KEVIN'S WEBSURFER HANDBOOK VIII
FOR CRYSTAL RADIO

RESONANCE and
RESONANT CIRCUITS

Kevin Smith
2012

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KJ Smith

NOTES:

INTRODUCTION

This handbook explores radio in general and crystal radio from the standpoint of RF Resonance. I again begin things with a brief walk down memory lane with an introduction to the subject of resonance and coupled circuits by Dr Hoag written in 1942. This represents an early, but already well-developed notion on the theory and application of resonance to radio apparatus.

The next few articles explore general theory concerning resonance with a strong emphasis on the mathematical base in order to help the reader better apply these concepts in theory own work, especially with respect to setting up excel spreadsheets. Note that while I like the pretty plots from SPICE simulations, I see this as a black box because authors almost never present the equations that make the plots possible, they are canned inside the program, bad form. I feel that, if I can plot it myself, I must be understanding it pretty well.

Then included are general discussion articles on resonance and coupled circuits. Most crystal radios of any pretentions to good performance will be coupled with separate antenna tuning. These articles should help explore this aspect of your radio!

I finally follow with an article on impedance matching and resonance as these are closely-related and need to be thought of together when designing, or at least understanding your radio.

Of general note, the web is a marvelous source of data and information. Many long-time crystal set builders, and many others have created dedicated sites to disseminate information and resources, to share their creations and knowledge. I am eternally in your debt. All of the material in this handbook is copyright for which I have not sought permission. Therefore this is not presented for publication or copy. It is only my personal resource. I encourage anyone finding this copy to pursue ON THE WEB the web pages identified within. I include the name of the author and web address of each section. I wish to sincerely thank every author presented for their excellent pages and ask forgiveness for my editing into this handbook.

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NOTES:

as shown in Fig. 24*b*. It will be noted that the use of link coupling has the effect of reducing the equivalent primary and secondary inductance, and hence of raising the resonant frequencies. In other respects, the use of link coupling gives the same behavior as obtainable from ordinary inductive coupling.

6. Coupled Circuits with Resonant Primary and Secondary Tuned to Slightly Different Frequencies.—When two circuits resonant at slightly different frequencies are coupled together, the behavior depends upon the coefficient of coupling and the relative and absolute circuit Q's. When $Q_p = Q_s$, the response curve of secondary current (or voltage) has a shape of the same type as would be obtained if the two circuits were both resonant at the same frequency and coupled with an increased coefficient of coupling. In other words, the effect of detuning the circuits for the case of equal Q's is equivalent, as far as shape is concerned, to increasing the effective coupling. The only difference is that detuning causes the absolute magnitude of the response curve to be less than when the same shape is obtained without detuning.

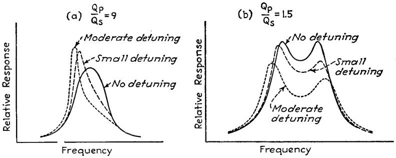

FIG. 25.—Curves illustrating the effect produced on the shape of the response curve by detuning primary and secondary circuits when the primary and secondary circuits do not have identical Q's.

Analysis shows that when $Q_p = Q_s$ the shape with detuning is as though the circuits were both tuned to the same frequency and coupled by an amount[1]

$$\left.\begin{matrix}\text{Equivalent coupling}\\ \text{corresponding to}\\ \text{detuned operation}\end{matrix}\right\} = \sqrt{k^2 + \left(\frac{\Delta}{f_0}\right)^2} \qquad (43)$$

where k = actual coefficient of coupling.
Δ = difference between resonant frequency of primary and secondary circuits.
f_0 = frequency midway between primary and secondary resonant frequencies.

In the more general case where the circuit Q's are not the same, the secondary response curve is no longer symmetrical about the mean frequency, as shown in Fig. 25. In cases where unsymmetrical peaks are obtained, the low-frequency peak will be depressed when the secondary is tuned to a higher frequency than the primary and the secondary Q is higher, or when the secondary is tuned to a lower frequency and the secondary Q is the lower. Otherwise the high frequency peak will be depressed.

Detuning produces much more effect upon the shape of the primary current curve than upon the secondary response, and if the detuning is at all large the primary

[1] See Harold A. Wheeler and J. Kelly Johnson, High Fidelity Receivers with Expanding Selectors, *Proc. I.R.E.*, Vol. 23, p. 594, June, 1935; or C. B. Aiken, *loc. cit.*
Equation (43) involves the assumption that Δ/f_0 is small compared with unity.

CONTENTS:

circuit, as shown in Fig. 17. In many cases where coupled resonant circuits are employed, the excitation of the primary circuit is obtained by applying the voltage in parallel with the primary circuit through a resistance, as shown in Fig. 23*a*. This arrangement can be termed *parallel excitation*, and corresponds to the case when the coupled circuits are excited from a vacuum tube.

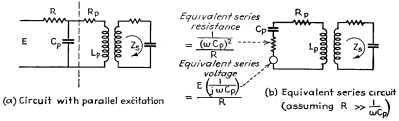

FIG. 23.—Actual and equivalent circuits for the case of parallel excitation of two coupled circuits.

The parallel excited circuit of Fig. 23*a* can be reduced by Thévenin's theorem to the equivalent arrangement shown in Fig. 23*b*. This is accomplished by considering that the portion of the circuit to the left of the dotted line is the equivalent generator circuit supplying the remainder of the circuit. The principal effect of parallel excitation is to introduce an added resistance in series with the primary circuit that becomes greater the lower the series resistance R used in the parallel feed. The equivalent

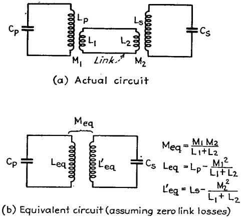

FIG. 24.—Actual and equivalent circuits for link coupling.

series voltage also varies somewhat with frequency, but in a limited range, such as the region in which the resonance phenomena occur, can be considered as being substantially constant.

Link-coupled Circuits.—Two resonant circuits are often coupled together by a link circuit, as shown in Fig. 24*a*. When this arrangement is reduced to an inductively coupled circuit by the method of Par. 8, the result for negligible losses in the link is

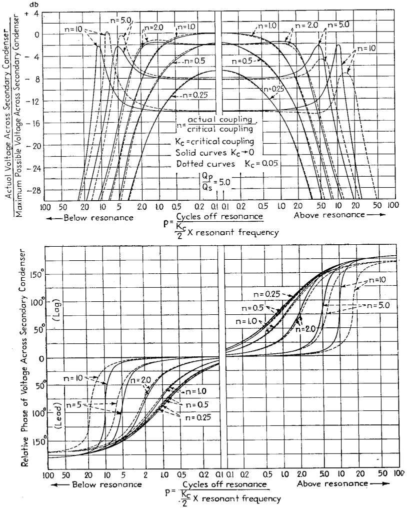

FIG. 22b.—Universal curves giving the phase and relative magnitude of the voltage across the secondary condenser for the case of two coupled circuits resonant at the same frequency and having a Q ratio of 5.

Parallel Feed in the Primary Circuit.—The analysis and discussion given above have all been for the case where the voltage was applied in series with the primary

BASIC RADIO

The Essentials of

ELECTRON TUBES AND THEIR CIRCUITS

By

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CHAPTER 6

RESONANT CIRCUITS

6.1 Series Resonance. When two bodies, or two currents, or two voltages, oscillate at the same frequency they are said to be in resonance.

A series *LCR* circuit can be built which resonates to a generator of given frequency. Then the current which flows through the circuit will have its greatest effective value. This is accomplished by choosing the inductive and capacitive reactances equal to each other. From the equations for the reactances, it is easily proven that the resonant frequency is given by

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \times 10^6}$$

where f_r is in cycles per second, $2\pi = 6.28$, $10^6 = 1$ million, L is the inductance in microhenries (μh), and C is the capacitance in micro-microfarads ($\mu\mu$f). This is one of the most important equations in radio. Note that the resistance of the circuit does not appear in the equation. At resonance, the only opposition to the flow of the current is that due to the resistance, i.e., $I_r = E/R$ and not $I = E/Z$.

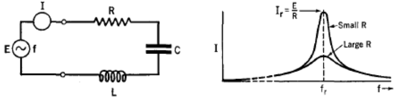


FIG. 6 A. A series circuit and its resonance curve

The current in the "tuned" or resonant series circuit is indicated at the peak of the "resonance" curve of Fig. 6 A. Note that if the total resistance of the circuit is large, the curve is broader and flatter, and vice versa.

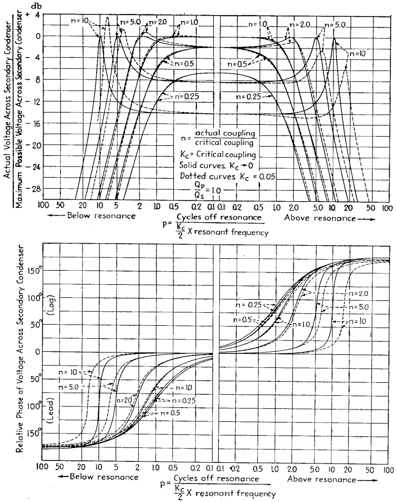


FIG. 22a.—Universal curves giving the phase and relative magnitude of the voltage across the secondary condenser for the case of two coupled circuits resonant at the same frequency and having a Q ratio of unity.

many cycles as do the peaks, this can be readily obtained by neglecting the circuit resistance and using Eq. (34).

When $Q_p = Q_s$, the value of E_c/E approximates the maximum possible response that can be obtained, as given by Eq. (38). On the other hand, when $Q_p \neq Q_s$, the peaks always have a height less than the maximum possible response in the secondary.

The ratio of the mean of the responses at the two peaks to the response at resonance is plotted in Fig. 21 for a number of Q ratios.

Universal Response Curves.—Equations (33) and (38) can be combined to give

$$\frac{\left\{\begin{matrix}\text{Actual voltage across} \\ \text{secondary condenser}\end{matrix}\right\}}{\left\{\begin{matrix}\text{Maximum possible volt-} \\ \text{age across secondary} \\ \text{condenser with} \\ \text{optimum coupling}\end{matrix}\right\}} = \frac{2n}{\left(1 + \frac{pk_c}{2}\right)^2 \left[(n^2 + 1) - (mp)^2 + j(mp)\left(\sqrt{\frac{Q_p}{Q_s}} + \sqrt{\frac{Q_s}{Q_p}}\right)\right]} \tag{42a}$$

where $n = \frac{k}{k_c} = \frac{\text{actual coefficient of coupling}}{\text{critical coefficient of coupling}}$.

$$p = \frac{\text{cycles off resonance}}{\left(\frac{k_c}{2}\right) \times \text{resonant frequency}}.$$

$$m = \frac{1 + \frac{pk_c}{4}}{\left(1 + \frac{pk_c}{2}\right)^2}.$$

Universal curves based on Eq. (42*a*) are given in Figs. 22*a* and 22*b* for $k_c = 0$, and $k_c = 0.05$. These curves, especially when considered in conjunction with Figs. 18, 19, and 21, provide a complete and accurate picture of the response that can be expected when two resonance circuits are tuned to the same frequency and coupled together.

Figure 22 shows that when both primary and secondary circuits have the same Q, the response curve separates into double peaks whenever the coefficient of coupling exceeds the critical value, and these peaks maintain a substantially constant height irrespective of their location. On the other hand, when the two circuits have different Q's, then if the coupling exceeds the critical value by only a small amount, the response curve still has a single peak, but with reduced response at resonance. Double humps do not occur until the coupling exceeds the critical value by an amount that depends upon the ratio Q_p/Q_s, and the peaks become slightly lower as the coupling increases.

Practical Calculation of Coupled Systems Involving Two Resonant Circuits Tuned to the Same Frequency.—When it is desired to obtain the response curve of a given system, the relative shape of the curve of voltage developed across the secondary condenser can be determined with an accuracy sufficient for all ordinary purposes by the use of the universal curves of Fig. 22, interpolating between these in accordance with the Q ratio involved. The absolute magnitude of the curve can then be obtained by calculating the response at the resonant frequency with the aid of Eq. (35).

An alternative procedure for the case where the curve shows two peaks is to calculate the response at resonance by Eq. (35), determine the location and height of the peaks by Figs. 19 and 21, or Eqs. (40) and (42), and then to note that the response beyond the peaks falls to the response at resonance when the frequency is $\sqrt{2}$ times as far from resonance as the coupling peaks. This information is sufficient to define the shape of the double peaked curve in the vicinity of resonance. If quantitative information is desired at frequencies differing from resonance by at least twice as

6.2 Parallel Resonant Circuits. Fig. 6 B shows a parallel circuit and its impedance curves. The curves are of the same shape as the current curves of a series circuit. Note that the impedance across the terminals of a parallel circuit is a maximum at resonance,[1] whereas it is a mini-

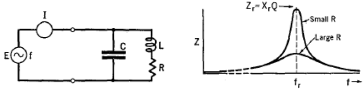


Fig. 6 B. A parallel circuit and its impedance curve

mum for the series circuit. The current in the LCR circuit itself is often very large at resonance and can cause considerable heating.

The impedance of a parallel circuit at resonance is given by

$$Z_r = \frac{(2\pi f_r L)^2}{R} = (2\pi f_r L)Q = X_r Q = \frac{L}{RC}$$

if all the resistance is in the coil (which is very nearly true in practice). The meaning of Q is given in Section 6.4. The resonant frequency f_r is given by the same equation as for a series circuit provided R is small in comparison with X. This is often true in radio frequency circuits.

6.3 Selectivity. The sharpness of the resonance curves is greater when the resistances are smaller. Sharpness of tuning is also called "selectivity." It shows the ability of the circuit to discriminate between voltages of different frequencies.

6.4 Q of a Circuit. As current surges back and forth in a resonant circuit, the electrical energy is alternately shifted back and forth between the magnetic field of the coil and the electric field of the condenser, losing a certain proportion of the total into the resistance at each alternation. In order to maintain oscillations, the generator must supply a small amount of energy each cycle. This is analogous to the small push we give to a swing; only a little push is needed at the end of each swing to keep it going. Electrically, the interest lies in the comparison between the total energy in the circuit to that which is dissipated (and must be re-supplied by the generator). This ratio is called the "Q" (quality) of the circuit. In practice, nearly all of a circuit's resistance is in the coil. It may then be shown that

[1] There are really three different ways of defining resonance in the case of a parallel circuit, but, if the resistance is low, the resonant frequency in all three cases is practically identical.

$$\boxed{Q = \frac{2\pi fL}{R} = \frac{X_L}{R}}$$

Practical radio frequency coils have Q's from 50 to 200 and occasionally 500. Audio frequency coils range from 1 to 10 and occasionally are as low as 1/2. The higher the Q, the better.

The rate at which current dies down in a resonant circuit, after the source has been disconnected, is called the damping of the circuit, whose measure is called "the decrement" δ. $Q = \pi/\delta$. A high Q circuit is lightly damped, has a small decrement, a sharp resonance peak, and a high selectivity.

6.5 The L to C Ratio. In order that a circuit shall resonate to a given frequency f_r, the product LC must have a definite value given by $LC = 1/(2\pi f_r)^2$. This does not tell us, however, whether the product shall be made up of a large L and a small C or vice versa. Within limits, an increase in the number of turns of wire in a coil increases its reactance faster than its resistance. Hence, for circuits alone, or for those connected to high load resistances, such as a vacuum tube, the coils should be made with a relatively large inductance, i.e., the L/C ratio should be large.

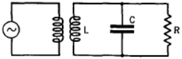


FIG. 6 C. If R is small, the LC circuit will be heavily loaded and will have a low Q. Making the ratio L/C small helps to keep Q large

When the load on a resonant circuit (R in Fig. 6 C) is small, say only a few thousand ohms, as in transmitters and induction heaters, a majority of the energy loss takes place in the load. The coil's resistance plays only a negligible role. In this case, it can be shown that L should be comparatively small and C large if Q is to be satisfactorily high.

6.6 Resonant Voltages. At resonance, the voltage across the inductance is numerically equal to that across the condenser, although they are always of opposite polarity. This voltage, at resonance, is equal to Q times the voltage in series with the circuit. The condenser must be built with an insulator which will not be punctured by this voltage.

6.7 Crystal Resonators. A properly cut and ground piece of quartz, located between two metal plates and placed in series with a high frequency generator, has a natural vibrational period dependent largely upon its thickness. It is equivalent to a series resonant circuit of very high Q.

