiments expanded to the next level. This was the result of an unexpected package that showed up in my mail box. I opened this offering to discover a 54 foot piece of the 660/46 Litz rope, plus a most interesting letter. My benefactor was Steve, AA7U. Steve's letter also mentioned the coil form that many of the crystal set guys use, a 4.5 inch OD coupler for large PVC pipe. I quickly purchased some. The part was found in the bins at Home Depot where it is their part number 405* and is marked "4 IN COUPLING HUBXHUB" and "(STYRENE)". The form cost was just over $1. Steve had done careful measurements of coils using this form and 660/46 Litz Rope.

I quickly wound the coil shown below.

The photo does not really tell us about coil length. This coil has the 44 turns spaced to occupy 3 inches of the 3.6 inch form length. The wire is held in place at the ends with holes in the form. Small cable ties are then used to keep the windings from moving. I wanted to preserve the integrity of the Rope for additional experiments, so did not want to apply any glue. I cut some small strips of Plexiglas with filed notches. These strips are then placed over a pair of holes drilled in the form so that three wire turns are covered. The cable tie is then inserted.

The results were outstanding; finally, Q>1000. The details with the EMRFD Q measuring scheme:
F=550 kHz Q=1199
1011 kHz Q=1375
1600 kHz Q= 973 Steve's results with a similar coil were 1276, 1426, and 1129 at approximately the same frequencies.

frequency. The tank is generally energized with a signal generator and measured with an oscilloscope or sensitive digital voltmeter. Other components often may include attenuators, SWR analyzers, Spectrum analyzers etc. Sources of error enter with coupling, loading, uncertainty as to the actual internal resistance of the source generator, and the need for several independent measurements which are then multiplied or divided together, adding additional error. All this works very well for the engineer with good bench practice, good equipment ($$), and the patience to perform the measurements several times to check repeatability. All these things plus the required anality coefficient the current author lacks.

In this paper I propose an alternative approach that takes an analytical look at the oscillating tank waveform and determines the coil Q from that. This technique dispenses with a majority of the equipment involved with traditional methods but does require a digital oscilloscope. In recent years the affordability of these scopes has greatly increased. If you have a digital scope, or access to one, or were looking for an excuse to purchase one, then read on, this technique may be for you.

Some Theory:

The inspiration for this technique is certainly not new. I first found the following simple circuit, (Figure 1) on page 22 of Bucher's 1919 "Wireless Experimenter's Manual". Having a good deal of experience capturing and evaluating damped radio oscillations for a Spark Gap transmitter, I immediately saw the utility of the circuit for a simple Q determination based on the damping decrement of the tank.

An Analytical Approach to the measurement of coil Q
http://www.lessmiths.com/~kjsmith/crystal/coilqm.shtml
Kevin Smith

Background:

Measuring the values of your radio components is an interesting and fun part of the Crystal Radio Hobby. Not only does it help you design better radios, but helps to understand the physics behind the wires and plates. The radio coil, one of the principal components is generally that part wound by hand. All other components, diode, variable capacitor are usually bought ready-made. The coil sits at the heart of the set and yet it remains devilishly difficult to measure and characterize. Even cheapo inductance meters, (I mean, my cheapo meter) do not properly measure the coil inductance at radio frequency. Coil quality, well that is a whole other matter. Quality, or the Q-factor as it is often called is a bench-intensive and measurement-intensive proposition that even the most dedicated radio fan may shy away from.

If you wish to know your coil Q-factor you have the choice of tracking down and paying a lot of money for an old HP or Boonton Q-Meter, or setting up a test bench and making the required measurements yourself. A number of techniques for measuring the Q of a coil have been published and excellent summaries can be found in "Q Factor Measurements on L-C Circuits" by Jacques Audet, and "Experiments with Coils and Q-Measurement" a web page by Wes Hayward.

All the techniques discussed involve variously setting up a parallel or series tank containing the coil to be tested and an accompanying capacitor to tune it to the needed measurement

Steve used a traditional 3 dB bandwidth measurement to establish Q, but with the coil tuned by the variable capacitor in a HP4342A Q meter. That Q meter stops at values of 1000.

Although I was pleased to have finally reached a Q goal, the measurement correlation was of equal significance to me. This data is further validation of the EMRFD Q measurement scheme. The Q values that are slightly lower than those that Steve obtained can be attributed to losses in the variable capacitor I used and to experimental error. These values are consistent with those reported in the crystalradio. eu web site listed above.

Recent examination of capacitor Q. (December 8, 2008.)

The earlier information regarding variable capacitor Q was confusing. That info, starting on page 8, has been modified to emphasize that my measurements are of total resonator Q. Although I have yet to do additional measurements, I did a few calculations that are illuminating.

We begin with analysis to derive a suitable formula. This is shown below in a MathCad work sheet.

Regarding the Q of an inductor where we measure just the resonator.

We may define the Q of a component as the ratio of the reactance at a frequency to the resistance, also measured at the same frequency. This is the usual model that is used.

$Q_i = \frac{X_i}{R_i}$ where the "i" subscript is L or C, indicating an inductor or capacitor. This becomes

$R_i = \frac{X_i}{Q_i}$ with i=L or C.

We now place the inductor and capacitor in series. They resonate at a frequency f with the usual resonance condition. We will evaluate Q values at this resonance, which is defined by:

$X_L = X_C$ $\quad \omega \cdot L = \frac{1}{\omega \cdot C}$ where $\omega = 2 \cdot \pi \cdot f$

The total resistance is $R = \frac{\omega \cdot L}{Q_L} + \frac{1}{Q_C \cdot \omega \cdot C}$

The Q of the resonator will merely be the value with this R, but with the assumption that all loss is within the inductor. Hence,

$$Q_{Res} = \frac{X_L}{R} = \frac{\omega \cdot L}{\left(\frac{\omega \cdot L}{Q_L} + \frac{1}{Q_C \cdot \omega \cdot C}\right)}$$

We divide top and bottom by ωL to obtain $Q_{Res} = \frac{1}{\frac{1}{Q_L} + \frac{1}{Q_C \cdot \omega^2 \cdot L \cdot C}}$

Because of resonace, this becomes

$Q_{Res} = \frac{1}{\frac{1}{Q_L} + \frac{1}{Q_C}}$ That is, Q values combine as resistors in parallel.

Assume that we know an inductor Q. The capacitor Q from a resonator measurement is then

$Q_c = \frac{Q_{Res} \cdot Q_L}{Q_L - Q_{Res}}$ Note that QL cannot equal resonator Q. Moreover we must have $Q_L > Q_{Res}$ to have a positive capacitor Q.

MathCad work sheet showing analysis.