Furthermore, in the very common case when the coefficient of coupling is small, Eq. (41a) becomes

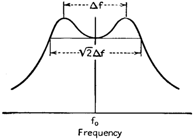


FIG. 20.—Relationship between band width, and width between secondary peaks existing when two circuits resonant at the same frequency are coupled together.

$$\frac{\text{Frequency at peak of secondary voltage}}{\text{Resonant frequency of tuned circuits}} = 1 \pm \frac{k}{2} \qquad (41b)$$

In practical work, one is commonly interested in the frequency band over which the response in the secondary circuit equals or exceeds the response at resonance. This band is illustrated schematically in Fig. 20, and can be shown to be equal to $\sqrt{2}$ times the width of the frequency band between coupling peaks.[1]

The height of the peaks of response in the secondary circuit can be obtained by substituting the value of γ_p from Eq. (40) into Eq. (33). This gives

Value of $\frac{E_c}{E}$ at peaks

$$= \frac{1}{\gamma_p^2}\sqrt{\frac{L_s}{L_p}}\frac{\sqrt{Q_pQ_s}}{\left[\sqrt{\frac{Q_p}{Q_s}} + \sqrt{\frac{Q_s}{Q_p}}\right]\left[1 - \frac{k_c^2}{4k^2}\left(\sqrt{\frac{Q_s}{Q_p}} - \sqrt{\frac{Q_p}{Q_s}}\right)^2\right]^{1/2}} \qquad (42)$$

It will be noted that the lower frequency peak is the highest peak. However, since

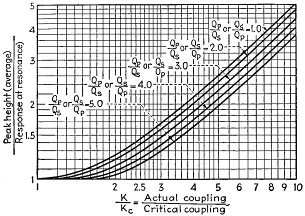


FIG. 21.—Universal curves giving the ratio of actual peak height to the response at resonance as a function of the coefficient of coupling for two coupled circuits resonant at the same frequency.

γ_p differs only slightly from unity in ordinary cases, it is customary to consider that the peaks are identical.

[1] See Aiken, *loc. cit.*

is given in Fig. 19 for various conditions, in terms of a parameter g given by the lower part of Fig. 19.

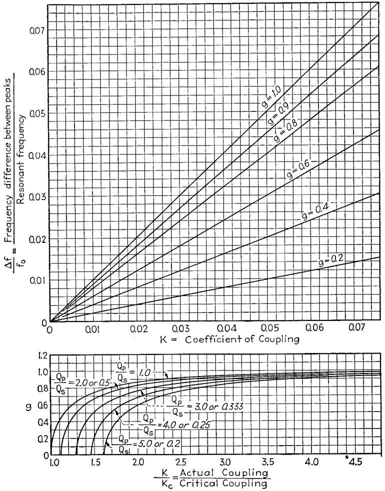


FIG. 19.—Curves from which the separation between peaks can be determined for the case of two coupled circuits resonant at the same frequency.

When the circuit Q's are not too unequal, and the coefficient of coupling greatly exceeds the critical value, Eq. (40) reduces to

$$\frac{\text{Frequency at peak of secondary voltage}}{\text{Resonant frequency of tuned circuits}} = \frac{1}{\sqrt{1 \pm k}} \tag{41a}$$

CHAPTER 7

COUPLED CIRCUITS

7.1 Introduction. It is sometimes desirable to transfer electrical energy from one circuit to a neighboring circuit. This can be accomplished, as in Fig. 7 A, by means of a transformer, wherein the source of energy is in the primary circuit and the load is in the secondary circuit. There are other coupling methods which employ a link circuit

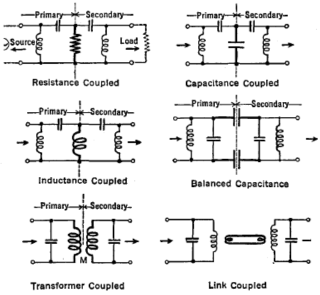


FIG. 7 A. Typical coupling methods

between the primary and the secondary. In still other types of coupling, the primary and the secondary circuits are actually connected together, using a coil or a condenser or a resistance, as the element common to both primary and secondary circuits.

The *transformer*, as shown in Fig. 7 A, differs in its action from the iron core transformer discussed in the chapter on a.c. circuits, in that only a few of the lines of force of the primary cut the wires of the secondary coil. The simple voltage step-up and impedance ratios applicable to the iron core case no longer hold true. The extent to which the lines of force of the primary cut the secondary is measurable, and is called the *mutual inductance*. Its symbol is M and its unit is the *henry*. When the secondary coil is placed close to the primary and on the same axis, the *coupling* will be great and M will be large. If the secondary coil is rotated, or is moved farther away from the primary, the voltages set up in it will be smaller, the coupling will be less, and M will represent a smaller number.

The coefficient of coupling is defined by the equation,

$$k = M/\sqrt{L_1L_2},$$

where the L's are the self-inductances of the primary and secondary circuits. If all of the lines of force from the primary were to cut the secondary, k would be equal to unity; the coefficient would be 100 per cent. This is the "tightest" possible coupling.

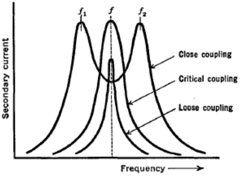


FIG. 7 B. Transformer coupling. The primary and secondary circuits were independently tuned to the same frequency (f)

When the circuits are tuned to the same frequency and are so related that but little energy is transferred from the primary to the secondary, they are said to be *loose coupled*. As the coupling is increased, the secondary increasingly "loads" the primary; and also, the primary increasingly loads the secondary. This loading action broadens the resonance curves of both circuits. At *critical coupling*, maximum energy is transferred and the resonance curve has quite a broad peak, as in Fig. 7 B. If the circuits are still closer coupled, the energy transferred from one circuit to the other decreases, and the resonance curve will have two peaks, one on either side of the single

This coupling k_c is often termed the *critical coupling*. The corresponding mutual inductance is

$$\omega M = \sqrt{R_pR_s} \tag{37}$$

With the mutual inductance for critical coupling, the resistance coupled into the primary circuit at resonance by the presence of the secondary equals the primary resistance. This gives the maximum possible secondary current. The resulting maximum possible voltage across the secondary condenser is

$$\text{Maximum possible } \frac{E_c}{E} = \frac{1}{2k_c}\sqrt{\frac{L_s}{L_p}} = \frac{\sqrt{Q_pQ_s}}{2}\sqrt{\frac{L_s}{L_p}} \tag{38}$$

where k_c is the critical coupling as given by Eq. (36). The voltage developed across the condenser at resonance with other couplings is less than value given by Eq. (38) by an amount depending upon the ratio of actual to critical coupling, as shown in Fig. 18.

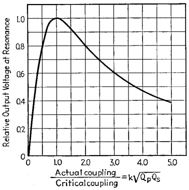


FIG. 18.—Relative output voltage obtained at resonance as a function of coefficient of coupling in the case of two circuits tuned to the same frequency and coupled together.

Conditions for Existence of Double Peak—Height, Location, and Width of Peaks.—The curve of response in the secondary circuit exhibits double peaks whenever the coefficient of coupling is sufficient to cause the right-hand side of Eq. (33) to show two maxima. This occurs when[1]

$$\frac{\text{Actual coefficient of coupling}}{\text{Critical coefficient of coupling}} \geqslant \left[\frac{1}{2}\left(\frac{Q_p}{Q_s} + \frac{Q_s}{Q_p}\right)\right]^{1/2} \tag{39}$$

It will be noted that when $Q_p = Q_s$, double peaks occur whenever the coupling exceeds the critical value, but if the Q's are different, double peaks do not occur until the coupling exceeds the critical value by an amount determined by the Q ratio.

When double humps occur, the location of the peaks in relation to the resonant frequency depends upon the actual coupling, the critical coupling, and the Q ratio. Analysis based upon Eq. (33) shows that to an accuracy sufficient for all ordinary purposes the location of the peaks is given by[2]

$$\left.\frac{\text{Frequency at peak of secondary voltage}}{\text{Resonant frequency of tuned circuits}}\right\} = \gamma_p$$
$$= \frac{1}{\sqrt{1 \pm k\left[1 - \frac{k_c^2}{2k^2}\left(\frac{Q_p}{Q_s} + \frac{Q_s}{Q_p}\right)\right]^{1/2}}} \tag{40}$$

where k_c is the critical coefficient of coupling as given by Eq. (36) and the remaining notation is the same as above. The ratio represented by the spacing between coupling peaks to the resonant frequency, as a function of the coefficient of coupling,

[1] Equation (39) assumes that the γ^2 multiplying the denominator of Eq. (33) is constant. The approximation that results is so small as to be of negligible consequence, but results in enormous simplification of the equations.

[2] Eq. (40) involves the same approximation as does Eq. (39), but again the error introduced is negligible and the simplification considerable.

The curve of secondary current is then somewhat broader in the immediate vicinity of the resonant frequency than the resonance curve of the secondary circuit, and the primary curve shows double peaks. With still greater coupling, the double humps in the primary current curve become more prominent, and the peaks spread farther apart. At the same time the secondary current curve begins to show double humps, with the peaks becoming progressively more pronounced and farther apart as the coefficient of coupling is increased.[1]

Basic Equations.—In practical work, the most important property of the coupled system under consideration is the ratio of voltage developed across the condenser in the secondary circuit to the voltage applied in series with the primary circuit. A useful expression for this important ratio is obtained by rearranging Eqs. (22) and (23) to give the relation[2]

$$\frac{\text{Voltage across secondary condenser}}{\text{Voltage applied in series with primary}} = \frac{E_c}{E}$$

$$= \frac{-1}{\gamma^2}\sqrt{\frac{L_s}{L_p}}\,\frac{k}{\left[k^2 + \dfrac{1}{Q_pQ_s} - \left(1 - \dfrac{1}{\gamma^2}\right)^2 + j\left(1 - \dfrac{1}{\gamma^2}\right)\left(\dfrac{1}{Q_p} + \dfrac{1}{Q_s}\right)\right]} \tag{33}$$

where $\omega = 2\pi$ times frequency.

$\omega_0 = 2\pi$ times resonant frequency.

$\gamma = \dfrac{\omega}{\omega_0} = \dfrac{\text{actual frequency}}{\text{resonant frequency}}$.

E = voltage applied in series with primary.

$Q_p = \omega L_p/R_p$ for primary circuit.

$Q_s = \omega L_s/R_s$ for secondary circuit.

$k = M/\sqrt{L_pL_s}$ = coefficient of coupling.

L_p = total inductance of primary circuit.

L_s = total inductance of secondary circuit.

This equation involves no approximations, other than assuming that the circuits are resonant at the same frequency.

In the special case where the circuit losses are zero, or where the frequency differs sufficiently from the resonance frequency to allow the losses to be neglected, Eq. (33) takes the form

$$\frac{E_c}{E}\text{ for zero losses} = \frac{-1}{\gamma^2}\sqrt{\frac{L_s}{L_p}}\,\frac{k}{\left[k^2 - \left(1 - \dfrac{1}{\gamma^2}\right)^2\right]} \tag{34}$$

Effect of Coupling on Response at Resonance—Critical Coupling.—The response at resonance is obtained by substituting $\gamma = 1$ in Eq. (33), which gives

$$\frac{E_c}{E}\text{ at resonance} = -\sqrt{\frac{L_s}{L_p}}\,\frac{k}{\left[k^2 + \dfrac{1}{Q_pQ_s}\right]} \tag{35}$$

This expression is maximum when the coefficient of coupling has the value

$$k_c = \frac{1}{\sqrt{Q_pQ_s}} \tag{36}$$

[1] When the circuit Q's are not equal, double humps do not appear in the curve of secondary current until the coupling is somewhat greater than the critical value. This is illustrated in the universal curves of Fig. 22.

[2] For derivation, see Terman, *op. cit.*, p. 81.

frequency to which the circuits were originally tuned. The closer the coupling, the greater the frequency separation of these peaks.

If the two circuits are not exactly tuned to the same frequency, their effect upon each other is not only to increase the effective resistance (the Q of the circuit decreases) but also to reduce the self-inductance. Hence the peak of the resonance curve shifts to a different frequency.

7.2 The Effect of " Neighboring Bodies." The secondary circuit need not necessarily consist of a coil, condenser, and resistance, as in the cases we have just discussed, but can consist of any metallic, and even a dielectric body, in the neighborhood of the circuit containing the alternating current. A piece of metal placed inside a coil will have small *eddy currents* induced in it. These currents, in turn, react upon the primary circuit to increase its resistance, i.e., lower its Q, and *decrease* its inductance and hence increase its resonant frequency. Because of the increased resistance, the generator must supply an additional amount of power (current squared, times resistance). Thus, if we use the symbol R_1^1 for the *effective resistance* of a circuit (its actual resistance plus that which is " reflected " from the load), we may write

$$\underbrace{I_1^2R_1^1}_{\text{Total Heat}} = \underbrace{I_1^2R_1}_{\text{Prim. Heat}} + \underbrace{I_2^2R_2}_{\text{Sec. Heat}},$$

where the term on the left side represents the total power which the source must supply. The first term on the right-hand side of the equation represents the heat developed in the primary circuit, and the second term represents the heat developed in the secondary, or load — be it a circuit or a metallic body or an insulator. Now, this state of affairs may be good or it may be bad. For example, it is " good " when the circuit is used as an *induction heater,* which consists of a radio frequency generator with a tuned circuit, the coil of which consists of a few turns of heavy copper wire. The coil is placed around a body which is to be heated; such, for example, as the metal electrodes sealed inside a vacuum tube during its process of manufacture, or of a human body in which it is desired to develop an " artificial fever." The eddy currents set up in the conductive parts of the body inside the coil cause the body to heat up and, in some cases, to an extent such as to melt it completely.

If an insulator is placed inside the coil, the high frequency electrostatic fields cause the electrical charges in its atoms and molecules to oscillate back and forth about their normal positions. This requires energy which can only come from the primary circuit. This causes the dielectric to heat up, and is spoken of as *dielectric loss*. That energy should be absorbed from a circuit by a neighboring dielectric is, in general, undesirable. The resistance of the primary circuit is effectively increased by these losses.

We may generalize the concept of resistance by defining it in the following manner:

$$\text{Generalized Resistance} = \frac{\text{Watts Lost}}{\text{Current Squared}},$$

where the "watts lost" include the heat losses in the circuit itself and in neighboring bodies (as ohmic, eddy current, or dielectric heating); in general, lost in any form whatsoever from the source of power.

7.3 Shielding. A metal shield can be used to prevent coupling between two circuits. Capacitive or electrostatic coupling can be prevented by shielding either the primary or secondary, or both, circuits with an enclosing metal container. The shield should be grounded and be made of material of low resistance. Also, metal shields can be used at radio frequencies to prevent magnetic coupling. Here, the eddy currents in the shield have magnetic fields which oppose the original field and more or less completely keep it out of the inside of the shield. The shielding effect is greater at the higher frequencies, is greater for more conductive materials, and also depends upon the thickness of the shielding material. At low frequencies, the eddy currents are so feeble that this method is not satisfactory. The best that can be done in this case is to surround the circuit with a complete shield of soft iron. This will partially divert the magnetic fields.

A shield changes the resonant frequency and the Q of the circuit being shielded. In shielding a coil, the spacing between the sides of the coil and the shield should be equal to at least half the diameter of the coil, and the distance of the shield from the end of the coil should be not less than the diameter of the coil. Copper and aluminum are satisfactory metals for shields.

7.4 Introduction to Filters. Various combinations of coils and condensers are used:

1. To separate currents of different frequencies from each other.
2. As a coupling unit between two circuits whose impedances are not equal to each other.

5. Two Resonant Circuits Tuned to the Same Frequency and Coupled Together.[1] When two circuits resonant at the same frequency are coupled together, the resulting behavior depends very largely upon the coupling, as shown in Fig. 17. When the coefficient of coupling is small, the curve of primary current as a function of frequency approximates closely the series resonance curve of the primary circuit considered alone. The secondary current at the same time is small and varies with frequency according to a curve having a shape approximating the product of the resonance curves of

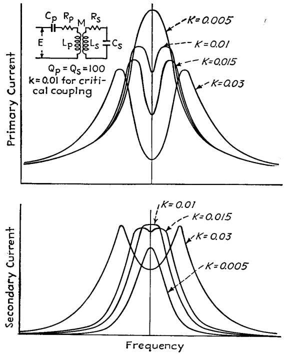


Fig. 17.—Curves showing variation of primary and secondary current with frequency for different coefficients of coupling when the primary and secondary are separately tuned to the same frequency.

primary and secondary circuits taken alone. As the coupling is increased, the curve of primary current becomes broader and the peak value of primary current is reduced. At the same time, the secondary current becomes larger and the sharpness of the secondary current curve is reduced. These trends continue until the coupling is such that the resistance that the secondary circuit couples into the primary at resonance is equal to the primary resistance. This coupling is called the *critical coupling*, and causes the secondary current to have the maximum possible value that it can attain.

[1] A very complete discussion of this subject is given by C. B. Aiken, Two-mesh Tuned Coupled Circuit Filters, *Proc. I.R.E.*, Vol. 25, p. 230, February; p. 672, June, 1937. See also E. S. Purington, Single and Coupled-circuit Systems, *Proc. I.R.E.*, Vol. 18, p. 983, June, 1930.

When ωL_p is not negligible compared with the primary resistance, the response curve still has the shape of a resonance curve, but the peak is shifted to a slightly higher frequency. This shift in location of the peak is due to the fact, expressed by Eq. (26), that maximum secondary current occurs when secondary is sufficiently

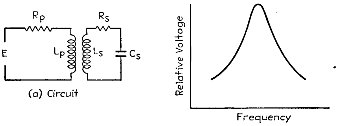


FIG. 15.—Inductively coupled circuit with tuned secondary and untuned primary.

detuned that its coupled reactance neutralizes the reactance of the primary. When the primary reactance is not negligible, this detuning is appreciable. The coupling required for maximum secondary current is also greater as the ratio $\omega L_p/R_p$ is increased.

Analysis of the output of coupled systems with untuned primary and tuned secondary can be most conveniently carried out by using the equivalent secondary circuit of Fig. 12*d*. Such equivalent circuits are shown in Fig. 16. These show how the asso-

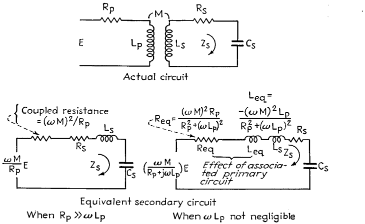


FIG. 16.—Equivalent secondary circuit for case of inductive coupling with tuned secondary and untuned primary.

ciation of the primary with the resonant secondary has the effect of increasing the effective secondary resistance and hence of lowering the effective Q, and also that an inductive primary neutralizes some of the secondary inductance and so raises the resonant frequency.

3. To shift the phase between voltage and current.
4. To alter the magnitude of voltage or current.

Figure 7 C shows some of the filters with which we are already familiar, together with their curves of output current at different input frequencies, their basic equations, and their names. See also Figs. 5 A and 5 B. More complicated combinations will serve better in separating neighboring frequencies and in more completely suppressing others. In other words, there are filters which have a *sharper cutoff.*

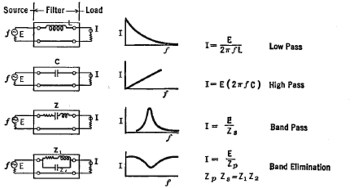


FIG. 7 C. Crude filters. Low-pass types use series inductance (top figure). High-pass use series capacitance. Band-pass use series *LC* circuits. Band-elimination use parallel *LC* circuits (bottom figure)

Of the host of possible *LC* combinations, a few simplified forms have been developed under the following assumptions and simplifications:

1. The resistances of all parts of the filter circuits are to be kept as low as possible. We shall assume that the resistances are zero. Such filters are called " ideal " or *non-dissipative.*
2. There shall be no batteries or other sources of electromotive force, nor vacuum tubes with their attendant amplification, in the filter. Such filters are said to be *passive.*
3. The inductances of all coils which contain iron cores shall be assumed to be the same for any of the currents which flow through them. Filters constructed with such inductances are said to be *linear.*
4. There shall be no magnetic coupling of the lines of force from any of the coils to any other coils in the circuits.

5. The output or load circuit shall consist of pure resistance (no coils or condensers), whose value shall be the same as that of the generator or input circuit.

It is possible to describe the characteristic of filters in terms of the output current as in Fig. 7 C, or in terms of the *ratio* of output to input currents, or the ratio of the output voltage across the load to the input voltage, or the ratio of the power output to the power input, or, as is more common and as we shall now do, in terms of the *attenuation* of the filter. The attenuation is expressed in decibels (Sec. 2.6) and is proportional to the logarithm of the ratio of the output to the input currents or voltages or powers. It is a measure of the losses which take place in the filter at the different frequencies.

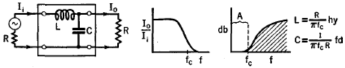


FIG. 7 D. An L-section, low-pass filter. The lower frequencies in the range "A" pass through, while those of higher value are more or less attenuated or lost in the filter

7.5 Low-Pass Filters. The circuit of Fig. 7 D is called an L-type, low-pass filter. It contains a series inductance and a shunt capacitance. From the attenuation (db.) curve in this figure, it can be seen that the choking action of the coil, aided by the bypass action of the condenser, has materially sharpened the curve over that of a single series inductance (Fig. 7 C), especially in the neighborhood of the *cutoff frequency*, f_c. In the equations given in Fig. 7 D, L is in henries, C is in

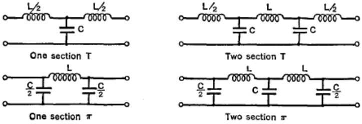


FIG. 7 E. — T- and π-type, low-pass filters

resistance of the tube. When the secondary is a resonant circuit, the curve of coupled impedance $(\omega M)^2/Z_s$ with variation in frequency has substantially the same shape and characteristics as the parallel impedance curve of the secondary, but the absolute magnitude is determined by the mutual inductance.

The important characteristic of this circuit is the curve of voltage developed across the secondary condenser as a function of frequency under conditions where $R_p >> \omega L_p$. Such a characteristic is shown in Fig. 15*b*, and has substantially the same shape as a resonance curve with the peak at the resonant frequency of the secondary and an effective Q somewhat lower than the actual Q of the secondary circuit.

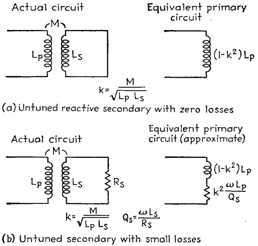


FIG. 14.—Inductively coupled circuits with untuned secondaries.

On the assumption that $R_p >> \omega L_p$, the response at resonance is

$$\frac{\left\{\begin{matrix}\text{Voltage across secondary}\\ \text{condenser at resonance}\end{matrix}\right\}}{\text{Voltage applied to primary}} = \frac{\omega M}{R_pR_s + (\omega M)^2} \times \omega L_s \tag{30}$$

With given primary and secondary circuits, the response at resonance is maximum when the coupled resistance equals the primary resistance, *i.e.*, when

$$\omega M = \sqrt{R_pR_s} \tag{31}$$

With greater or less coupling, the response is less.

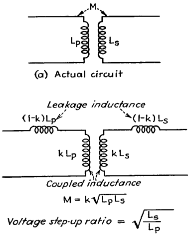
The relationship between the effective Q of the response curve and the actual Q of the secondary circuit is, still assuming that $R_p >> \omega L_p$

$$\frac{\text{Effective } Q \text{ of amplification curve}}{\text{Actual } Q \text{ of tuned circuit}} = \frac{1}{1 + \frac{(\omega M)^2/R_s}{R_p}} \tag{32}$$

The effective Q of the response curve approaches the actual Q of the secondary circuit when the coupling is very small, and is exactly one-half of the actual Q when the coupling corresponds to the value giving maximum response.

Inductive Coupling Considered as a Transformer.—Two coils inductively coupled represent the general case of a transformer, and can be reduced to the equivalent transformer circuit of Fig. 13. Here the primary and secondary inductances are divided into leakage and closely coupled components. The leakage inductances correspond to magnetic flux lines that produce no linkages to any secondary circuit, while the coupled inductances are taken as having unity coefficient of coupling.

In a conventional transformer with iron core, the coefficient of coupling between primary and secondary is so large that the leakage inductances represent only a very small fraction of the total circuit inductances. Under these conditions it is convenient to analyze the behavior in terms of the turn-ratio, leakage inductance, etc., as is commonly done with 60-cycle power transformers. However, in the more general case where the coefficient of coupling may be small, most of the inductance is leakage, and turn ratio has relatively little significance. It is then preferable to use the general method of analysis given above.



(b) Same circuit shown as transformer

FIG. 13.—Equivalent circuit of a transformer expressed in terms of primary and secondary inductances and coefficient of coupling.

4. Analysis of Some Simple Coupled Circuits. *Untuned Secondary Consisting of a Pure Reactance.*—This case corresponds to a coil serving as the primary, with a metal mass of low resistivity serving as the secondary. In this case, which is illustrated in Fig. 14*a*, one has

$$\text{Coupled reactance} = -\frac{(\omega M)^2}{\omega L_s} = -k^2\omega L_p \quad (29)$$

where k is the coefficient of coupling between the coil and the secondary. The presence of the secondary reduces the inductance as viewed from the coil terminals to an equivalent value $(1 - k^2)L_p$.

It will be noted that the amount of reduction in inductance is determined solely by the coefficient of coupling and not by the number of primary turns, etc. It will be also observed that the effect of the secondary decreases rapidly as the coefficient of coupling becomes small.

Untuned Secondary Having Both Resistance and Reactance.—This corresponds to the practical case of a coil in a shield, or near a metal panel or other mass, with the resistance of the metal secondary taken into account. For such a secondary, the coupled impedance as given by Eqs. (24) and (25) consists of a resistance and a negative reactance. The effect of the secondary upon the primary is hence to lower the equivalent inductance and increase the equivalent resistance of the primary coil as viewed from its terminals.

Under practical conditions, a secondary consisting of a shield or other metal object is usually of copper or aluminum, so that the secondary reactance is considerably greater than the secondary resistance. Under these conditions, the equivalent primary circuit has the values and form given in Fig. 14*b*.

Tuned Secondary, Untuned Primary.—This circuit, illustrated in Fig. 15*a*, is of importance because it represents the equivalent circuit of the transformer-coupled tuned radio-frequency amplifier, with the primary resistance R_p representing the plate

farads, R is in ohms, and f_c is in cycles per second. Note that the generator and the load both have the same resistance.

Referring to Fig. 7 E, we see that several " T "-sections or " π "-sections can be connected in series with each other to form two, three, or more section filters. The cutoff becomes increasingly sharp the more sections used. Then, of two frequencies near f_c, the lower one will get through and the higher one will not. The same design equations are used as for the L-section low-pass filter. Note that, in Fig. 7 E, only one-half the inductance or one-half the capacitance is used in certain places while the full value is used at others.

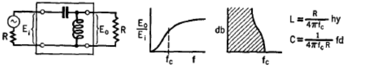


FIG. 7 F. An L-section, high-pass filter. L is in henries, C is in farads, R is in ohms and is the same at the input and output. The cutoff frequency, f_c, is in cycles per second

7.6 High-Pass Filters. Whereas the low-pass filters of the preceding section permitted the passage, more or less, of all frequencies below the cutoff value, and attenuated or suppressed, more or less, all frequencies above this value, high-pass filters do just the reverse. A single L-section high-pass filter is shown in Fig. 7 F and is seen to con-

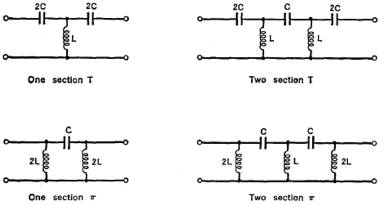


FIG. 7 G.—T- and π-type, high-pass filters

sist of a series condenser, whose reactance is large for low frequencies and small for higher frequencies, and a shunt inductance, which readily bypasses low frequencies but forces the higher frequencies to continue on out to the load R.

Figure 7 G shows T- and π-type, one- and two-section high-pass filters which offer greater discrimination to frequencies near the cutoff. The design equations are the same as for the L-section.

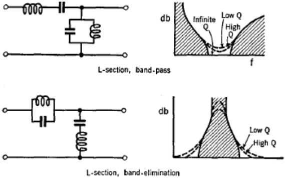


FIG. 7 H. Simple band-pass and band-elimination filters

7.7 Band-Pass and Band-Elimination Filters. It is possible to transmit a certain range of frequencies through a *band-pass filter*, and more or less completely suppress all lower and all higher frequencies. A simple filter of this type, together with its attenuation curve, is given in Fig. 7 H.

It is also possible to construct filters which will, more or less, transmit all frequencies except those which lie within a certain range. A simple form of *band-elimination* or *band-suppression filter* is also given in Fig. 7 H, together with its attenuation curve.

With multi-section filters of these types, the separation of adjacent frequencies can be made much sharper than with the simple L-section. On the other hand, in practice, the coils are not pure inductances but have some resistance. Now Q is our measure of the inductive-ness versus resistance of a coil, i.e., $Q = 2\pi fL/R$. In Fig. 7 H, the dotted lines show in what manner the attenuation is affected when low- and high-Q coils are used. Similar unsharpening effects occur in practice with all filters.

circuit leads to the same results as the first form, since the extra voltage that is considered as acting in the second case is just compensated for by the voltage drop in the extra impedance considered as existing in the secondary circuit.

The resistance and reactance components of the coupled impedance are

$$\text{Resistance component} = \frac{(\omega M)^2}{R_s^2 + X_s^2} R_s \tag{24}$$

$$\text{Reactance component} = -j \frac{(\omega M)^2}{R_s^2 + X_s^2} X_s \tag{25}$$

where R_s and X_s are, respectively, the resistance and reactance components of the secondary impedance Z_s. The effect that the secondary has on the primary circuit is exactly as though the resistance and reactance given by Eqs. (24) and (25) had been inserted in series with the primary circuit. The energy and reactive volt-amperes consumed by the primary current flowing through this hypothetical resistance and reactance represent the energy and reactive volt-amperes that are transferred to the secondary circuit.

Conditions for Maximum Secondary Current.[1]—In dealing with coupled circuits, it is desirable to keep in mind the adjustments giving maximum secondary current. The behavior in the more important cases is summarized below for the case of two circuits inductively coupled, with resistance and reactance in each circuit.

Case 1. Maximum Secondary Current with Variation of Secondary Reactance.—When frequency, mutual inductance, primary impedance, and secondary resistance are constant, the secondary current is maximum when

$$X_s = \frac{(\omega M)^2}{R_p^2 + X_p^2} X_p \tag{26}$$

where X_p and X_s = reactance of primary and secondary circuits, respectively.
R_p and R_s = resistance of primary and secondary circuits, respectively.
ω = 2π times frequency.
M = mutual inductance.

Case 2. Maximum Secondary Current with Variation of Primary Reactance.—If the primary rather than secondary reactance is varied, maximum secondary current is obtained when

$$X_p = \frac{(\omega M)^2}{R_s^2 + X_s^2} X_s \tag{27}$$

Case 3. Maximum Possible Secondary Current.—The secondary current has its maximum possible value when the adjustments are such that the resistance and reactance coupled into the primary circuit by the secondary are equal to the resistance of the primary circuit and the negative of the primary reactance, respectively; *i.e.*, when

$$\left.\begin{aligned} \frac{(\omega M)^2}{R_s^2 + X_s^2} R_s &= R_p \\ -\frac{(\omega M)^2}{R_s^2 + X_s^2} X_s &= -X_p \end{aligned}\right\} \tag{28}$$

It will be noted that in order to obtain maximum possible secondary current, it is necessary that two independent variables be adjusted. One of these is commonly the mutual inductance; the other, the reactance of one or both circuits. Unless the mutual inductance satisfies the relation $\omega M \geq \sqrt{R_p R_s}$, it is impossible to satisfy the condition for maximum possible secondary current.

[1] For a more detailed discussion, see G. W. Pierce, "Electric Oscillations and Electric Waves," Chap. XI, McGraw-Hill, New York, 1920.

circuit as a result of a voltage applied to that circuit produces magnetic flux that induces a voltage in the coupled circuit, resulting in induced currents and a transfer of energy from the first or primary circuit to the coupled or secondary circuit.

Analysis of Inductively Coupled Circuits.—The behavior of inductively coupled circuits can be determined by making use of the following principles:[1] *First,* as far as the primary circuit is concerned, the effect of the presence of the coupled secondary circuit is exactly as though an impedance $(\omega M)^2/Z_s$ had been added in series with the primary, where

M = mutual inductance.
$\omega = 2\pi f$.
Z_s = series impedance of secondary circuit when considered by itself.

Second, the voltage induced in the secondary circuit by a primary current of I_p has a

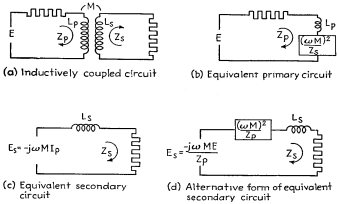


Fig. 12.—Inductively coupled circuit and also equivalent primary and secondary circuits.

magnitude of ωMI_p and lags the current that produces it by 90°. *Third,* the secondary current is exactly the same current that would flow if the induced voltage were applied in series with the secondary and if the primary were absent.

The preceding analysis shows that the actions taking place in the primary and secondary circuit are as though one had the equivalent circuits shown in Figs. 12*b* and 12*c*. Here the primary circuit consists of the actual primary impedance Z_p in series with an additional impedance $(\omega M)^2/Z_s$ that takes into account the effect the secondary produces on the primary current. This extra impedance $(\omega M)^2/Z_s$ is termed the *coupled impedance.* The equivalent secondary circuit of Fig. 12*c* consists of the voltage $-j\omega MI_p$ induced in the secondary by the primary current, in series with the secondary impedance Z_s as shown.