Armed with the final formula in the above sheet, we can examine the capacitor Q data in more detail. We did this with Excel, beginning with a summary of the data and a "plot"

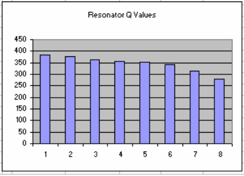
2Layer Bank	32/38	9.40	327	219
3Layer Bank	32/38	13.00	336	162
2Layer Bank	24	14.20	323	143
2Layer Bank	28	16.80	323	121
4Layer Bank	32/38	19.50	360	116
3Layer Bank	24	21.00	333	100
3Layer Bank	28	24.00	333	87
4Layer Bank	24	29.00	336	73
4Layer Bank	28	32.50	336	65
Double Layer	28	inf	355	0

showing the results. The plot is just a bar chart and serves only to illustrate that most of the results are close to each other. Only one resonator Q was below 300, the result of a single bearing variable capacitor. None of the resonator Q values exceeded 400. This is shown below.

Capacitor Q Determinations. L=38t #18 enameled wire on FT-114A-61 toroid. 8-Dec-08

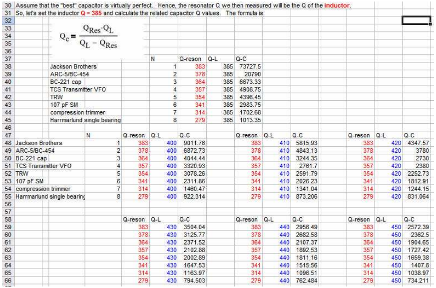
All C close to 100 pF. The data has been sorted with descending Q-resonator.

Capacitor	N	Measured Resonator Q
Jackson Brothers	1	383
ARC-5/BC-454	2	378
BC-221 cap	3	364
TCS Transmitter VFO	4	357
TRW	5	354
107 pF SM	6	341
compression trimmer	7	314
Harrmarlund single bearing	8	279



Sorted and plotted Capacitor impact on Resonator Q.

Having generated the bar chart, we proceeded to use the spread sheet to do some “what if” investigations. The first analysis assumed that out best capacitor was very good, so the highest resonator Q that we measured was almost the inductor Q value. If that really was the case, we can then calculate capacitor Q values. This case is at the top of the following page with QL=385.. After that, we assumed even higher inductor Q values and calculated the resulting capacitor Q results. See the next page.



30 Assume that the "best" capacitor is virtually perfect. Hence, the resonator Q we then measured will be the Q of the inductor.

31 So, let's set the inductor Q = 385 and calculate the related capacitor Q values. The formula is:

$$Q_c = \frac{Q_{Res} \cdot Q_L}{Q_L - Q_{Res}}$$

Row		N	Q-reson	Q-L	Q-C
37		N	Q-reson	Q-L	Q-C
38	Jackson Brothers	1	383	385	73727.5
39	ARC-5/BC-454	2	378	385	20790
40	BC-221 cap	3	364	385	6673.33
41	TCS Transmitter VFO	4	357	385	4908.75
42	TRW	5	354	385	4396.45
43	107 pF SM	6	341	385	2983.75
44	compression trimmer	7	314	385	1702.68
45	Harrmarlund single bearing	8	279	385	1013.35

Row		N	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C
47		N	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C
48	Jackson Brothers	1	383	400	9011.76	383	410	5815.93	383	420	4347.57
49	ARC-5/BC-454	2	378	400	6872.73	378	410	4843.13	378	420	3780
50	BC-221 cap	3	364	400	4044.44	364	410	3244.35	364	420	2730
51	TCS Transmitter VFO	4	357	400	3320.93	357	410	2761.7	357	420	2380
52	TRW	5	354	400	3078.26	354	410	2591.79	354	420	2252.73
53	107 pF SM	6	341	400	2311.86	341	410	2026.23	341	420	1812.91
54	compression trimmer	7	314	400	1460.47	314	410	1341.04	314	420	1244.15
55	Harrmarlund single bearing	8	279	400	922.314	279	410	873.206	279	420	831.064

Row	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C
58	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C	Q-reson	Q-L	Q-C
59	383	430	3504.04	383	440	2956.49	383	450	2572.39
60	378	430	3125.77	378	440	2682.58	378	450	2362.5
61	364	430	2371.52	364	440	2107.37	364	450	1904.65
62	357	430	2102.88	357	440	1892.53	357	450	1727.42
63	354	430	2002.89	354	440	1811.16	354	450	1659.38
64	341	430	1647.53	341	440	1515.56	341	450	1407.8
65	314	430	1163.97	314	440	1096.51	314	450	1038.97
66	279	430	794.503	279	440	762.484	279	450	734.211

Spreadsheet showing a variety of capacitor Q values resulting from assumed inductor Q (in blue) and the measured resonator Q values (in red.)

The results are interesting. If the best measured Q is indeed mostly that of the inductor, we see that most of the capacitor Q values are high with even the worst still over 1000. If we slowly allow the inductor Q to increase in our "thought experiment," the capacitor Q values begin to drop. However, the best of the lot are still several thousand. While none of this conjecture is really hard data, it certainly makes us feel more comfortable with inductor Q values close to 400 and that it is valid to assume high capacitor Q.

Examination of the central equation is revealing. It is clear that if the inductor Q becomes much higher, we will be able to

Spider	165/46	1.94	232	750
Basket weave	175/46	2.56	265	650
spider	660/46			641
Spider	100/45	2.41	238	620
Spider	100/45	2.44	241	620
Spider	100/45	1.73	149	540
spider	175/46	4.94	373	474
Loose Basket	32/38	5.30	317	376
Spider	40/44	4.16	248	375
Spider	40/44	4.16	225	340
Spider	40/44	2.93	154	330
Spider	32/38	7.50	331	277
Loose Basket	24.00	7.60	317	262
Basket weave	22.00	6.82	265	244
Loose Basket	28.00	8.50	317	234
Spider	24.00	9.50	327	216
Basket weave	32/38	10.40	332	201
Spider	28.00	10.60	327	194
Honeycomb	32/38	12.00	355	186
Spider	28.00	12.00	330	173
Basket weave	24.00	13.80	323	147
Basket weave	28.00	16.40	323	124
Honeycomb	24.00	18.50	347	118
Honeycomb	28.00	27.50	347	79
ferrite rod	50/46	2.58	250	610
toroid	22	3.93	240	384
toroid	18	3.20	195	383
toroid	22	8.20	449	344
toroid	175/46	4.82	244	318
toroid	50/46	5.23	248	298
ferrite rod	22		80	
ferrite rod	22	16.94	158	58.6

Radio-Frequency Resistance and Inductance

Qu = 2pifL / Rs

		1Mhz		
Coil Type	Wire	Rs	L	Q
		awg	ohm	uH
solenoid	660/46	0.90	200	1400
solenoid	660/46	0.91	200	1375
solenoid	175/46	2.88	265	579
solenoid	12	2.98	230	485
solenoid	14	3.28	230	440
solenoid	16	3.52	230	410
solenoid	18	4.01	230	360
solenoid	50/46	4.89	275	353
solenoid	32/38	6.20	327	331
solenoid	20	4.66	230	310
solenoid	16	7.60	319	264
solenoid	28	8.80	360	257
solenoid	22	5.78	230	250
solenoid	24	8.10	319	247
solenoid	28	10.10	360	224
solenoid	28	10.30	360	220
solenoid	50/46	10.84	378	219
solenoid	24	6.72	230	215
solenoid	28	9.60	327	214
solenoid	28	9.70	319	207
solenoid	26	8.03	230	180
solenoid	28		327	
Basket weave	660/46	1.03	186	1134
Spider	660/46	1.38	241	1000++
Basket weave	660/46	1.08	186	1082
Spider	660/46	0.90	150	1000+
spider	660/46			816

obtain much better data about the capacitors. The most recent inductor built has a Q that is three times higher than the one used for the capacitor difference. The next experiment is obvious and will be in the next update to this note.