An alternative form of equivalent secondary circuit is given in Fig. 12*d*. This is obtained by applying Thévenin's theorem and noting that when the secondary is opened, the open circuit voltage that appears is $-j\omega MI_p$, while when the applied voltage is short-circuited, the equivalent impedance as viewed from the opened secondary terminals is $Z_s + (\omega M)^2/Z_p$. This second form of the equivalent secondary

[1] These rules for handling coupled circuits are direct consequences of the mesh equations, which for this case can be written as

$$E = I_pZ_p + j\omega MI_s \qquad (22)$$
$$\text{Induced voltage} = -j\omega MI_p = I_sZ_s \qquad (23)$$

It is well known that a quartz slab cut from its crystal in proper fashion and mounted between two metal plates acts like a circuit of very high Q. Hence, in circuits where the frequency of the currents is comparable with the natural vibrational frequency of the quartz, it is possible to obtain very sharp cutoff filters of all types.[1]

[1] Details are given by W. P. Mason and R. A. Sykes, in *The Bell System Technical Journal,* Volume XIX, page 221, April 1940.

Radio Engineers Handbook
F.E. Terman, 1943
Section 3: Circuit Theory
pp 148 - 163
Inductively Coupled Circuits

3. Inductively Coupled Circuits.—When mutual inductance exists between coils that are in separate circuits, these circuits are said to be inductively coupled. The effect of the mutual inductance is to make possible the transfer of energy from one circuit to the other by transformer action; *i.e.*, the alternating current flowing in one

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By L. Van Warren MS CS, AE - AE5CC

www.wdv.com/.../SoftRadio/BlazingFastSDR/BFSDR-Chapter1.docx

"Every day sees humanity more victorious in the struggle with space and time."
– Guglielmo Marconi

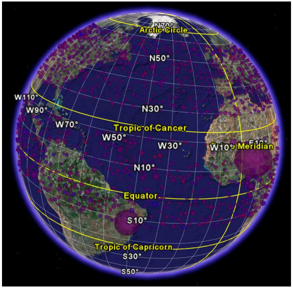

Figure 1: Earth Footprints of Celestial Radio Sources

Introduction
There are lots of books, articles and websites describing Software Defined Radio (SDR). My goal, in this introductory work, is to give you blazing fast access to a working set of concepts you can use to decide when and how SDR will be useful to you. It will start simply and build essential ideas step-

by-step. This book has two goals. The first is to provide a working overview of SDR. The second is to make hardware and software prototyping easier for the uninitiated.

This will not be a mathematically intensive development but rather a plug and “play” approach. Each chapter will start with interactive simulation and end with real devices - devices you can explore and interconnect. The interested reader should visit the references provided in the final chapter to clarify the more sophisticated ideas. Running each simulation is easy and highly recommended.

The book is divided roughly in half. In the first chapters, essential radio hardware issues will be discussed. For the foreseeable future SDR has not eclipsed the entire radio. Front-end RF hardware is still required to gather, sample and downconvert the signal. In the latter chapters we will transition to software-based concerns, while keeping an eye on hardware and instrumentation that will make our lives easier and our understanding more complete.

To demonstrate hardware concepts, we will be using a set of PostCardKits™. The pattern is this. We will use simulation to understand the theory behind each PostCardKit™. Then we mix and match the postcards to configure different kinds of radios. Pretty fun and exciting! Later we will mix and match software blocks to accomplish the same objective.

http://www.electronics-tutorials.ws/accircuits/parallel-circuit.html

Ben Gurion University
Mutual Inductance
Pdf downloaded from BGU Lab 7 web page
http://www.ee.bgu.ac.il/~intrlab/lab_number_7/

Ben Gurion University
EE Lab 7
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Impedance of R, C and L in parallel Calculator
http://keisan.casio.com/exec/system/1258032708

DesignSoft, Inc.
Resonant Circuits
http://www.tina.com/English/tina/course/28resonant/resonant.htm

Georgia State U: HyperPhysics - Electricity and Magnetism
RLC Parallel Circuit
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/rlcpar.html

Hoag, J Barton, 1942
Basic Radio
http://www.tubebooks.org/Books/hoag.pdf

Kuphaldt, Tony R.
Resonance
http://www.allaboutcircuits.com/vol_2/chpt_6/1.html

where Ze is the equivalent primary impedance
Z1 is the source impedance (primary) and
Z2 is the secondary load.

From Reuben Lee, 1947.
http://www.vias.org/eltransformers/lee_electronic_transformers_07b_22.html

Equivalent primary impedance:
Ze = Z1 + (XM^2 / Z2)

where Ze is the equivalent primary impedance
Z1 is the primary circuit impedance and
Z2 is the secondary circuit impedance and
XM = jwM

From Bob Weaver: 2016
http://electronbunker.ca/eb/Bandspreading_3.html

Net impedance:
Ze = Zca || (Z1 + ((wM)^2 / (Z2 + Zcb))

where Zca Primary circuit Z capacitor = -j / wCa
Zcb Secondary circuit Z capacitor = -j / wCb
Z1 Primary circuit Z inductor = j wL1
Z2 Secondary circuit Z inductor = j wL2

Bibliography:

AspenCore , 2016
The Parallel RLC Circuit

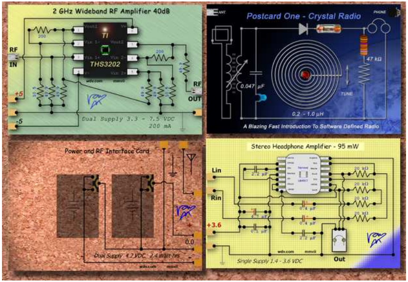


Figure 2: Mix and Match Postcards for Chapter 1

In the second chapter on amplifiers the crystal radio card is improved using an audio amplifier PostCardKit™, an RF amplifier PostCardKit™, and two kinds of power cards. One power card features rechargeable lithium batteries; the other uses solar cells for recharging and direct power. How green is that?! PostCardKits™ are flat, lead-free evaluation cards, printed on high quality paper with conductive ink. PostCardKits™ can be stamped and mailed, or mailed in envelopes to maintain pristine appearance. The first chapter is introduced with a crystal radio set. This card functions without any external or battery power. It receives AM radio stations. A first attemp on a file card pulled in stations from Asia and Central America.

You can hear fainter stations if you add an audio amplifier card. You can receive more stations if you add the RF amplifier. These additional cards require power. We will reuse the audio, RF amplifier and power cards in later chapters in novel ways. For example, the audio card is a stereo amplifier used for a special kind of station hunting called binaural radio.

Laterwe will develop radio software on a PC. Towards the end, we will extend the power of the hardware and software and reach for the stars.

The first PostCardKit™, Crystal Radio, utilizes a germanium diode for signal detection. It demonstrates the simplest effective combination of discrete components. It consists of an inductor, a capacitor, a resistor, a germanium diode detector and a piezoelectric crystal earphone.

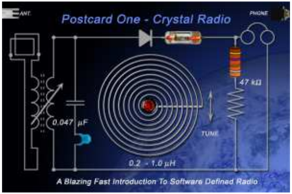


Figure 3: Crystal Radio PostCardKit™

Don’t let our simple start fool you; We will be moving many these functions into software and Software Defined Radio (SDR) can do sophisticated things.

(A + jB) / (C + jD)

1)

$$\frac{(A+jB)}{(C+jD)} * \frac{(C+jD)}{(C+jD)} = \frac{AC + jAD + jBC + j^2BD}{C^2 + D^2}$$

Remember that $j^2 = -1$, so

2)

$$\frac{(A+jB)}{(C+jD)} * \frac{(C+jD)}{(C+jD)} = \frac{AC + jAD + jBC - BD}{C^2 + D^2}$$

3)

$$\frac{(A+jB)}{(C+jD)} * \frac{(C+jD)}{(C+jD)} = \mathbf{\frac{AC - BD}{C^2 + D^2} + j\,\frac{AD + BC}{C^2 + D^2}}$$

Parallel Combination

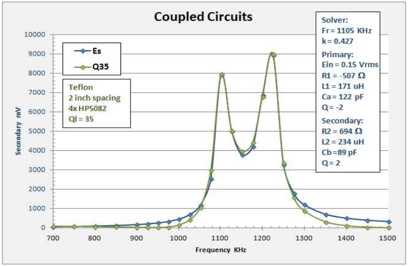
Zeq = (Z1*Z2) / (Z1+Z2)

Appendix 2:

Coupled Impedance Formulae:

From Ben Gurion University EE Lab 7
http://www.ee.bgu.ac.il/~intrlab/lab_number_7/Two%20inductively%20coupled%20RLC%20circuits.pdf

Equivalent primary impedance:
Ze = Z1 - ZM^2 / Z2



Just for fun, I let solver run with k, Ca, E, Cb, R1 and R2. Adding R1 to solver's program really allowed it to nail the data with a close-fit model. The price? Look at the resistivity values it chose. R1 = -439 ohms and R2 = 719 ohms. Well, at least R2 is not negative! These results may be telling me something important, but as yet I am too dense to understand. Or, they may be just what they appear, totally bogus.

Appendix 1:

Big Fun with Complex Algebra

Multiplication
(A + jB) * (C + jD) = **(AC – BD) + j(AD + BC)**

Division
multiply both top and bottom by the conjugate of the bottom.

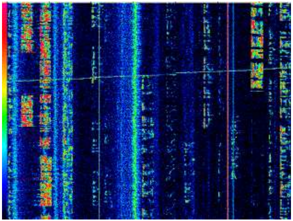


Figure 4: Visualization of Radio Spectrum

The Crystal Radio PostCardKit™

The crystal radio is the simplest of all radios. In World War II, Allied GI's used paper clips set against rusty razor blades to form crude diode junction receivers that the Nazi forces could not detect. These were dubbed "Foxhole receivers". Crystal radios have a long and colorful history documented in Wikipedia and various radio collections documented on the web.

Attach the earphone provided to the jack in the upper left corner of the postcard. You probably won't hear anything unless you live close to a powerful AM radio station. An antenna and a ground will improve your reach considerably. Just as a picture in the dark cannot be seen, a radio without an antenna cannot be heard. Lighting is half of art. The antenna is half of radio. You can learn more about antennas in the ARRL Antenna Book. It is highly recommended.

Putting Up the Litz

There is one errand to run before heading back to the easy chair. It is essential to route the Litz wire provided around a wall or ceiling to create an antenna. With antennas, bigger is usually better. I use a fold of masking tape to make a tiny hangar that holds the antenna on the wall. You can stick a clear pushpin through the tape to secure the antenna. Suspend the wire from the four corners of the room so it is up and out of the way. The wire provided is fine, so it is a quite aesthetic. When you are done, tin or sand the ends of the wire so that all the strands are conducting and install them in the connector provided. Now you have an antenna.

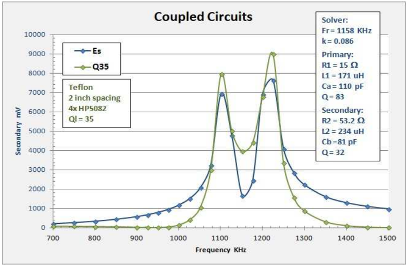
This square wire loop is a versatile omnidirectional antenna. If you wrap the antenna more than once around the room, the inductance will increase and the resonant frequency will drop according to:

$$f_{resonant} = \frac{1}{2\pi\sqrt{LC}}$$

Formula: Resonant Frequency

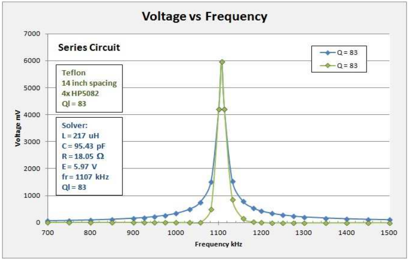
This formula informs us that those stations you manage to receive will be lower in the band, and lower in frequency. Start with one trip around the average sized room. Take your time getting this antenna right, it will serve you well. If you live on top of a hill, you will get better reception, but since radio waves bounce off the ionosphere, you will usually hear something unless you live in a salt mine. Using the clip provided, attach your antenna to the upper left hand corner your PostCardKit™ by the ANT. symbol. That was the hard part. You will also need a good ground. Grounding is

happens at the -3dB point. My simulation suggests a resistive loss of 18 ohms and reproduced the Q = 83. Pretty interesting. But, before congratulating myself too quickly...



Here I look at the over coupled case. As before, I am just unable to reproduce the steep flanks of my measured data with such a simple coupled circuit model. At a 2 inch separation, and looking only at the lower peak (recall my re-peaking), I had measured a Q = 35. The model does a fairly poor job of simulating this circuit. Note that in my simulation, I kept the known values of inductance for the two coils. Additionally I found that solver gave unacceptable values for R1 and so I pegged it at 15 ohms. I had solver look for k, Ca, E, Cb, and R2.

The first thing to note on the above plot is that the over-coupled curve is not centered with respect to the loosely coupled curves. This is a result of my A) not knowing at the time about exactly how the resonance curve splits with over coupling and B) my technique of re-peaking the set before each set of measurements. It is obvious that after moving the ATU to 2 inches separation, on re-peaking the output, I must have subtracted capacitance shifting the curve upward in frequency until the I got a peak at my desired measurement frequency of 1100 KHz. I should have made my resonance study earlier!



Looking now at just the measured data from the 14 inch spacing and comparing it to a curve generated with my series resonance spreadsheet. The data in green is my radio which, at this spacing gave me a set loaded Q = 83. I used excel's Solver routine on the central three datapoints to search for model parameters L, C and R. The steep flanks of the measured data were impossible to reproduce, but at least I could see what

discussed in an essential book on radio: The ARRL Radio Amateur's Handbook.

Figure 5: Ideal Reception

Using the Crystal Radio

Now that the radio has a good antenna, you should be able to hear more stations. You can tune the radio with the knob in the center. Remarkably, it needs no power! You might want to keep a logbook of the signals you hear, the time of day along with any tweaks you have made to the radio or antenna. Low frequency signals travel better at night than in the daytime. Some high frequency signals are the opposite. Where do we look when an aircraft is lost? The radio logbook.

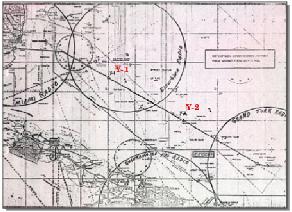


Figure 6 – Keeping a log enables discovery.

After you log a few entries it will be time to improve the radio. We will do that with the amplifiers mentioned above. Now for a little about how the crystal radio works.

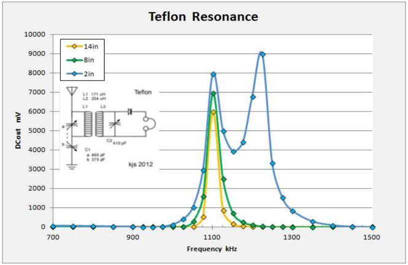
Basic Parts

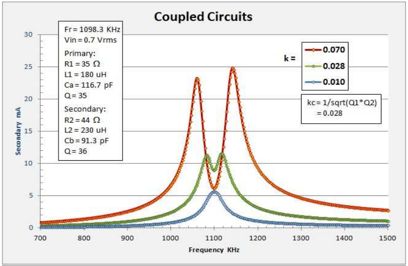
Picking up the card and shining the light on it you will notice it contains but five parts! The radio contains a diode, a resistor, an inductor, and two capacitors. Here is the schematic for the crystal radio including the Tidy TINA meter for RF power gain. The meter is used to optimize performance – it doesn't appear in the final circuit.

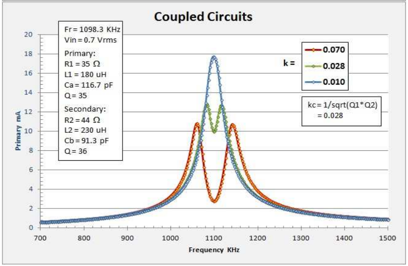
should give pause to note just how little signal one deals with in crystal sets. Note that critical coupling is when the secondary current attains its greatest value, not necessarily where the double hump disappears. Kc = 1/sqrt(Qp*Qs).

A copy of the coupled circuit spreadsheet can be found here: Coupled_kjs.xlsx

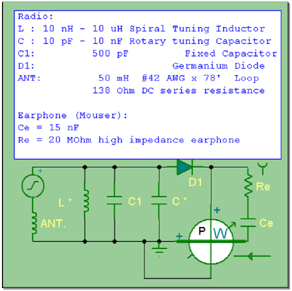
What else can one do with these spreadsheets? One of my motivations in building spreadsheets is that it allows me to compare measurements from actual radios to their theoretical counterparts. This allows me to better understand my sets and approach a best guess on certain parameters I might not otherwise be able to measure. The following few plots take data from a double tuned set that I have made frequency versus DC out at three differing coupling distances, 2 inches between coils (over coupled), 8 inches (presumed near critical), and 14 inches (under coupled).







In each plot I show curves with coupling factors of 0.070 (over-coupled), 0.028 (critically coupled), and 0.010 (loosely coupled). I find it amazing just how quickly the induced secondary current falls off with lowered coupling. These plots look quite different from most "generic" cartoon plots and



Example 0: Crystal Radio Schematic and Values

Just as in the Richter scale of earthquakes and the Fujitsu scale of tornadoes, we use a logarithmic scale when comparing the intensity of radio signals. This scale is measured in decibels (dB). This makes for much more reasonable comparisons. If two signals differ by a factor of two, they are about 3 dB apart. If they differ by a factor of four, they are 6 dB, and so on. Logarithmic scales turn multiplication into addition. This is useful when we want to talk about very large or small numbers. To convert a factor of 1000 to dB, you first count zeros to get 3. This corresponds to log(1000) = 3. Then you multiply by 10. 3 x 10 = 30 dB. So if two signals differ in power by a factor of 1000, then they are 30 dB apart:

In short dB = 10log(P), where P is power. Can you feel the power?

Tip - Use dB to compare the power of radio signals.

The resistor Re and capacitor Ce simulate the earphone. To really understand the crystal radio, we must understand the principles of the parts. If you are already an expert skip this quick review, but you might want to glance at the gain curves for voltage, current and power.

Resistors (units: Ohms) dissipate energy as heat. They impede the flow of electrical current, causing a voltage drop across the terminal ends. I once asked my dad if it wouldn't be better if a circuit had no resistors at all because of this energy loss. He said "No" and then paused for a moment and said, "Yes". The voltage drop E across a resistor is R times the current I, using Ohm's law. You can think of an Ohm of resistance as the volt of force required to make an ampere of current flow.

$$\boxed{E = R \times I}$$

Formula: Ohm's Law

Power (Watts) is voltage times current. Is your resistor rated for the power passing through it? Touch it and see, but don't get burned.

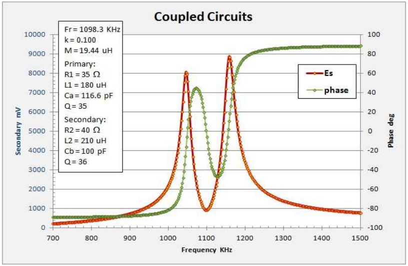
$$\boxed{P = E \times I}$$

Formula: Power

With Ohms Law and Power, you can derive six others! Two other handy resistor formulas:

Xs = wLs - 1 / (wCs)
Rcoupl = (wM)^2 * Rs / (Rs^2 + Xs^2)
Xcoupl = -1 * (wM)^2 * Xs / (Rs^2 + Xs^2)
Real = Rp + Rcoupl Imag = wLp + Xcoupl - 1/(wCp)
w = 2pi * frequency

Where the subscripts "s" means secondary circuit and "p" means primary circuit. M is the Mutual Induction of the two circuits defined as: M = sqrt(Lp*Ls) and is measured in Heneries. The Coupling Coefficient k indicates the amount of inductive coupling. This is a fractional value between 0 (no coupling) and 1 (perfect coupling). The Coupling Factor for the circuit then is defined as follows: M = k * sqrt(Lp*Ls).

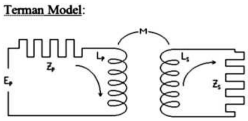


The plot above presents a generic coupled circuit with loose coupling (k = 0.1). I show the current induced in the secondary and the phase of the circuit impedance. Below are two interesting plots with the secondary current and primary current each at three different coupling factors.

value, reactance consists of inductive XL and capacitive XC parts. The parts are not necessarily the same and so adding them together does not necessarily equal zero. Note that, at resonance, they are equal in magnitude but opposite in sign (by convention, XC is negative) and they do add to zero. Where they have differing magnitudes, the combined impedance Z is the vector sum of the resistance and total reactance (XL + -XC). Z is the hypotenuse of a right triangle with an interior angle phi. This is the phase of the circuit. Where the phase is negative the circuit is capacitive, where the phase is positive the circuit is inductive. Where XL-XC is very large with respect to R, the phase approaches (but never reaches) plus or minus 90 degrees.

A copy of the parallel circuit spreadsheet can be found here: Parallel_kjs.xlsx

Time to move to the big leagues with coupling of two circuits. Here I am afraid my skills prove inadequate and I have turned to Mike Tuggle for help and, ultimately, a spreadsheet to make the needed calculations. For this I wish to publically acknowledge and thank Mike. Also to apologize to him for the significant liberties I have taken with said spreadsheet.

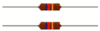


The model is based on Terman 1947 p149, shown at left. Mike builds a number of equations, the essential ones of which I used as follows:

Add two resistors in series to obtain the equivalent resistance:

$$\boxed{R_{series} = R_1 + R_2}$$

Formula: Series Equivalent Resistance

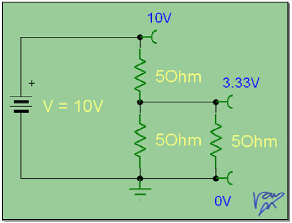


Use the product over sum for resistors in parallel:

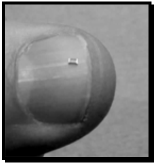
$$\boxed{R_{parallel} = \frac{R_1 R_2}{R_1 + R_2}}$$

Formula: Parallel Equivalent Resistance

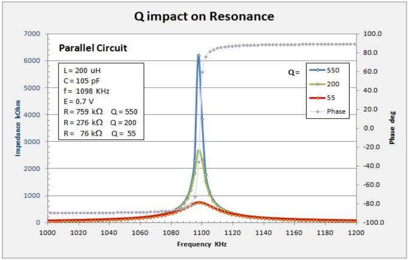
The current flow in circuit loops and the voltage drop across circuit elements can be computed using Kirchoff's Laws and Thevenin Equivalent circuits. The programming of these laws is already done for you in a tidy program called TINA-TI, a free download from the TI web site. I highly recommend it. Here is a classic voltage divider, simulated in TI's TINA-SPICE



Circuit 1: Classic Voltage Divider Solved in Tidy TINA.

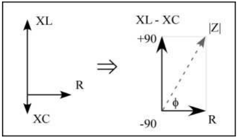
 - Ordy
Figure 7: No room for color codes on surface mount resistors!

w = 2pi * frequency



This plot above for a parallel circuit looks, as it ought, just like that for the series circuit with the same values of L and C. Note though that parallel R is significantly higher than in the series case. Parallel resistance is not a measure of the circuit loss but rather how "tight" it is, (spoken as a non-engineer). You should see that Q values now go up with increasing value of parallel resistance.

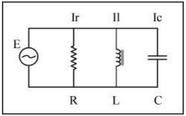
Additionally in this plot I include the phase of the impedance (for the Q = 550 case). The diagram at left indicates why it is



necessary to keep track of the real (resistance R) and imaginary (reactance X) parts of the equations. While resistance is a "real" plot on the chart the current versus frequency for the same tank with three variations of resistivity which gives three different values of Q. It is immediately clear that keeping resistance in the circuit low good of the quality of the set.

A copy of the series circuit excel spreadsheet can be found here: Series_kjs.xlsx

Having proved that I can simulate a simple series circuit it is time to take it up a notch with a PARALLEL circuit. Analysis of a parallel is inherently more complicated as it involves



imaginary, or complex numbers. It is necessary to keep close track of the real and imaginary parts of the calculations. Also, the math dealing with complex numbers is different, especially the rules for multiplication and division. See my Appendix at the bottom of this page.

I take the tack of starting with the impedances (Z) of the components and combining them in parallel. The symbol "||" means to combine in parallel. Formulas for this computation as follows:

Zs = Zc || Y
R || Zl
Y = (R * Zl) / (R + Zl)
Zs = (Zc * Y) / (Zc + Y)

Where:
Zl = jwL and
Zc = -j / wC

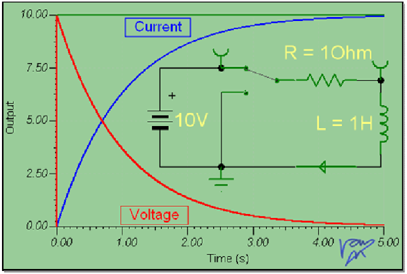
Notes on Resistors.
1) Always measure the value of a resistor before using it in a prototype circuit. Make sure your volt-ohm meter has a fresh battery.

2) In radio sections that operate at high frequencies we want resistors whose value does not vary with frequency. Thin-film and metal film resistors are preferred to wirewound resistors, which are really just lossy miniature inductors! Speaking of inductors…

Inductors (units: Henries)
store energy as a magnetic field. They are usually coils of wire or other conductive material in various shapes. Inductors have a direct current (DC) response and an alternating current (AC) response. These responses can be steady state or transient. Let's throw the switch!

When the switch is closed on the circuit below, an equal and opposite voltage is "induced" in the inductor. This is induced voltage is called "back EMF". After several time constants, the circuit reaches its "steady state". The magnetic field is established and this opposing voltage disappears. If the switch is opened, the magnetic field collapses and sparks can ensue! There was an old saying, "nature abhors a vacuum". Magnetic fields don't like suddenly open switches. This is an important principle when working with sensitive semiconductor components. Fried!

To track voltage in Tidy TINA we add a pin connection . To track current we add an arrow connection seen at the bottom of the circuit. By convention positive current flows from positive to negative. Electron vacancies or “holes” move in this direction, but real electrons flow the other way. Thanks Ben Franklin!

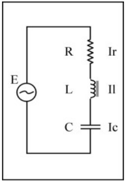


Circuit 2: Inductor Transient DC Response

Tidy TINA shows us two curves if we request a Transient Analysis. The top curve shows the current in the circuit. Since a coil is a conductor, keeping the switch on drains the battery. The bottom curve shows the induced voltage as it decays over time. This transient DC response ends with the steady-state DC response. What about AC, the stuff of which radio signals are made?

Consider the same circuit as before, but this time, we replace the DC battery with an AC signal generator and simplify the circuit to obtain:

Starting with a series RLC circuit. This circuit is straightforward and the math is pretty easy. The formula for the current through a SERIES circuit is:

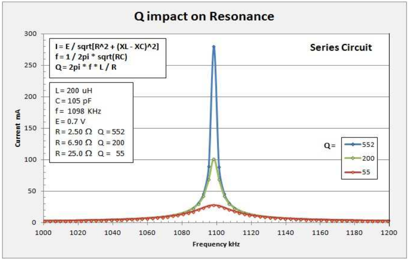


I = V / sqrt[R^2 + (XL - XC)^2]

Since reactances XL = 2pifL and XC = 1/2pifC, the above formula becomes:

I = V / sqrt[R^2 + (2pifL - 1/(2pifC))^2]

The resonance response with input values often discovered in a crystal set, L = 200 uH, C = 105 pF, and R = 6.9 ohms peaks at a frequency of 1098 KHz with a Q factor of 200.



The above plot shows first that it is indeed straightforward to evaluate a circuit in excel, a series circuit anyway. Further, it gives an indication as to what visualizations can be made. I increases with frequency while capacitive reactance decreases. Resonance is that frequency where the two reactance's cancel each other leaving only resistance in the circuit. A little fun derivation may be in order.

Looking at reactance's where f = frequency, L = inductance, and C = capacitance:

XL = 2pifL ... Inductive reactance
XC = 1/2pifC ... Capacitive reactance, so:
2pifL = 1/2pifC ... condition of resonance

taking f out of the denominator:
2pif^2L = 1/2piC

divide both sides by 2piL
f^2 = 1/2^2pi^2LC

take the square root of each side
f = sqrt(1) / sqrt((2pi)^2LC)

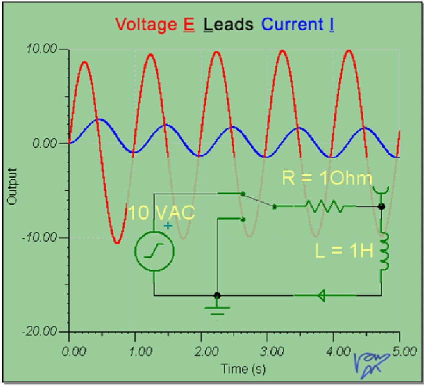
and simplifying...

f = 1 / 2pi sqrt(LC)

This is the general formula for resonance.

In all the plots from here on I will show sweeps of frequency with impedance or current as the dependant variable. The point of resonance on each plot should be apparent as that point where the dependant variable reaches its maximum value.

The AC signal causes the inductor's magnetic field to repeatedly collapse and expand in alternating directions. Ohm's law is constantly running, but now there is a delay. This delay is caused by the union of magnetic field workers whose boss is Maxwell and whose contact is binding. Forget that. Remember this. Voltage E Leads Current I in an inductor.



Circuit 3: Inductive AC Response

ELI the ICE man reminds us that voltage leads current by 90° in an inductor. This is called phase shift. Radio is all about keeping track of phase. We draw voltage in red and current in blue on the same graph so we can see their relationship in time. But what about frequency?

Frequency Response: What Happens When Inductance and Resistance Change?
Now we can play with the inductor and resistor values and see what happens in our circuit. We will measure this by comparing the gain of various configurations. Gain is the amplitude of the signal in a circuit. Gain comes in three flavors, voltage gain, current gain and power gain. Power gain is voltage gain times current gain. We want to know how the gain changes as we change the frequency of our input signal. We can determine this quickly with Tidy TINA. First, we fix the resistor at 1 Ohms and set the inductor to 1 microHenries. Then we ask TINA to compute the AC Transfer Characteristics. Voila! We get a graph that yields major insight.

A graphical look at Resonance
Kevin Smith

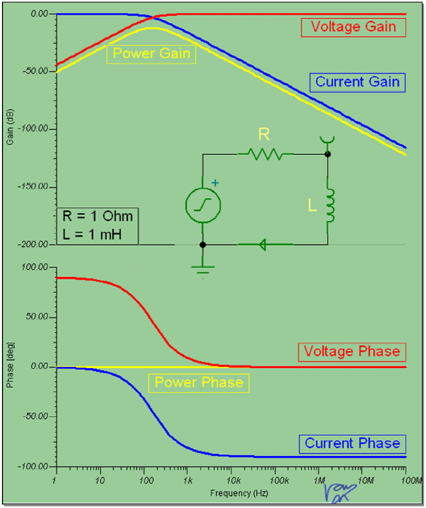
http://www.lessmiths.com/~kjsmith/crystal/resonance.shtml

I write this page in order to review, in my own mind mostly, those things which go into radio frequency resonance. My goal was to cast into spreadsheet form the formulas and equations that determine resonance for simple circuits that one often encounters in crystal radio.

On may ask why create spreadsheets to do what SPICE or Mathematica can easily do already. As I am addressing a crowd who enjoys building their own radios when such can easily and cheaply be had at a local store, I think the answer to my rhetorical question is clear. But more than just "doing it myself", there are good reasons to do so. I have read numerous textbook and online discussions of resonance. Most will feature "resonance curves" either generic cartoons or SPICE plots with no indication of the actual formulas that created the plot. I start to think of SPICE as a kind of black box that magically produces interesting results even to those with little to no understanding of the math inside.

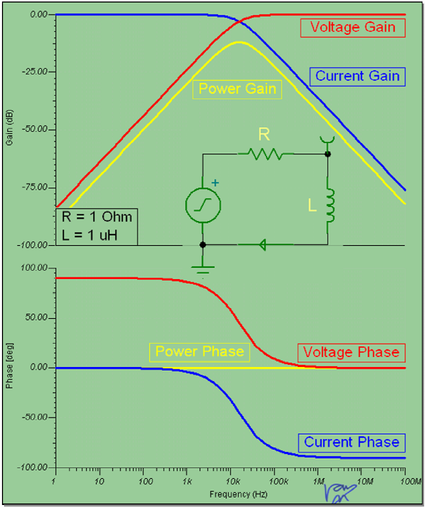
Additionally, those occasional plots with actual scales are for values and magnitudes of R, L, and C that have little to do with medium wave broadcast band work. In order to set or adjust my expectations, I prefer to see things in the scales I typically deal with. Finally, while SPICE and other software may be essential to evaluate complicated circuits, those typically found in crystal radio are really not beyond spreadsheet work.

Something about resonance. An RLC circuit contains both inductive L and capacitive C reactance. Inductive reactance



Circuit 4: One milliHenry Inductor Frequency Response

Next we want to know what happens if the inductance changes, say, to a thousandth of its value. That would be 1 microHenry (uH). Again, TINA computes the AC transfer characteristic, sweeping the frequency from 1 to 100 MegaHertz. This feels more like radio! Out pops our next graph. Decreasing the inductance has shifted all our gain curves to higher frequencies.



Circuit 5: One microHenry Inductor Frequency Response

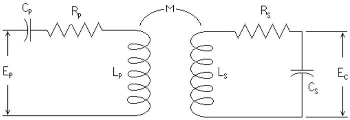
Maximum possible secondary current and maximum possible voltage across C_S, when:

$$\mathbf{E_C / E_P = (1/ 2k_c) * \sqrt{(L_S L_P)} = [\sqrt{(Q_P Q_S)} / 2] * \sqrt{(L_S L_P)}}$$

- goes to zero at unity coupling
- equals 3.01 dB (50 percent loss) at critical coupling
- becomes infinite at zero coupling.