Acknowledgements

Thanks go to my son Roger, KA7EXM, to colleagues Rick, KK7B, and Bob, N7FKI, for numerous discussions. Bob also provided some critical parts for my experiments. Special thanks go to Steve Ratzlaff, AA7U, for the crucial Litz wire that so greatly enhanced the experiments.

Do = Diameter of the finished cable over the strands in inches = 0.056
K = Constant depending on the number of strands = 2

Rac/Rdc = 1.0003 +2(660*0.0016 / 0.056)^2 * (0.0016*1000 / 10.44)^4 = 1.393

Therefore the AC Resistance of 660/46 litz wire is approximately:

The A.C. resistance is: 1.39 * 7.27 = 10.13 ohms/1000ft.

A 5inch diameter basket weave coil made from 660/46 Litz wire having an inductance of 230 uH will typically require some 40 turns or about 53ft of wire. At 10.13 Ohm/kft, that comes to an AC wire resistance of 0.53 ohms for the coil alone. Were all the losses represented by series resistance of the wire, the coil would have, at 1 MHz an unloaded Q = 2700! Naturally, wire resistance is not the only source of loss in the tank. There is a capacitor, metallic objects intruding into the magnetic field of the coil, diaelectric losses, eddy current and other losses. Its no wonder that the best coils just top out above Q = 1000 or so.

This is where I leave things. Time to get measuring. Below I provide my input data, have at it!

Kevin Smith

*Note that in 1925 the importance of coil Rs was understood, but the factor Q was apparently not used. I have taken (quite painfully) the L and Rs data from their plots and calculated the resulting Q. I recommend to those interested to download the pdf of their paper.

The following calculation is made to determine the AC resistance of the Litz wire used in a coil:
From different Litz wire manufacturing sites one can find tables and data allowing the calculation of the resistance of your favorite litz wire.

The formula for the D.C. resistance of any Litz construction is:

$$Rdc = Rs\,(1.0515)^{Nb} * (1.025)^{Nc} / Ns$$

Where:
Rdc = Resistance in Ohms/1000 ft.
Rs = Maximum D.C. resistance of the individual strands (4544 for 46awg wire)
Nb = Number of Bunching operations (assume = 2)
Nc = Number of Cabling operations (assume = 1)
Ns = Number of individual strands (assume = 660)

$$Rdc = 4544\,(1.015)^2\,(1.025)^1 / 660 = 7.27 \text{ ohms} / 1000 \text{ ft.}$$

The ratio of AC resistance to DC resistance of any Litz construction is:

$$Rac/Rdc = S + K\,(N\,Di / Do)^2 * G$$

Where:
S = Resistance ratio of individual strands when isolated (1.0003 for 46awg wire)
G = Eddy Current basis factor = (Di * sqrt(f) / 10.44)^4
F = Operating Frequency in HZ (assume 1MHz)
N = Number of strands in the cable = 660
Di = Diameter of the individual strands over the copper in inches = 0.0016

http://digital.library.unt.edu/ark:/67531/metadc66512/m2/1/high_res_d/metadc66512.pdf

DEPARTMENT OF COMMERCE
BUREAU OF STANDARDS
George K. Burgess, Director

TECHNOLOGIC PAPERS OF THE BUREAU OF STANDARDS, No. 298
[Part of Vol. 19]

RADIO-FREQUENCY RESISTANCE AND INDUCTANCE OF COILS USED IN BROADCAST RECEPTION

BY

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October 22, 1925

RADIO-FREQUENCY RESISTANCE AND INDUCTANCE OF COILS USED IN BROADCAST RECEPTION

By August Hund and H. B. De Groot

ABSTRACT

This paper gives experimental data on the radio-frequency resistance and inductance of certain "low loss" coils within the range of broadcast frequencies. The coils are of different shapes and wound with different kinds of wire. The results are plotted in graphs so that the reader can use them for selecting a coil for a desired purpose. For the data to apply it is necessary that the coil be constructed in accordance with the information given in a table. The dimensions are such that the coils are suitable for modern broadcast reception. A discussion of the important characteristics of coils is given. Of the coils measured the loose basket weave coil and the single-layer coil have the lowest radio-frequency resistance. Where a binder is required for holding the turns in position, collodion introduces the least amount of resistance. For the entire range of broadcast frequencies (500 to 1,500 kc) No. 32-38 litz has somewhat smaller resistance than No. 24 AWG d. c. c. wire; however, for most work No. 24 wire is suitable.

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I. INTRODUCTION

The purpose of this paper is to present data on the radio-frequency resistance and inductance of coils within the range of frequencies used in radiotelephone broadcasting. The coils are of different shapes and wound with different kinds of wire. The method of measurement used is indicated in Figure 1; it was described in detail by one of the authors [1] in 1924. The effective resistance R was found by direct comparison with a standard variable resistance R_s

[1] August Hund, "Measurements at radio frequency," Elect. World, 84, pp. 998-1000; Nov. 8, 1924.

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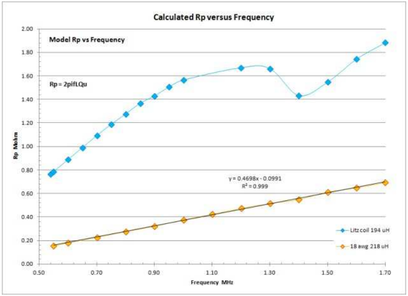
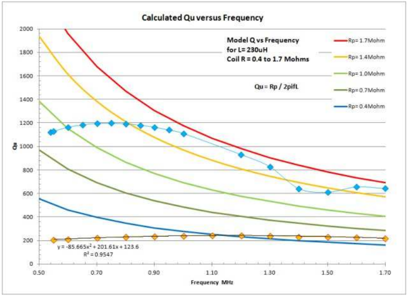
also includes Q vs f curves at constant Rp values. It is clear that, while tuning across the BCB spectrum, the Rp of the coil changes continuously.



Calculating the coil Rp from the data (Q:C:f), I have reproduced the above graph of the coil Rp vs frequency. It is evident that the trend follows a perfectly straight line for the low-Q coil. I would expect a similar straight-line relation for the litz coil but again things look strange. My impression here is that of a much steeper Rp vs f relation where the calibration on the Q-meter breaks down near 1MHz (Rp ~1.5 MOhm) and the data wanders until a second linear trend (with about the same slope) is re-established between 1.4 to 1.7MHz.