If primary power held constant by adjusting the primary voltage as coupling is varied, then secondary voltage, current, power and power transfer all increase with increasing coupling. There are no maxima at critical coupling -- just the characteristic 3 dB power transfer loss.

<u>Coupled Resonant Circuits</u>, from Terman, p. 154, ff.



The coupling coefficient is:

$$\mathbf{k = M / \sqrt{(L_P\, L_S)}}$$

Voltage ratio:

$$\mathbf{E_C / E_P = -\sqrt{(L_S\, L_P)} * k / [k^2 + 1/ (Q_P\, Q_S)]}$$

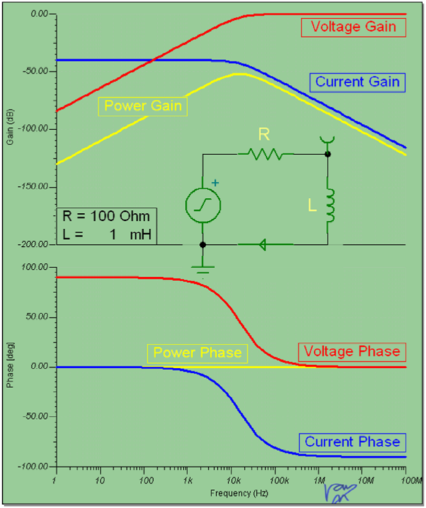
Voltage ratio is maximum when:

$$\mathbf{k = k_c = 1 / \sqrt{(Q_P\, Q_S)}}, \quad \text{and}$$

$$\mathbf{\omega M = \sqrt{(R_P\, R_S)}}$$

Now we can run different cases for hours, and trust me, I have. The trick is to focus on essential relationships. What happens if we change the resistor value but not the inductor?

Let's return the inductor to 1 milliHenry and change the resistance from 1 to 100 Ohms. What happens? How does increased input resistance affect frequency response?



Circuit 6: Inductive Frequency Response – Increased Series Resistance

If we compare Circuit 4 and Circuit 6, there is a loss in current gain as a direct consequence of the resistor. That makes sense. So to first order, we observe that with respect to voltage we have a high pass filter – so named because high frequencies are

Resistance and reactance components of the <u>coupled impedance</u> (see 1.) are:

Resistance component: $$\frac{(\omega M)^2}{R_S^2 + X_S^2} * R_S$$

Reactance component: $$\frac{-j\ (\omega M)^2}{R_S^2 + X_S^2} * X_S$$

R_S and X_S are resistance and reactance components of Z_S.

The effect of the secondary on the primary is exactly as if these components had been inserted, in series, into the primary circuit. The energy and volt-amperes consumed by the primary current flowing through this hypothetical resistance and reactance represent the energy and reactive volt-amperes that are transferred to the secondary circuit. (Alternatively, $\mathbf{Z_P + (\omega M)^2 / Z_S}$ can be calculated directly using standard complex algebra.)

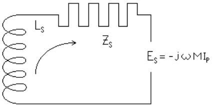
When primary and secondary are both resonant at the same frequency, X_S equals zero, the reactance component equals zero, and coupled impedance equals the resistance component, $\mathbf{(\omega M)^2 / R_S}$.

If, further, the coupling is critical, the coupled impedance is simply R_P.

If input voltage is held constant and coupling is varied, then primary and secondary currents and powers and secondary voltage all change with coupling.
Secondary voltage, current and power all reach maximum values when coupling is critical. Power transfer loss (from primary to secondary):

Z_S = series impedance of secondary when considered by itself

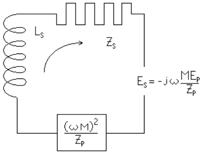
2. The voltage induced into the secondary by primary current, I_P, has a magnitude, ω M I_P, and lags the current that produces it by 90 degrees. The equivalent secondary circuit is:



(I_P is the primary current after the coupled impedance from the secondary has been factored in.)

3. The secondary current is exactly the same current that would flow if the induced voltage, E_S, were applied in series with the secondary and with the primary absent.

An alternate form of the equivalent secondary circuit is:



(Z_P is the primary series impedance before the coupled impedance from the secondary is factored in.)

passed and low frequencies are blocked. With respect to current, we have a low¬¬-¬¬¬¬pass filter, and with respect to power, we have a band-pass filter. Interesting, no?

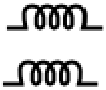
Inductor Calculations

Inductors are like resistors when it comes to equivalent circuits.

Adding two inductors in series gives the equivalent inductance of the pair:

$$\boxed{L_{series} = L_1 + L_2}$$

Formula: Series Equivalent Inductance



Use the product over sum for inductors in parallel:

$$\boxed{L_{parallel} = \frac{L_1 L_2}{L_1 + L_2}}$$

Formula: Parallel Equivalent Inductance

Transformers are inductors that are magnetically coupled by their proximity to each other. We will discuss them in more detail later.

Inductors have a kind of imaginary AC resistance called inductive reactance that has units of Ohms.

$$\boxed{X_L = 2\pi f L}$$

Formula: Inductive Reactance

Inductive circuits have a time constant that we alluded to above. This is the time it takes for the current to build up to 63.2% of its steady state value. The units are seconds.

$$\boxed{t_L = \frac{L}{R}}$$

Formula: Inductive Time Constant

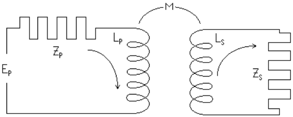
By choosing the right value of inductors, we can tailor the frequencies we block or pass using “analog filtering”. More on that and its upscale digital cousin in a moment. Take a break. Don’t become incapacitated!

Capacitors (Farads)

Chapter 2-3 Coupled Circuits

<u>Inductively Coupled Circuits</u> -- from Terman, p. 149, ff. See also Landee, Davis and Albrecht, Section 13.

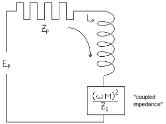
<u>Terman Model</u>:



Pertinent mesh equations:

$\mathbf{E_P = I_P Z_P + j\omega M I_S}$
$\mathbf{E_S = -j\omega M I_P = I_S Z_S}$ induced voltage in secondary

1. The effect of the presence of the secondary, coupled to the primary, is to add a series impedance, $\mathbf{(\omega M)^2 / Z_S}$, to the primary.



M = mutual inductance
$\omega = 2\pi * \text{frequency}$

- the coupled impedance components are added to the primary circuit impedance components to give the complex impedance of the primary.

- magnitude of the total primary current

- the current flow divides going through "Lp", thus the actual current through the L-R branch (through L) is proportional to the ratio of the L-R branch admittance magnitude to the overall "Lp" admittance magnitude:

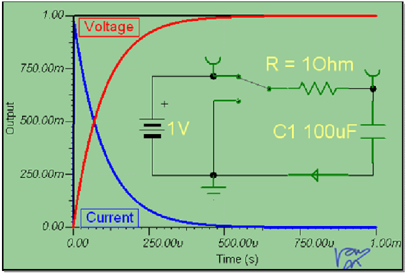
L-R branch current = total primary current * | Y (L-R branch) | / | Y (Co branch) + Y (L-R branch) | (Note, the L-R branch current can actually <u>exceed</u> the total primary current due to the "tank" effect of the Co || (L-R) network.)

- the voltage induced into the secondary Es is calculated (step 2, above)

The voltage |V2| across load resistor Rload is Es times the ratio Rload / |total sec. impedance|.

store energy as an electric field. They consist of plates of foil separated by an insulating or dielectric material. Like their inductive counterparts, capacitors have a direct current (DC) response and an alternating current (AC) response.

When the switch is closed, there is a surge of current until charge accumulates on the plates of the capacitor. After several time constants, the circuit reaches “steady state”. The electric field is established and the current surge disappears. If the switch is opened, nothing happens but the capacitor remains fully charged! A large capacitor can shock you!



Circuit 7: Capacitor Voltage and Current Vs. Time, Transient DC Response

This simulation uses a 1-Farad capacitor, which is physically large, about the size of a large soup can. In radio, we typically work with much smaller values as we shall soon see. The principles and response curves are similar; the time constants are much shorter. Remember these units and abbreviations; you will use them often, especially nano and pico.

Unit	Abbrev.	“Of a Farad”	Multiplier	Comment
Farad	F	1	1	Huge!
milliFarad	mF	1 thousandth	10-3	Big!
microFarad	µF	1 millionth	10-6	Pwr. Sup.
nanoFarad	nF	1 billionth	10-9	Various
picoFarad	pF	1 trillionth	10-12	RF freq.

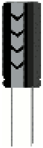


Table 1: Unit Prefixes, Abbreviations and Multipliers

Consider the same circuit as above, but we replace the DC battery with an AC signal generator like so.

The complex impedance and admittance of primary and secondary coils "Lp" and "Ls" are:

$$\mathbf{Z = R / [(1 - \omega^2 LCo)^2 + \omega^2 Co^2R^2]}$$

$$\mathbf{+\ j * \omega [L(1 - \omega^2 LCo) - CoR^2] / [(1 - \omega^2 LCo)^2 + \omega^2 Co^2 R^2]}$$

$$\mathbf{Y = R / [R^2 + \omega^2 L^2] - j * \omega[L(1 - \omega^2 LCo) - CoR^2] / [R^2 + \omega^2 L^2]}$$

The **spreadsheet** requires the following **input data**:

L, **Co** and, at each frequency, **R** for coils "Lp" and "Ls." (L's are true inductances.)

Coupling coefficient **k** or mutual inductance M.

Residual circuit resistances (if any) **Rpri** and **Rsec**.

The spreadsheet proceeds by calculating

- the complex impedance and admittance of the primary coil "Lp"

- the admittance calculation is checked by calculating the admittances of the parallel legs in "Lp": the Co leg and the series L-R leg.

- the complex impedance of secondary coil "Ls"

Resistance and reactance components of the **coupled impedance** are:

Resistance component: $$\frac{(\omega M)^2}{R_S^2 + X_S^2} * R_S$$

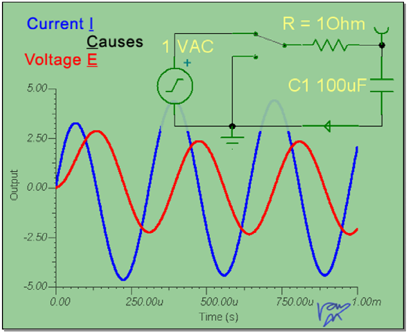
Reactance component: $$\frac{-j\ (\omega M)^2}{R_S^2 + X_S^2} * X_S$$

$\mathbf{R_S}$ and $\mathbf{X_S}$ are the resistance and reactance components of $\mathbf{Z_S}$.

The effect of the secondary on the primary is exactly as if these components had been inserted, in series, into the primary circuit. The energy and volt-amperes consumed by the primary current flowing through this hypothetical resistance and reactance represent the energy and reactive volt-amperes that are transferred to the secondary circuit. (Alternatively, $Z_P + (\omega M)^2 / Z_S$ can be calculated directly using standard complex algebra.)

2. The voltage **Es** induced into the secondary by primary current, $\mathbf{I_P}$, has a magnitude, $\boldsymbol{\omega}$ **M** $\mathbf{I_P}$, and lags the current that produces it by 90 degrees. $\mathbf{I_P}$ is the primary current <u>after</u> the coupled impedance from the secondary has been factored in.

3. The secondary current is exactly the same current that would flow if the induced voltage were applied in series with the secondary and with the primary absent.



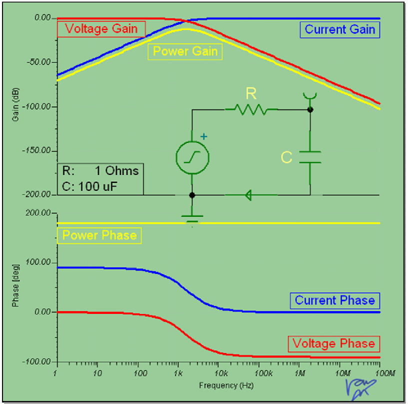
Circuit 8: Capacitive AC Response

The AC signal causes the capacitor's electric field to repeatedly collapse and expand in alternating directions. Ohm's law is constantly running, and again there is a phase delay. Current Causes Voltage in a capacitor. The word "Causes" is just a hack so that we remember the C for Capacitor in the famous ELI-the-ICE-man phrase that reminds us that voltage leads current in inductors and current leads voltage in capacitors.

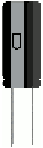
Frequency Response: What Happens When Capacitance and Resistance Change?

Just like before we play with capacitance and resistor values to see what happens in our circuit.

Again, we will measure this by comparing the gain of various configurations. First, we fix the resistor at 1 Ohms and set the capacitor to 100 uF. Then we ask TINA to compute the AC Transfer Characteristic. A graph again provides insight:

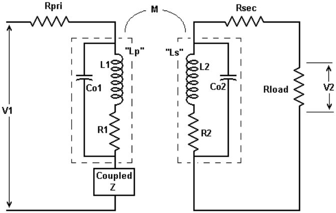


Circuit 9: 100 uF Capacitor Frequency Response



As before we want to know what happens if the capacitance changes, this time to a hundredth of its value. We set the capacitor to 1 uF (1 microFarad). Again, TINA computes the AC transfer

Terman Inductive Coupling Model
Mike Tuggle
(Personal Comm)



While the primary and secondary circuit impedances involve the Co || (L-R) networks "Lp" and "Ls", we assume that the coupling between circuits is a function solely of true inductances L1 and L2 and their mutual inductance M.

The **basis of the calculation** is drawn from F.E.Terman, Radio Engineers' Handbook, 1st ed., 1943, p. 148 ff:

1. The effect of the presence of the secondary, coupled to the primary, is to add a series impedance, **coupled** $\mathbf{Z = (\omega M)^2 / Z_S}$, to the primary, where:

$\mathbf{M}$ = mutual inductance
$\boldsymbol{\omega} = 2\pi * \text{frequency}$
$\mathbf{Z_S}$ = series impedance of secondary when considered by itself

v3 = jω M23 i2

The voltage v1 induced in L1 will be:

v1 = jω M12 i2

Since voltages in series must add up, the voltage across the entire coil v1 will be:

v1 = v2 + v3

Putting these voltages in terms of i2 from the previous relationships, we get

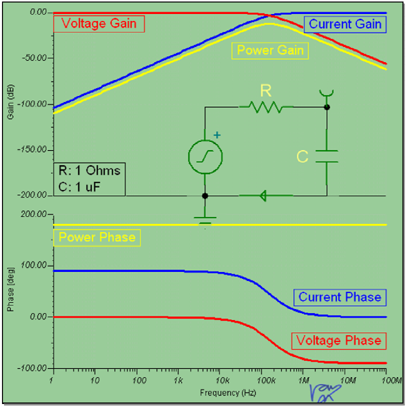
jωM12 i2 = jωL2 i2 + jωM23 i2

Taking out the common factor jωi2, this simplifies to

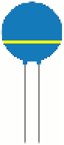
M12 = L2 + M23

This page last updated: January 3, 2016

characteristic, sweeping the frequency from 1 to 100 MegaHertz. That radio feeling is coming on strong.

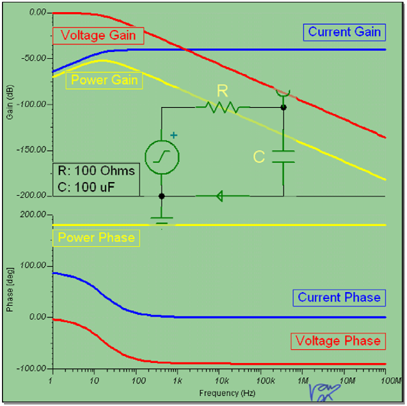


Circuit 10: 1 uF Capacitor Frequency Response



Are you starting to see a pattern? What happens if we change the value of the resistor but not the capacitor? Let’s return the capacitor to 100 uF and change the resistance from 1 to 100 Ohms.

What happens? How does increased series resistance affect circuit frequency response?



Circuit 11: Capacitive Frequency Response – Increased Series Resistance

If we compare Circuit 9 and Circuit 11, there are two effects of keeping the same capacitor and increasing the resistance. Current gain decreases. That makes sense. The second effect is to shift the curves to the left. It looks like we increased the capacitance, but we didn't.

components. To develop formulae to find the required component values is beyond the scope of this discussion. In this case we are better off availing ourselves of a spreadsheet using these formulae, and then plugging in various values until we have the desired frequency range. If it turns out that the practical values of inductor and bandspread capacitor we have to work with don't give a narrow enough frequency range, we can of course, reduce the range of the bandspread capacitor as we did in Parts 1 and 2 by adding a padder and/or trimmer.