Now time for a few calculations, just how good can a coil Q get?

tuned a big litz coil with cheapo capacitors. Having an idea of your set's Ql will allow a better selection of the proper diode for matching.



The above plot is from a series of measurements on two of my coils kindly made for me by Steve Ratzlaff, AA7U. The blue diamonds are the measurements on a 660/46 Litz basket beast and the yellow diamonds are measurements on a modest 18awg tapped solenoid wound of a cardboard form. The data from the low Q coil form a lovely continuum across the BCB band while the litz coil data above 1MHz or so seem to be declining and erratic. Steve measured the coil a great many times and found good consistant results. The measurements are excellent but I must say, I am a bit suspicious that the high-Q coil may be at the limit of the HP Q-meter calibration, or perhaps 660/46 litz Q tops out at 0.7-0.8 MHz and crashes above that. (I have not yet found time to persue this). The plot

after the test coil and the condenser C had been tuned to resonance. The apparent inductance was calculated from the formula

$$L=\frac{25350}{f^2C}$$

where the inductance L is in microhenries, frequency f in kilocycles per second, and the measured resonance capacity C of the series condenser is in microfarads.

II. PROCEDURE

The method employed was to compare at radio-frequency coils of several types commonly used in receiving sets. The coils had been adjusted to the same self-inductance at a frequency of 1 kc per

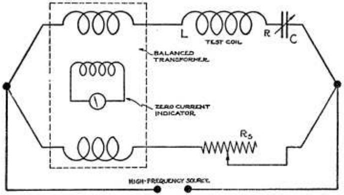


FIG. 1.—*Arrangement for the determination of radio-frequency resistance and inductance*

second. This represents the minimum value of the apparent inductance, larger values being obtained at the broadcast frequencies. The direct-current resistance was different for the various coils, since some shapes required more wire than others for the same inductance at 1 kc.

The quality of coils can be considered in terms of a number of different properties,[2] each of which is of importance in the use of

[2] It may help the reader to recall that the function of the coil is essentially to introduce inductance in the circuit, a given amount of inductance being introduced with the minimum possible length of wire and of resistance. On account of the capacity action of a coil the apparent inductance is usually much larger at broadcast frequencies, since the decrease due to skin effect action is small in comparison. The capacity action of a coil tends to transfer more or less energy across the turns than along them, thus turning the coil with increasing frequency gradually into a condenser with a resistance which is due to the insulation between the turns.

Technologic Papers of the Bureau of Standards, Vol. 19

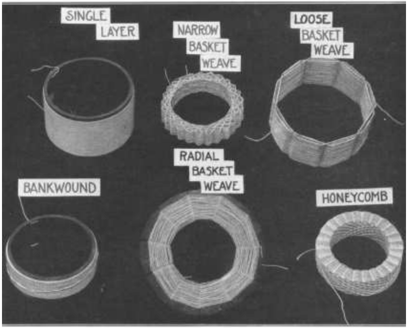
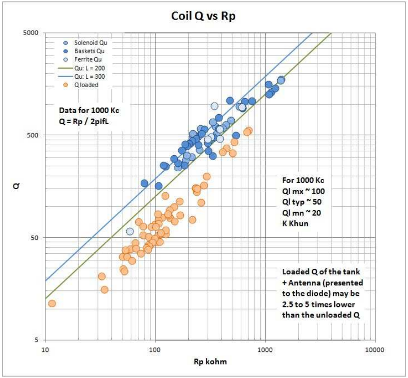


FIG. 2.—*Low-loss coils used for measurement*



On the plot I also post in orange circles my estimate of what the loaded Q presented to the diode (Antenna + Tank) might look like. Loaded Q will always be lower than the unloaded Q by several times. I have estimated that high-end sets (big litz, silver-plated ceramic insulated caps, best wiring practices) may lower the Q by about 2 1/2 times (B Tongue's performance set has Q = 700). At the low end (vintage components, small solid wire coils, taps) the load may lower Q by up to 5 times. Ken Khun in his excellent web book states that typical sets at 1Mhz have a loaded Q between 20 and 100, 50 typical and this is where most of the data falls. I scaled the divisor by Qu to produce the above plot but note that this is merely an estimate. Somewhere some bloke has no doubt cardboard form, sealed, of course. With various "open" coils, spiderweb, basket weave, diamond weave, etc. you may expect to double that.. possibly. The advantage in open coils derives from first the separation between adjacent wire turns which reduces self-capacitance in the coil, and secondly from the obvious lack of a form. All materials used in the coil will have some amount of dielectric losses associated with them and the less material used the better. Air core coils are best in this respect. I am not here to speak about ferrite coils as I have no experience with them. Frankly, they seem (to me) a bit like cheating.

The following graph presents data that I have scoured off the web. My primary sources include Hund and Groot, 1925*, Wes Hayward, Dave Schmarder, Ken Khun, Mike Tuggle, Steve Ratzlaff, and Dick Kleijer. 10,000 thanks for those who post their data on the web! The following plot gives the Q value as a function of coil Rs (series resistance. In striving for a high Q coil, in effect one is eliminating losses and lowering the series resistance as much as possible. There is more to it of course. The Q formula Qu = 2pifL / Rs tells us that Q is also a function of the coil inductance L and the frequency f of the measurement. All measurements chosen for plotting are made around 1 Mhz and most of the coils are in the L = 200 - 350 uH range. So, despite the formula, in this plot the coil Rp is the main driver. From the plot it should be apparent that winding a coil with Q = 200 or better should be a no-brainer. If your coil Q is less, you just aren't trying. At the high end, Q's > 1000 seem to be pretty extreme and these coils are expensive. You better be using big Litz 660/46 and use a basket design (although the two best coils on the plot were solenoids).

the coil. For use in the tuned circuits of a receiving set the important characteristics are:

(*a*) The radio-frequency resistance. A low value of resistance at the actual frequencies used is desirable.

(*b*) The magnitude of the ratio $\frac{L}{R}$ should not be unreasonably small compared with the value obtained at 1 kc. This ratio enters into the sharpness of resonance[3] which a circuit containing the coil would have.

It should be noted that L denotes here the apparent inductance, which is always larger at radio than at audio-frequencies. It increases very rapidly as it approaches the natural[4] frequency of the coil, but this corresponds to a frequency range within which the coil is of little practical value.[5]

(*c*) The percentage increase of radio-frequency resistance to the direct-current resistance. This value should not be unreasonably large.

(*d*) The percentage decrease of the ratio $\frac{L}{R}$ at radio-frequencies with respect to the value at audio-frequencies (1 kc). This value should not be unreasonably large.

(*e*) The apparent inductance. This should not be too large compared with the value at 1 kc, because the increase is mostly due to the coil capacity.

All five properties show certain merits of a coil, for which reason their variation with frequency are plotted in Figures 7 to 10. It is of importance that a coil have a radio-frequency resistance which is comparatively low as long as it does not require a shape and size of coil which is unusually bulky. The curve for properties *a* and *b* (figs. 8 and 10) are therefore of importance for rapid inspection of the results. The curves corresponding to *e* (fig. 7) give a means for evaluating the capacity of a coil.