Acknowledgement

I would like to express my thanks to Mike Tuggle who provided valuable feedback and also performed SPICE simulations verifying the formulae.

Appendix

Derivation of Mutual Inductance Between a Coil and a Section of the Same Coil

Using the same notation as used previously, L1 is the inductance of the entire coil, L2 is the inductance of the bottom section, and L3 is the inductance of the top section.

If an AC voltage source v2 is impressed across L2, the current i2 will be:

$$i2 = v2/(j\omega L2)$$

and hence

$$v2 = j\omega L2\ i2$$

The voltage v3 induced in L3 will be:

$$(1-k^2)L_1L_2C_AC_B\omega^4 - L_1C_A\omega^2 - L_2C_B\omega^2 + 1 = 0$$
(31)

where the coupling coefficient k is given by:

$$k = \frac{L_1 + L_2 - L_3}{2\sqrt{L_1L_2}}$$

and then (31) can be solved for frequency using the usual quadratic formula:

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

where:

$x = \omega^2$
$a = (1-k^2)L_1L_2C_AC_B$
$b = -(L_1C_A + L_2C_B)$
$c = 1$

Having calculated x from the above, the resonant frequency F will be

$$F = \frac{\sqrt{x}}{2\pi}$$

where F is in Hz, and the other component values are given in Farads and Henries.

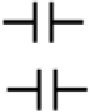
For convenience, the calculation may be done using picofarads and microhenries, but the resulting frequency will then be in Gigahertz, and therefore will need to be multiplied by 1,000,000 to get kHz. Hence:

$$F = \frac{10^6\sqrt{x}}{2\pi}$$

In contrast to Parts 1 and 2, where we selected an arbitrary frequency range and then developed formulae to find the necessary components values, here we have developed a formula to find the resonant frequency from a given set of

We observe that our capacitor drains more slowly when the resistance is higher. In an opposite sense to inductors, capacitors are a low-pass filter with respect to voltage and a high-¬¬¬¬pass filter with respect to current. With respect to power, we have a band-pass filter as before.

Capacitor Calculations
Capacitors are the opposite of inductors and resistors when it comes to equivalent circuits.



Because it looks like increasing plate area, adding two capacitor values gives the PARALLEL equivalent capacitance:

$$\boxed{C_{parallel} = C_1 + C_2}$$

Formula: Parallel Equivalent Capacitance

Use the product over sum for capacitors in SERIES:

$$\boxed{C_{series} = \frac{C_1 C_2}{C_1 + C_2}}$$

Formula: Parallel Equivalent Inductance

There isn't the capacitive equivalent of a transformer.

Capacitive circuits have a time constant. This is the time it takes for the voltage to build up to 63.2% of its steady state value. The units are seconds.

$$\boxed{t_C = RC}$$

Formula: Capacitive Time Constant

Capacitors also have a kind of imaginary AC resistance called capacitive reactance that has units of Ohms.

$$\boxed{X_L = \frac{1}{2\pi f C}}$$

Formula: Capacitive Reactance

That's it for capacitance right now. Consult the references in the last chapter if you want to delve in deeper than time allows here.

Summary – Capacitance and Inductance:

The figure below summarizes what we have just discovered by direct simulation. Inductors and Capacitors are the inverses of each other. This is an idea as deep as the electron itself.The left

aiding or opposing, and the sign and value of M12 will be self-evident.

If we are dealing with the single tapped winding of the circuit in figure 1, then we can directly measure the values of L1, L2 and L3.

Then the mutual inductance between L2 and L3, is

$$M_{23} = \frac{L_1 - L_2 - L_3}{2}$$

And it can be shown (see Appendix below) that the mutual inductance between L1 and L2 is

$$M_{12} = M_{23} + L_2$$

Hence:

$$M_{12} = \frac{L_1 + L_2 - L_3}{2}$$

Now that we have determined M12 by one of the above methods, the coupling coefficient is then given by:

$$k_{12} = \frac{M_{12}}{\sqrt{L_1 L_2}}$$

or combining these two formula, we get:

$$k_{12} = \frac{L_1 + L_2 - L_3}{2\sqrt{L_1 L_2}}$$

This now gives us all the information required to calculate resonant frequency of the bandspread tank circuit.

Summary

The relationship between component values, and resonant frequency of a tapped coil bandspread circuit has been given by formula (31):

$$x = \omega^2$$
$$a = (1 - k^2) L_1 L_2 C_A C_B$$
$$b = -(L_1 C_A + L_2 C_B)$$
$$c = 1$$

Note, that a quadratic equation will generally have two solutions because the square root portion can be either positive or negative. For normal situations, the form given above will provide the correct frequency. The other result is equally valid mathematically, and will give a much higher resonant frequency which we may interpret as a parasitic. Therefore, it may be prudent to ensure that related circuit components are selected to prevent operation at undesired frequencies.

Now, we can plug the component values into the above formula and solve for ω2, and consequently solve for ω and frequency.

The only value that will not be readily known is the coupling coefficient. However it can be calculated from the other coil parameters.

If we are dealing with the isolated windings of the circuit in figure 2, then we can measure the inductance of the two windings separately, and then connect them in series, and measure them again. Using the following formula, the mutual inductance can be calculated:

$$L_{SERIES} = L_1 + L_2 \pm 2M_{12}$$

Hence:

$$M_{12} = \pm \frac{L_{SERIES} - L_1 - L_2}{2}$$

The contribution due to M12 will be positive or negative depending on whether the polarity of the series connections are

column shows a capacitive circuit and the right column shows its inductive counterpart. The component values are summarized in the lower left corner of each diagram. As before voltage gain is red, current gain is blue, and power gain is yellow.

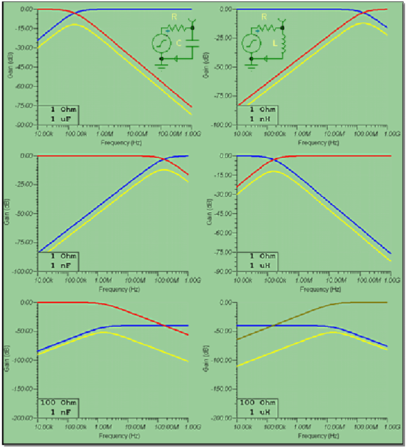
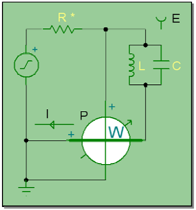


Figure 8: Side-By-Side Comparison - Capacitance and Inductance

RLC Behavior – Parallel Case

Consider an RLC circuit where the inductor and capacitor are in parallel. The following figures catalog voltage, current and power gain as we vary component values. You can reproduce these results in Tidy TINA and see which values result in which curves, a worthwhile bit of fun. The flat line in each image is the gain for the source, set to 1 milliVolt to simulate a strong radio station.



Circuit 12: RLC Circuit – Parallel LC

$$Z_N = Z_{CA} \,||\, \frac{(k^2-1)\omega^2 L_1 L_2 + j\omega L_1 Z_{CB}}{j\omega L_2 + Z_{CB}} \qquad (26)$$

Next, we invert the formula so that we can combine the parallel impedances:

$$\frac{1}{Z_N} = \frac{1}{Z_{CA}} + \frac{j\omega L_2 + Z_{CB}}{(k^2-1)\omega^2 L_1 L_2 + j\omega L_1 Z_{CB}} \qquad (27)$$

Also note that the reciprocal of the impedance is the admittance, and at parallel resonance the admittance 1/ZN (neglecting any resistive components) will be zero. Hence:

$$\frac{1}{Z_{CA}} + \frac{j\omega L_2 + Z_{CB}}{(k^2-1)\omega^2 L_1 L_2 + j\omega L_1 Z_{CB}} = 0 \qquad (28)$$

Capacitive reactance is given by:

ZC = – j/(ωC) = 1/(jωC)

Substituting this for the ZCx terms in the formula:

$$j\omega C_A + \frac{j\omega L_2 + \frac{1}{j\omega C_B}}{(k^2-1)\omega^2 L_1 L_2 + \frac{L_1}{C_B}} = 0 \qquad (29)$$

Multiplying everything by the denominator of the fraction:

$$j\omega C_A((k^2-1)\omega^2 L_1 L_2 + \frac{L_1}{C_B}) + j\omega L_2 + \frac{1}{j\omega C_B} = 0 \qquad (30)$$

and then multiplying by jωCB we get a relatively simple formula:

$$(1-k^2)L_1 L_2 C_A C_B \omega^4 - L_1 C_A \omega^2 - L_2 C_B \omega^2 + 1 = 0 \qquad (31)$$

Since there are no ω or ω3 terms we can solve this as yet another quadratic equation:

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

where

Note that when we convert from impedance to capacitance, 1/n2 becomes n2. Also, since n=N2/N1, and we assume that because N2 represents the tapped part of the coil in the original circuit, it is smaller than N1. Hence, n will be less than 1. Therefore, when CB is transferred to the primary side, its effective value becomes smaller.

We have demonstrated that if the coupling coefficient is 1, then the equivalent circuit is reduced to the secondary impedance multiplied by the turns ratio and transferred to the primary in parallel with the primary inductance and capacitance, which agrees with traditional transformer theory. The capacitor CB on the secondary side can be scaled by multiplying by n2 and placed in parallel with CA on the primary side. However, it's worth noting one important fact. In traditional transformer theory, the self inductance of the primary is generally considered much higher than the transferred impedance from the secondary side, and is often neglected. However, we can't neglect it in our case, since it forms the inductive part of the resonant circuit.

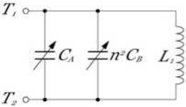


Figure 5

Since we have carried the effect of the coupling coefficient k through to formula (21), we can go back a few steps and see the effect of a coupling coefficient less than 1.

Starting with (21) again:

$$Z_N = Z_{CA} \,||\, \frac{-\omega^2 L_1 L_2 + j\omega L_1 Z_{CB} + \omega^2 k^2 L_1 L_2}{j\omega L_2 + Z_{CB}} \qquad (21)$$

First we combine the common ω2L1L2 terms

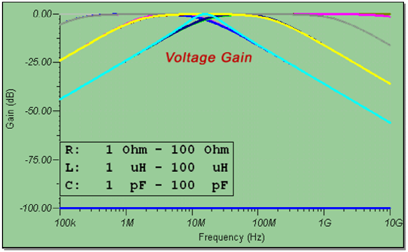


Figure 9: RLC Circuit – Parallel LC – Voltage Gain

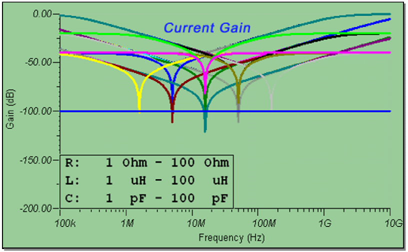


Figure 10: RLC Circuit – Parallel LC – Current Gain

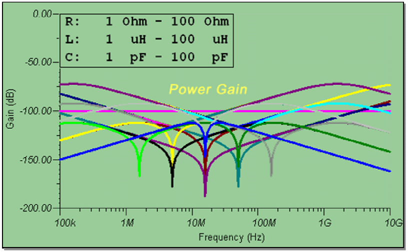


Figure 11: RLC Circuit – Parallel LC – Power Gain

RLC Behavior – Series Case

Consider an RLC circuit where the inductor and capacitor are in series. Note the difference in the response curves in series versus parallel components. You can right click a specific curve in Tidy TINA to discover the RLC values that gave rise to it. One thing you will notice is that while the parallel case is a band-stop filter for RF power, the series case is a band-pass in most cases. The peakiness of the filter is the Q or Quality Factor of the resonant circuit. More on that later. Notice that series resistance hurts performance of the band-pass filter, turning it into a band-stop filter! Not good for tuning in your favorite crystal radio station. Again, we set the source to 1 mV to simulate a strong station. We will make those conditions more severe later.

Taking everything on the right to a common denominator:

$$Z_N = Z_{CA} \,||\, \left(\frac{j\omega L_1(j\omega L_2+Z_{CB})}{j\omega L_2+Z_{CB}} + \frac{\omega^2 k^2 L_1 L_2}{j\omega L_2+Z_{CB}}\right)$$

$$Z_N = Z_{CA} \,||\, \left(\frac{j\omega L_1(j\omega L_2+Z_{CB})+\omega^2 k^2 L_1 L_2}{j\omega L_2+Z_{CB}}\right) \qquad (20)$$

Remembering that **j2=-1**:

$$Z_N = Z_{CA} \,||\, \frac{-\omega^2 L_1 L_2 + j\omega L_1 Z_{CB} + \omega^2 k^2 L_1 L_2}{j\omega L_2+Z_{CB}} \qquad (21)$$

Notice that in the numerator, the first and last terms will cancel each other out when k=1. Let's assume for the moment that this is the case. Then we are left with:

$$Z_N = Z_{CA} \,||\, \frac{j\omega L_1 Z_{CB}}{j\omega L_2+Z_{CB}} \qquad (22)$$

Also when k=1, the n2 relationship exists between inductance values as previously mentioned:

L2/L1 =(N2/N1)2=n2

where we define n as the turns ratio. Substituting L2/n2 for L1

$$Z_N = Z_{CA} \,||\, \frac{1}{n^2}\frac{j\omega L_2 Z_{CB}}{j\omega L_2+Z_{CB}} \qquad (23)$$

Notice that we now have a fraction in the form of AB/(A+B) which is the formula used when combining parallel impedances. Therefore, this is equivalent to having A and B in parallel. Hence:

$$Z_N = Z_{CA} \,||\, \frac{1}{n^2} Z_{CB} \,||\, \frac{1}{n^2} j\omega L_2 \qquad (24)$$

or

$$Z_N = Z_{CA} \,||\, \frac{1}{n^2} Z_{CB} \,||\, j\omega L_1 \qquad (25)$$

So, for the case of k=1 (which would generally apply in the situation of a coil on a ferromagnetic core), we can redraw our circuit as shown in figure 5.

From this we can define Z', the coupled impedance as:

$$Z' = \frac{(\omega M)^2}{Z_2+Z_{CB}} \qquad (16)$$

Using equation (16) we can simplify the circuit of figure 3 into a single loop as shown in figure 4.

In summary then, the effect that the current flowing in the secondary circuit has on the primary circuit can be treated as another voltage drop VZ' due to an impedance Z' applied to the primary circuit as shown in figure 4. Here, Z' is placed in series with the other primary circuit components, and completely accounts for the effect due to the secondary circuit. We have now reduced the original two loop circuit to a single loop circuit.

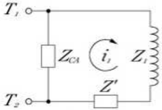


Figure 4

The net impedance ZN of the primary circuit is:

$$Z_N = Z_{CA} \mid\mid (Z_1 + \frac{(\omega M)^2}{Z_2+Z_{CB}}) \qquad (17)$$

where || indicates components combined in parallel.

Substituting (11) into (17) we put M in terms of L1 L2 and k.

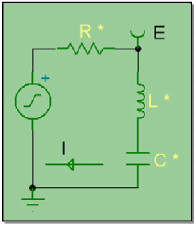
$$Z_N = Z_{CA} \mid\mid (Z_1 + \frac{\omega^2 k^2 L_1 L_2}{Z_2+Z_{CB}}) \qquad (18)$$

If we neglect any resistance in the circuit, then the impedances are purely reactive. The impedance of an inductor is given as:

Z=jωL

Hence:

$$Z_N = Z_{CA} \mid\mid (j\omega L_1 + \frac{\omega^2 k^2 L_1 L_2}{j\omega L_2+Z_{CB}}) \qquad (19)$$



Circuit 13: RLC Circuit – Series LC

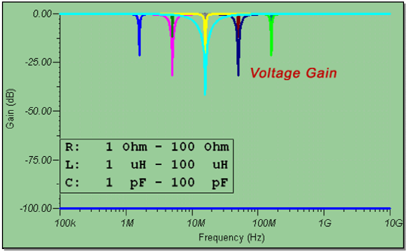


Figure 12: RLC Circuit – Series LC – Voltage Gain

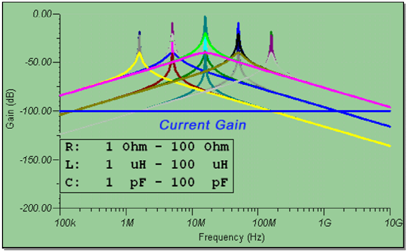


Figure 13: RLC Circuit – Series LC – Current Gain

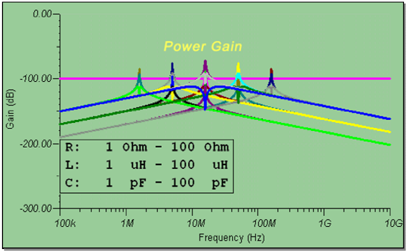


Figure 14: RLC Circuit – Series LC – Power Gain

Now, let us assume that a sinusoidal alternating voltage VT is applied across terminals T1 and T2. This causes an alternating current i1 to flow in the primary circuit. This is illustrated in figure 3, which shows the circuit components as impedances.