III. DESCRIPTION OF TEST SAMPLES

The comparisons were carried out only for coils of so-called "low-loss" type and did not include shapes which are seldom used for radio work at broadcast frequencies. As an exception, an ordinary two-layer coil was measured in order to illustrate the unusual changes taking place in such a coil. The various shapes of test coils are shown in Figure 2.

The inductance of all coils was adjusted to 291 microhenries at 1 kc, which is of the order of magnitude common in receiving equip-

[3] Circular No. 74 of the Bureau of Standards, Radio Instruments and Measurements, p. 36.

[4] The apparent increase of L of a coil is mostly due to the capacity of the coil, which effect is, in general, much larger than the decrease of inductance due to a nonuniform current distribution in the conductor.

[5] There are several natural frequencies of a coil which do not bear definite harmonic ratios to each other.

ment for tuning to broadcast frequencies. For the sake of brevity the different shapes of coils are designated by capital letters, as is shown in Table 1. The kind of wire used is indicated by subscripts. Thus, D_{28} indicates a honeycomb coil using No. 28 (AWG) d. c. c.

TABLE 1.—*Key for the test coils, using no binder*

Kind of coil	Symbol	Kind of coil	Symbol
Single layer	A	Narrow basket weave	F
Radial basket weave on cardboard	B	Loose basket weave	T
Radial basket weave on hard rubber	C	Bank wound, two-layer	N
Honeycomb	D	Bank wound, three-layer	G
Two-layer	E	Bank wound, four-layer	H

wire, T_{16} indicates a loose basket weave using No. 16 (AWG) d. c. c. wire, and N_L a two-layer bank-wound coil using litz. All litz used was 32–38 d. c c. wire and corresponds roughly to the cross section of No. 23 AWG wire. In this wire 32 No. 38 AWG enameled copper wires are braided together. In order to have comparative tests on various binders, six single-layer coils were wound of the same size as A_{28} and coated with the materials indicated in Table 2, which gives the designations used.

TABLE 2.—*Key for test coils, using binder*

Binder used	Symbol	Binder used	Symbol
Shellac	K_{28}	Spar varnish	P_{28}
Commercial insulating varnish A	L_{28}	Collodion	Q_{28}
Paraffin	M_{28}	Commercial insulating varnish B	R_{28}

Detailed information on all coils is given in Table 3. The resistance at 1 kc is, for the coils used, practically equal to the direct-current resistance. For this reason the direct-current resistance is utilized for evaluating the ratio $\frac{L_o}{R_o}$ at 1 kc.

TABLE 3.—*Details of construction of test coils*

Coil		Wire AWG	Core	Dimensions of winding	Direct-current resistance R_o in ohms	$\frac{L_o}{R_o}$ at 1 kc in 10^{-4} henries in ohms	Remarks
Symbol	Type						
A_{28}	Single layer	No. 28 d. c. c.	Hard rubber	81 mm diameter, 31.5 mm long, about 55 turns.	3.15	9.24	
A_{24}	do	No. 24 d. c. c.	do	82 mm diameter, 45 mm long, about 60 turns.	1.44	20.2	
A_{16}	do	No. 16 d. c. c.	do	163 mm diameter, 67 mm long, about 40 turns.	.28	103.8	With about twice the diameter as A_{28} makes a coil of about the same proportions.

Coil Q:
http://www.lessmiths.com/~kjsmith/crystal/coilq.shtml
Kevin Smith

Introduction:

I have been building and studying crystal radios for some time now and slowly begin to learn a few things about these marvelous sets. In this section I begin my exploration of coil, coil quality and that mysterious dimensionless factor, Q... This page thus is a bit preliminary as I have not yet begun any measurements to determine the Q of my coils. Please bear with me.

My purpose here is to present some facts and data that has resulted from my explorations of the web. I have often wondered at what the quality factor of my coils should be and as often realized that I really do not have any expectations as to what is possible. Now, having dome some research, I can with some confidance say that this is solved. Coils as used in crystal radio broadcast band reception typically employ coils with Q factors ranging from 100+ (pretty lousy) through the several 100's (decent) and on up to 1000 or more for those remarkable Big-Litz wonders ($$$). Knowing the expected (or actual) Q of your coil is important in so far as it impacts the choice of diode to be used in the set. On my page of Diode Calibration I present a summary graphic which indicates how the diode Rd value relates to the tank parallel resistance Rp. This Rp in turn is a function of the coil Q. Schezzam! So here we are.

In your set construction, with a good effort and good engineering practice one can easily expect to wind a solenoid coil in the Q = 200 range without much trouble, even with a

TABLE 3.—*Details of construction of test coils*—Continued

Coil		Wire AWG	Core	Dimensions of winding	Direct-current resistance R_o in ohms	$\frac{L_o}{R_o}$ at 1 kc in 10^{-6} henries in ohms	Remarks
Symbol	Type						
A_L	Single layer	No. 32-38 d. s. c. litz.	Hard rubber	81 mm diameter, 59 mm long, about 65 turns.	1.25	23.27	
B_{28}	Radial basket weave.	No 28 d. c. c.	Cardboard 1.5 mm thick, 11 spokes, with slots 2 mm wide.	Inside diameter 65 mm.	3.08	9.45	
C_{28}	do	do	Hard rubber 3 mm thick, 11 spokes, with slots 1 mm wide.	Inside diameter 64 mm.	3.59	8.11	
C_{24}	do	No. 24 d. c. c.	do	do	1.65	17.62	
C_L	do	No. 32-38 d. s. c. litz.	Hard rubber 3 mm thick, 13 spokes, with slots 2.5 mm wide.	do	1.24	23.46	
D_{28}	Honeycomb	No. 28 d. c. c.	Air, and just enough collodion to hold coil together.	Inside diameter 57 mm; four diagonal retarded winding; 25 pins 2 mm in diameter.	3.31	8.79	Pins are set on inside diameter and coil builds up along radius.
D_{24}	do	No. 24 d. c. c.	do	Inside diameter 55 mm; otherwise as above.	1.35	21.55	Do.
D_L	do	No. 32-38 d. s. c. litz.	do	do	1.11	26.21	Do.
E_{28}	Double layer.	No. 28 d. c. c.	Hard rubber	81 mm inside diameter.	2.75	10.58	Very poor co...
F_{28}	Narrow basket weave.	do	Air, and just enough collodion to hold coil together.	Outside diameter 76 mm; 25 pins of 1.5 mm diameter; four diagonal retarded winding.	2.97	9.8	Pins are set on outside diameter and coil builds up along axis.
F_{24}	do	No. 24 d. c. c.	do	do	1.30	22.38	Do.
F_L	do	No. 32-38 d. s. c. litz.	do	do	1.07	27.2	Do.
G_{28}	3-layer bank wound.	No. 28 d. c. c.	Hard rubber, coil held together by collodion.	Inside diameter 81 mm; length of coil 10 mm.	2.70	10.78	
G_{24}	do	No. 24 d. c. c.	do	Inside diameter 81 mm; length of coil 14 mm.	1.17	24.87	
G_L	do	No. 32-38 d. s. c.	do	Inside diameter 81 mm; length of coil 17 mm.	.88	33.06	
H_{28}	4-layer bank wound.	No. 28 d. c. c.	do	Inside diameter 81 mm; length of coil 7 mm.	2.46	11.83	
H_{24}	do	No. 24 d. c. c.	do	Inside diameter 81 mm; length of coil 11 mm.	1.14	25.51	
H_L	do	No. 32-38 d. s. c. litz.	do	Inside diameter 81 mm; length of coil 12 mm.	.86	33.85	
K_{28}	Single layer.	No. 28 d. c. c.	Hard rubber, using shellac as a binder.	Same as A_{28}	3.11	9.36	Used for binder test.
L_{28}	do	do	Hard rubber, using commercial insulating varnish A as binder.	do	3.20	9.09	Do.
M_{28}	do	do	Hard rubber, using paraffin as a binder.	do	3.14	9.27	Do.

TABLE 3.—*Details of construction of test coils*—Continued

Coil		Wire AWG	Core	Dimensions of winding	Direct-current resistance R_0 in ohms	$\frac{L_0}{R_0}$ at 1 kc in 10^{-6} henries in ohms	Remarks
Symbol	Type						
N_{28}	2-layer bank wound.	No. 28 d. c. c.	Hard rubber, coil held together by collodion.	Inside diameter 81 mm.	2.65	10.975	
N_{24}	do	No. 24 d. c. c.	do	Inside diameter 81 mm; length of coil 20 mm.	1.25	23.27	
N_L	do	No. 32–38 d. s. c. litz.	do	Inside diameter 81 mm; length of coil 25 mm.	.95	30.63	
P_{28}	Single layer.	No. 28 d. c. c.	Hard rubber, using spar varnish as a binder.	Same as A_{28}	3.14	9.27	Used for binder test.
Q_{28}	do	do	Hard rubber, using collodion as a binder.	do	3.11	9.36	Do.
R_{28}	do	do	Hard rubber, using commercial insulating varnish B as a binder.	do	3.21	9.07	Do.
T_{28}	Loose basket weave.	do	Air, and just enough collodion to hold coil together.	9 diagonal alternating winding; 9 pairs of pins set on a circle of 92.5 mm diameter; diameter of pins 2.5 mm; and small spacing between a pair of pins 10 mm.	3.19	9.13	
T_{24}	do	No. 24 d. c. c.	do	do	1.42	20.5	
T_L	do	No. 32–38 d. s. c.	do	do	1.21	24.05	

Figures 3 to 6 show how coils of these types are wound. Figure 3 illustrates the method used for winding a "three-layer bank wound"

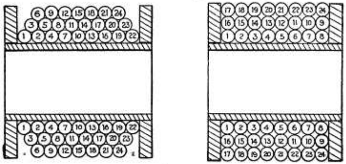


FIG. 3

Three-layer bank winding

Ordinary three-layer winding (useless for currents of the broadcasting range of frequency)

7. All the insulating materials which were used as binders caused very slight increases in the resistance of the coils. Collodion seems best, and also has the inherent advantage of drying rapidly after application to the coil. This is of especial advantage in the construction of a bank-wound coil.

WASHINGTON, May 25, 1925.

V. SUMMARY

1. The various experimentally obtained curves given in this paper can be used as design bases for comparing coils of six types for any frequency in the broadcast range. For these data to apply it is necessary that the coils be constructed in accordance with the information given in Table 3. The coil dimensions are such that the coils are applicable to modern broadcast reception. A statement of the important characteristics of coils is given in the introduction.

2. The curves shown in Figures 9 and 11 give the changes of resistance and of $\frac{L}{R}$ with frequency. High values of these ratios do not in all cases correspond to high values of radio-frequency resistance. In some cases, for instance, a particular coil has a relatively high value for $\frac{\Delta R}{R_o}$, although the actual radio-frequency resistance is not large, because the direct current resistance is comparatively low.

3. The curves of Figures 8 and 10 give the actual radio-frequency resistance and ratio of inductance to resistance at various frequencies.

4. Of the coils measured the loose basket weave coil and the single-layer coil, and next to them the radial basket weave coil wound on hard rubber, have the lowest radio-frequency resistance. The four-layer bank-wound coil and the honeycomb winding have the highest resistance. This can not, however, be generalized to other frequency ranges. For instance, for low-frequency sets (20 to 100 kc) the multilayer bank-wound coil and the honeycomb coil have relatively low resistance, and besides are good coils mechanically while the loose basket weave coil has no special advantage and the single-layer coil can not be used on account of excessive size.

5. There appears to be little reduction of resistance at the lower frequencies in spacing the turns, so that the advantage of getting a smaller resistance is small compared with the disadvantage of requiring a coil twice as long.

6. The use of Nos. 32–38 litz gives coils of somewhat lower resistance than coils wound with solid wire of the same cross section. No. 24 AWG solid wire has less resistance than No. 28 wire, and No. 16 wire for a certain range has less resistance than Nos. 24 and 28 wire. If solid wire is used it does not appear necessary to use wire larger than No. 24 AWG. This conclusion can not, of course, be extended outside the broadcast frequency range; for instance, No. 16 solid wire would be better for frequencies above 5,000 kc.

coil in comparison with an ordinary three-layer coil. Figure 4 illustrates the method used in making the "narrow basket weave" coil, in which the coil is built up along the pins, which are removed after

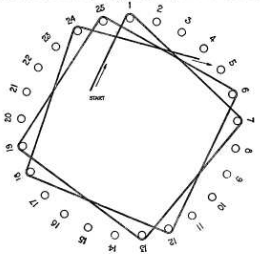


Fig. 4.—*Narrow basket weave. Four diagonal retarded winding (using 25 pins set on a circle. Coil builds up along axis)*

the coil is finished. Figure 5 illustrates the method used in winding the " loose basket weave" coil, which requires a pair of pins for each corner. A spacing of 10 mm between the two pins of a pair was

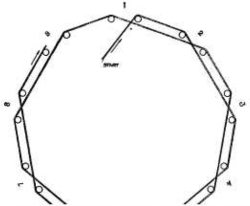


Fig. 5.—*Loose basket weave. Nine diagonal alternating winding (using nine pairs of pins set on a circle. Coil builds up along the axis)*

56961°—25——2

used. Figure 6 shows the method used in making the "honeycomb" coil. The zigzag winding is illustrated by the view of the entire cylinder surface. The winding is built up along the radii of the coil.

IV. RESULTS

The results are exhibited in graphical form. Figure 7 gives curves for the apparent self-inductance of the various test coils as a function of frequency. As on all curve sheets, the dotted curves indicate the different coils using No. 32–38 litz. It is seen that the "loose basket weave" type of coil and the "single-layer" coils give the lowest apparent inductance over the entire range of broadcast frequencies (500 to 1,500 kc). This indicates that the coil capacity is comparatively low, while the ordinary two-layer coil (E_{28}) acts more or less like a condenser, since the coil capacity is exceedingly large.

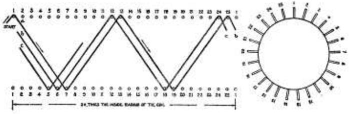


FIG. 6

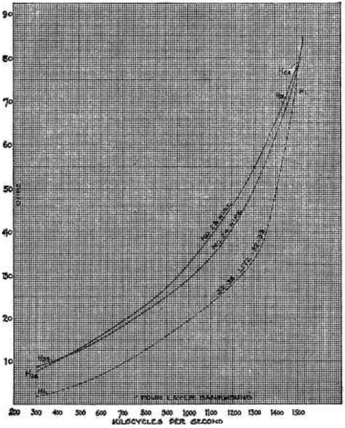
Honeycomb coil. Four diagonal retarded winding (using 25 pins. Coil builds up along radius) *Cross-section of winding frame with the 25 pins*

Figure 8 gives the curves for the radio-frequency resistance. It is seen again that the values of the resistance vary greatly. Naturally, the ordinary two-layer coil (E_{28}) has very large radio-frequency resistance within the broadcasting range. Though its direct-current resistance is only 2.75 ohms, the effective resistance at 500 kc is 162 ohms, at 580 kc it is 465 ohms, and at 748 kc it is 1,800 ohms. The resistance increases very rapidly until it is mostly due to the dielectric resistance across the insulation between the layers. A three-layer coil wound in the ordinary way (ordinary multi-layer coil) and adjusted, like all coils discussed in this paper, to 291 microhenries at 1 kc has a radio-frequency resistance of several thousand ohms at a frequency as low as 400 kc.

Figure 9 gives the curves for the ratio of the increase of the radio-frequency resistance over its direct-current value to the direct-current value. It is seen that the loose basket weave and the single-layer coils have low values; next comes the radial basket weave coil,

TABLE 5.—*Variation with frequency of radio-frequency resistance (in ohms) for different binders*

Frequency in kilocycles	A_{28}, no binder	Q_{28}, collodion	R_{28}, commercial insulating varnish B	L_{28}, commercial insulating varnish A	K_{28} shellac	M_{28} paraffin
300	3.9	3.8	3.9	3.8	3.7	3.9
400	4.4	4.3	4.4	4.3	4.1	4.6
500	5.1	4.9	5.0	4.8	4.8	5.2
600	5.9	5.5	5.7	5.3	5.6	6.0
700	6.7	6.3	6.6	6.0	6.6	6.8
800	7.6	7.1	7.5	6.8	7.6	7.8
900	8.5	8.0	8.5	7.7	8.8	9.0
1,000	9.4	8.9	9.6	8.8	10.1	10.3
1,100	10.3	10.0	10.8	10.1	11.4	11.8
1,200	11.3	11.0	12.0	11.5	12.8	13.3
1,300	12.2	12.1	13.3	13.1	14.2	15.0
1,400	13.2	13.2	14.6	14.7	15.7	16.7
1,500	14.2	14.4	15.9	16.4	17.2	18.5



some cases the effective resistance of a coil using a binder came out smaller by a fraction of an ohm than for the coil using no binder. The binder seemed to make an accurate resistance adjustment more difficult.

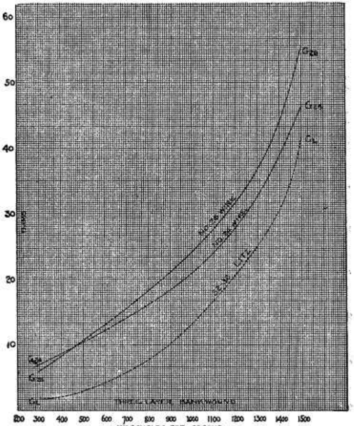


FIG. 18.—*Radio-frequency resistance of three-layer bank-wound coils*

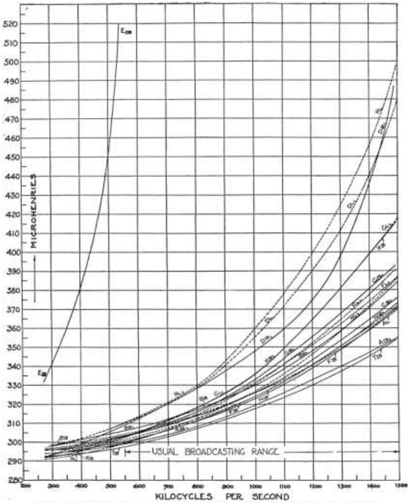


FIG. 7.—*Apparent inductance at different frequencies (minimum inductance is 291 microhenries for all coils)*

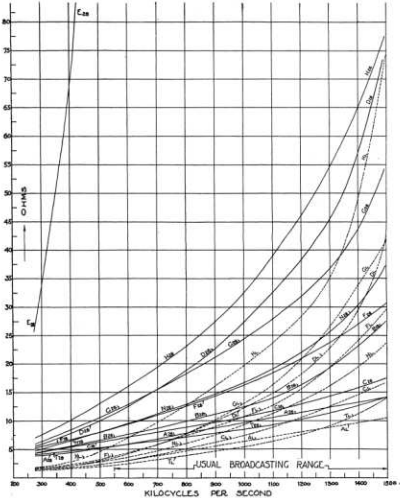


FIG. 8.—*Radio-frequency resistance at different frequencies*

3. EFFECT OF BINDER ON THE EFFICIENCY OF COILS

The binders used in this test were each applied to a single-layer coil, and just enough of it to cover the entire surface. The varying nature of the different binders probably resulted in different thick-

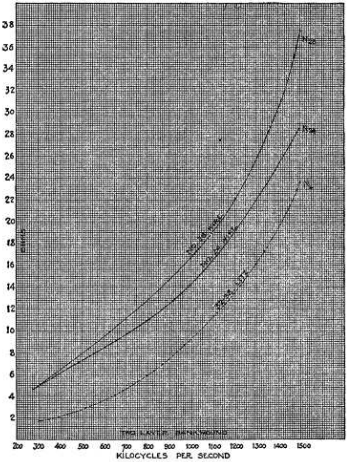


FIG. 17.—*Radio-frequency resistance of two-layer bank-wound coils*

nesses. The coils in every case were dried thoroughly. The measurements were a little difficult when a binder was used, even though great care was taken to have the binder dried. For instance, in

Table 4 shows that it is not so serious to have a few strands broken, since the radio-frequency current apparently finds its way back

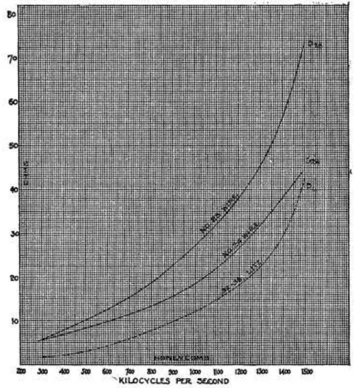


FIG. 16.—*Radio-frequency resistance of honeycomb coils*

across the strands. Even if, say, six strands are broken, the radio-frequency resistance is 3.4 ohms as compared with 3.1 ohms for perfect strands.

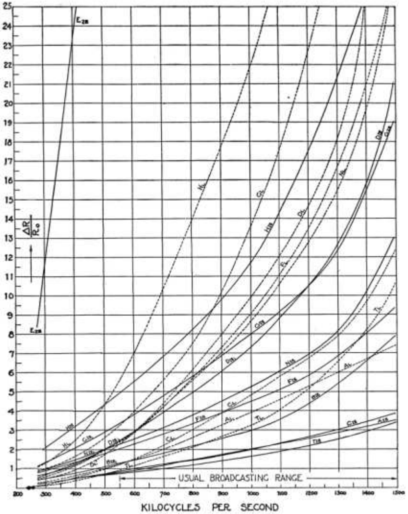


FIG. 9.—*Ratio of the increase of resistance over its direct-current value to the direct-current value*

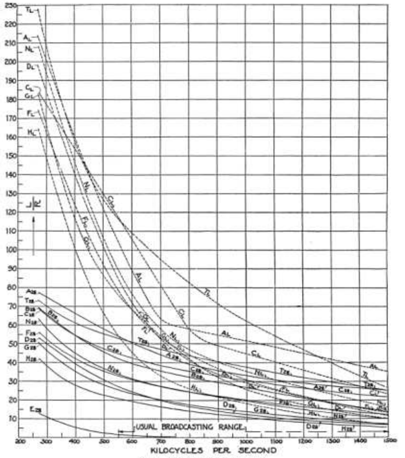
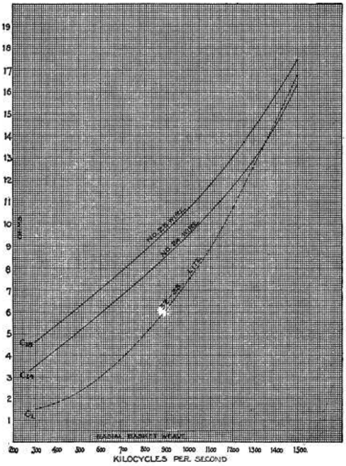


FIG. 10.—*Ratio of inductance to resistance*

TABLE 4.—*Resistance of litz wire Nos. 32–38, at 750 kc*

Broken strands	Resistance	Broken strands	Resistance	Broken strands	Resistance	Broken strands	Resistance
	Ohms		*Ohms*		*Ohms*		*Ohms*
0	3.1	8	3.6	16	5.4	24	9.5
1	3.2	9	3.8	17	5.6	25	10.8
2	3.2	10	3.8	18	6.1	26	13.5
3	3.3	11	4.2	19	6.4	27	14.4
4	3.3	12	4.4	20	7.4	28	16.5
5	3.3	13	4.4	21	7.6	29	21.7
6	3.4	14	4.4	22	7.8	30	42.4
7	3.5	15	4.7	23	8.4	31	51.6

Litz used is Nos. 32-38 and has 32 strands of No. 38 AWG enameled wire braided together.



convenient use in receiving sets. Figure 12 shows a single-layer coil the turns of which are spaced by a distance equal to the diameter of the wire. There seems to be no material reduction in resistance except at the higher frequencies.

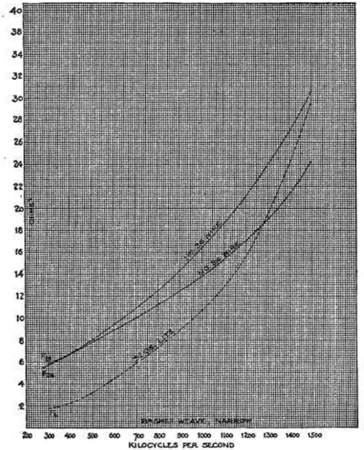


FIG. 14.—*Radio-frequency resistance of narrow basket-weave coils*

2. EFFECT OF BROKEN STRANDS IN LITZ WIRE

The question is often asked concerning litz wire as to the effect on the resistance of some of the strands being broken or not connected at the respective ends.

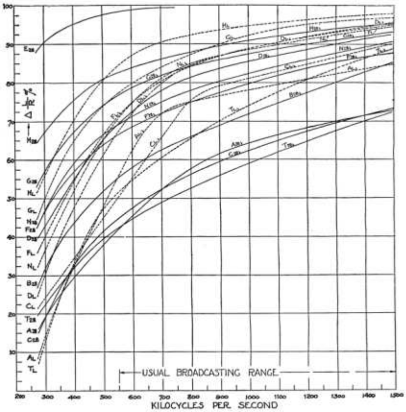
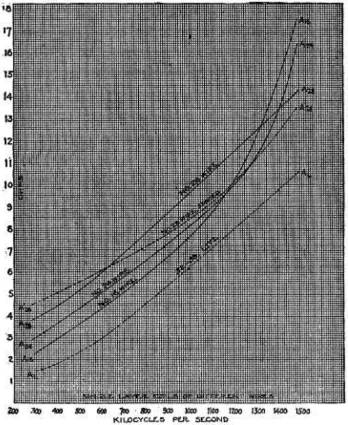


FIG. 11.—*Percentage decrease in the ratio of inductance to resistance below the value at 1 kc*

using hard rubber for the core, while the four-layer bank wound, three-layer bank wound, and the honeycomb coils show considerable percentage increases.

Figure 10 gives the curves for the ratio $\frac{L}{R}$. It is always smaller than the ratio $\frac{L_o}{R_o}$ at 1 kc. Since the apparent inductance and the radio-frequency resistance increase with the frequency, this ratio in certain cases has only small changes although both L and R are changing rapidly. Nevertheless, these curves show about the same comparisons as the resistance curves.

Figure 11 gives the curves for the percentage decrease in the ratio $\frac{L}{R}$ at different frequencies.



1. EFFECT OF THE KIND OF WIRE ON THE RESISTANCE OF A COIL

In order to show the effect of the size of the wire, curves for the resistance for No. 28, No. 24, and No. 16 d. c. c. are plotted and compared among themselves and against No. 32–38 litz. These comparisons are shown in Figures 12 to 19.

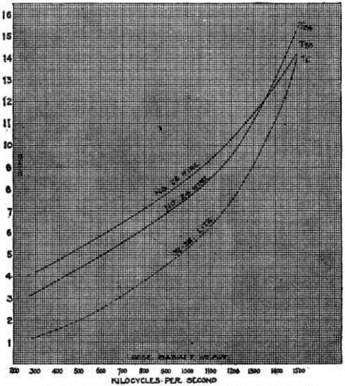


FIG. 13.—*Radio-frequency resistance of loose basket-weave coils*

It will be noted that in all cases litz, which corresponds roughly to No. 23 solid wire as regards its cross section, had the lowest effective resistance. If solid wire is used, it appears unnecessary to use wire larger than No. 24, although No. 16 gives, for the lower frequencies, resistances which are slightly lower. Such a large size of wire would, however, make the winding of certain types of coils more difficult and the size of the finished coil too large for