From the previous definition of mutual inductance, then this alternating current induces a voltage VZ2 in the secondary circuit across Z2, which in turn causes a current i2 to flow in the secondary circuit.

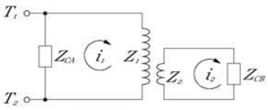


Figure 3

Now, current i2 in the secondary also induces a voltage VZ1 back in the primary across Z1.

The voltage and current relationships are given by the following formulae:

$$V_T = i_1 Z_1 + j\omega M i_2 \quad (12)$$

$$i_2 = \frac{V_{Z2}}{Z_2 + Z_{CB}} \quad (13)$$

$$V_{L2} = -j\omega M i_1 \quad (14)$$

The second term in equation (12) shows that the effect of the secondary circuit corresponds to a voltage drop. We can substitute in equations (13) and (14) to get this voltage drop in terms of i1.

$$V_T = i_1 Z_1 + \frac{(j\omega M)(-j\omega M)i_1}{Z_2 + Z_{CB}}$$

or

$$V_T = i_1 Z_1 + \frac{(\omega M)^2}{Z_2 + Z_{CB}} i_1 \quad (15)$$

$$L_2/L_1 = (N_2/(N_2+N_3))^2 = (N_2/N_1)^2$$

etc.

However, except as specifically noted below we will not make any assumptions about the value of the coupling coefficient.

Note that L1 is not equal to L2+L3 (except when k=0). The inductance of the entire coil is, in fact, given by:

$$L_1 = L_2 + L_3 + 2M_{23}$$

We will leave the circuit of Figure 1 for the time being, and deal with an equivalent coupled circuit consisting of completely separate windings as shown in Figure 2.

In this circuit, we will still assume that even though the coils are separate, they are still inductively coupled. Notice also, we keep the same terminology between figures 1 and 2 such that CA is still connected across L1 and CB is still connected across L2. We will also assume that L1 has the same number of turns in both figures and L2 has the same number of turns in both figures. Consequently, for the purpose of this analysis, both circuits will behave essentially the same. The reason for separating the bandspread section of the circuit into an electrically isolated winding is to make the circuit analysis a bit simpler to follow.

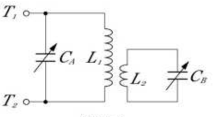
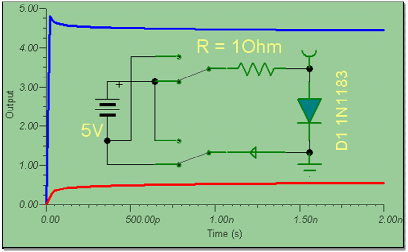


Figure 2

Diodes
do not store energy like capacitors and inductors. They are one-way valves for the flow of current. They are arguably the most important single component in radio because of the multiple purposes they serve. Diodes are semiconductors consisting of a P-N junction doped to attain desireable characteristics. Like their siblings, diodes have a direct current (DC) response and an alternating current (AC) response. These responses can be steady state or transient. Let’s throw the switch!

Notice that the circuit below uses two single pole double throw switches connected so that we can switch the polarity on the diode. We retain the series resistor as a current-limiting resistor, although in a real circuit, say with a light-emitting diode, the value would be considerably higher, between 500 and 2000 Ohms to prevent the diode from burning out.

In this simulation we assume the diode can take whatever the flow of current is and we observe the transient DC response for two cases. The first case when both switches are down, corresponds to the normal polarity of DC voltage seen in previous examples. The diode is positioned so that this is a forward voltage corresponding to the direction in which the diode allows current to flow. The current curve is blue and the forward voltage curve is red. Notice that the diode takes only a few picoseconds for the diode to switch on. The time it takes a diode to turn on is an imporant parameter of the diode, especially for radio work.



Circuit 14: Diode Transient DC Response – Forward Current

Now let's reverse the position of both switches to simulate flipping a double pole double throw (DPDT) switch. This reverses the polarity of the battery. What do think the curves will look like?

$$V_2 = \omega M i_1 \qquad (10)$$

and conversely

$$V_1 = \omega M i_2$$

where the subscripts 1 and 2 designate the parameters of the first and second coil respectively. M is the mutual inductance between the two coils, and the voltages and currents are sinusoidal at an angular frequency ω=2πf. (ω is the lowercase Greek letter omega.) If the two coils are perfectly coupled, then

$$M = \sqrt{L_1 L_2}$$

If they are not perfectly coupled then we introduce a coupling coefficient into the relationship thus:

$$M = k\sqrt{L_1 L_2} \qquad (11)$$

where k ranges between zero (no coupling at all) and one (perfect coupling).

From Figure 1:

L3 is the inductance of the upper section of the coil, and has N3 turns

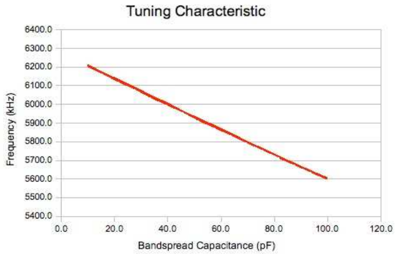
L2 is the inductance of the bottom section of the coil, and has N2 turns

L1 is the total inductance of the whole coil and has N1=N2+N3 turns

If the coupling coefficient is one, then there is a direct relationship between number of turns and inductance, and the following relationships apply:

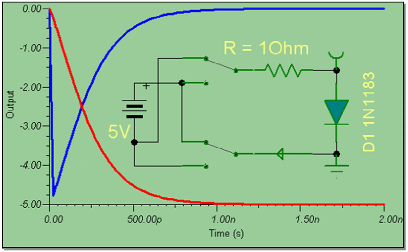
L3/L2=(N3/N2)2

linear relationship between frequency and tuning capacitance, as the following graph illustrates:



In this analysis we will treat the coil as an autotransformer keeping in mind that traditional transformer theory diverges somewhat from loosely coupled coil theory. With transformer theory, the coupling coefficient between windings is nearly unity, which allows for a few simplifications. Therefore, in this analysis, we will need to adjust the model to account for the fact that the sections of the coil are loosely coupled. (This work is based largely on the information provided in the classic texts: 'Radio Engineering' and 'Radio Engineer's Handbook,' both by Frederick E. Terman.)

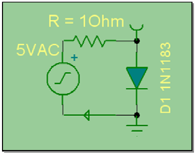
When two coils are inductively coupled, then a current in one coil will induce a magnetic flux in the other coil and vice versa. When the current in one coil changes, the flux changes, and the changing flux will induce a voltage in the other coil according to this relationship:



Circuit 14: Diode Transient DC Response – Reverse Current

This case shows the diode in the direction it does not want to conduct. There is a momentary surge of current until the diode turns off. Note that it takes this diode longer to turn off than it does to turn on – in the simulation at least. About a nanosecond. Do these curves remind you of anything familiar?

Now let's replace the DC battery with an AC signal generator and simplify the circuit to. We don't need the DPDT switch, because the AC signal generator is doing that for us. The simplified circuit looks like this:



Circuit 15: Diode AC Circuit

Let's run the signal generator at a low frequency, say 60 Hz. This is the frequently encountered in power supplies running from wall current in the US after a step-down transformer. We obtain the expected and classic waveform for half-wave rectification of an AC signal.

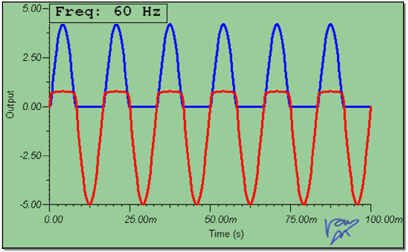


Figure 15: Diode 60 Hz Frequency Response

Now let's run the frequency up to the high end of the audio sampling spectrum, say 44 kHz. Notice that we start

Using a Bandspread Capacitor
Robert Weaver

http://electronbunker.ca/eb/Bandspreading_3.html

Connected to a Tap on the Tank Coil

Here in Part 3, we take off in another direction. We are going to analyze a method of bandspreading where the bandspread capacitor is connected to a tap on the tank inductor, and a bandset capacitor is connected across the entire coil. This configuration is shown in Figure 1.

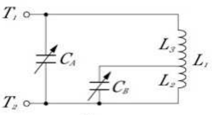


Figure 1

This is a system that has been used on both commercial and homebrew shortwave receivers dating back to the 1930's. The circuit is as shown in figure 1. In this circuit, the main "Bandset" variable capacitor is CA, and may be either fixed or variable, while the "Bandspread" capacitor CB is variable.

In essence, the goal is to find a relationship between circuit impedance across terminals T1 & T2 and frequency for given values of capacitance and inductance.

Before we go any further, we may ask ourselves why we want to do something that looks quite a bit more complicated than the bandspread arrangements we have already looked at. The answer, as we will discover, is that this gives a remarkably is no longer maximum at resonance. As we have already noted, at a frequency slightly below resonance the coupled impedance is *inductive*, whereas the impedance of the series tuned primary is *capacitive*. Therefore there are two frequencies, one slightly above the resonant frequency and one slightly below where the coupled reactance tunes the primary reactance to give minimum total primary impedance and therefore maximum primary current. This double hump in the primary current tends to flatten the peak of the secondary current response curve since the emf induced into the secondary at the resonant frequency is not quite as great as at the two hump frequencies either side of resonance.

If the tuned circuits are overcoupled, the increase in coupled impedance moves the primary current humps further apart. This in turn causes such a decrease in primary current at resonance that the secondary current also starts to show a double humped curve as shown in Fig. 19.15. In practice a coefficient of coupling of 1.5 times the critical coefficient of coupling produces such a slight dip in the secondary current at resonance that the resulting secondary current resonance curve has a very desirable flat top with steep skirts.

encountering some switching noise as we approach the switching speed of the diode.

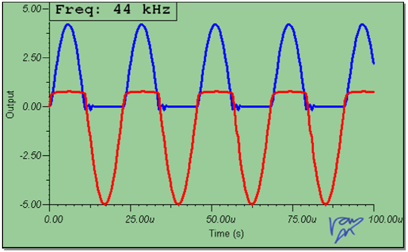


Figure 16: Diode 44 kHz Frequency Response

Finally let's run the diode at a frequency we might encounter in our crystal radio, say the middle of the AM band:

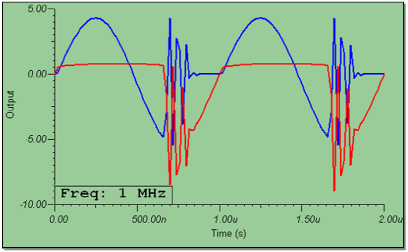


Figure 17: 1N1183 Diode 1 MHz Frequency Response

Now our signal is dominated by switching noise. This particular diode, can't switch fast enough to rectify the signal. Notice the ringing. The only way to see it is to integrate using the Gear method with a 6th order integration. Changing the diode to a faster 1N4150 largely eliminates the ringing.

a critical coupling for a given pair of tuned circuits at which maximum energy transfer from primary to secondary takes place.

At resonance, $Z_p = R_p$

and the coupled $Z = \dfrac{(\omega M)^2}{R_s}$

Therefore for critical coupling,

$$M^2 = R_pR_s/\omega^2$$

But $M^2 = K^2L_pL_s$ (19.32)

Therefore $K^2 = \dfrac{R_p}{\omega L_p} \times \dfrac{R_s}{\omega L_s}$

from which $K_c = \dfrac{1}{\sqrt{Q_pQ_s}}$ (19.40)

With critical coupling, the secondary current attains its greatest value. But with the coupled impedance rising at resonance, the primary current

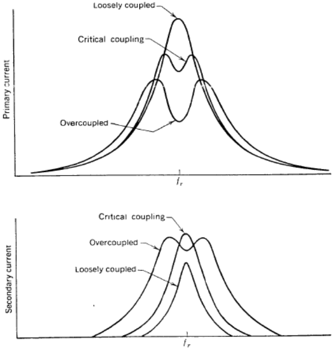


Fig. 19.15. Effect of coefficient of coupling on the resonance curves of a tuned transformer.

Considering now the effect of tuning the secondary circuit, the total secondary circuit impedance [$\mathbf{Z}_s + \mathbf{Z}_L$ in Equation (19.39)] will be minimum and equal to R_s at resonance. Therefore the secondary circuit behaves like a series resonant circuit. Since $(\omega M)^2$ is essentially constant over the small range of frequencies near resonance, when $\mathbf{Z}_s + \mathbf{Z}_L$ becomes a *minimum*, the coupled impedance becomes a *maximum* as shown in Fig. 19.14. And when the source frequency is slightly lower than the resonant frequency, X_C, is greater than X_L, and the secondary impedance becomes capacitive. But dividing a *capacitive* impedance into $(\omega M)^2$ with its 0° angle results in an *inductive* coupled impedance. Therefore, as far as the signal source is concerned, the secondary behaves as if it were a *parallel* resonant circuit in *series* with the primary winding.

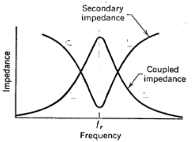


Fig. 19.14. Effect of a tuned secondary circuit.

The primary impedance by itself in the circuit of Fig. 19.13 is attempting to become a minimum at resonance in order to allow maximum primary current to flow. But the coupled impedance is attempting to raise the primary impedance at resonance, thus limiting the maximum primary current. The extent to which the coupled secondary tuned circuit affects the primary resonance curve depends on the degree of coupling between the coils. When the coupling is very loose, the mutual inductance is very small and even at resonance the coupled impedance is *smaller* than the resistance of the primary circuit. Under these circumstances, the only effect that the coupled secondary has on the shape of the primary current resonance curve is to limit its peak value slightly. The secondary current tends to have the usual series resonance curve. But with loose coupling, since the primary current rises to a maximum at resonance and since the secondary induced emf depends on the primary current, the secondary resonance curve is much sharper than that of a single tuned circuit having the same Q.*

We will recall from earlier studies that maximum transfer of energy occurs when the load resistance is equal to the resistance of the source. Applying this to magnetically coupled tuned circuits, maximum energy transfer will take place when the coupled resistance is equal to the primary circuit resistance. Since the coupled resistance depends on the mutual inductance, which in turn depends on the coefficient of coupling, there is

* The bandwidth is approximately $\Delta f \approx K f_r$.

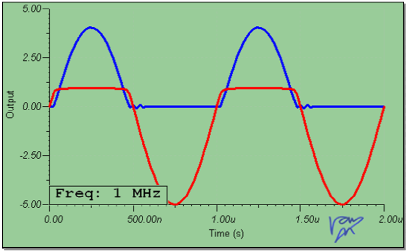


Figure 18: 1N4150 Diode 1 MHz Frequency Response

Diode Calculations

For low frequency applications diodes can be stacked in series.

Connecting two diodes in series doubles the peak reverse voltage (PRV) rating:

$$\boxed{PRV_{series} = PRV_1 + PRV_2}$$

Formula: Series Equivalent Inductance

Connecting two diodes in parallel doubles the current rating:

$$\boxed{I_{parallel} = I_1 + I_2}$$

Formula: Series Equivalent Inductance

It the first case it is necessary connect a high value resistor across eacg diode to minimize transients and equalize slight differences in the characteristics of the diode. One rule of thumb is to multiply the PRV of the diode by 400. In the second a low-value resistor, usually less than an ohm, is connected in series with the pair of diodes.

Summary

This concludes chapter one. We have seen a simple radio, a crystal radio, and how each of the parts work. Now we will look in detail at fundamentals of software defined radio, including software and hardware.

the vector division, dividing an impedance with a + angle into $(\omega M)^2$ with its 0° angle results in a *capacitive* coupled impedance. Since $\mathbf{Z}_p$ is largely *inductive*, the *capacitive* impedance coupled into the primary circuit via the mutual inductance adds the coupled resistance component in series with the primary resistance component but *reduces* the total primary reactance. As a result, the total impedance becomes smaller as the secondary is loaded with a resistance, thus allowing more primary current to flow in order to transfer energy to the secondary circuit. This then checks with our first approach to transformer action in which we discovered that primary current must increase when a transformer is loaded in order to keep the amplitude of the mutual flux sine wave constant.

Example 8: If the resistance of the windings can be neglected, determine the total input impedance to the transformer of Examples 6 and 7, (a) with the secondary open circuit, and (b) with a 50 ohm resistance connected to the secondary.

Solution:

(a) $\mathbf{Z}_{in} = \mathbf{Z}_p = \omega L_p = 377 \times 5 = \mathbf{+j1885\ ohms}$

(b) $\text{Coupled } \mathbf{Z} = \dfrac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L} = \dfrac{(377 \times 1.67)^2}{+j(377 \times 0.8) + 50}$

$$= \frac{396000\underline{/0^\circ}}{304\underline{/80.5^\circ}} = 1300\underline{/-80.5^\circ} = 213 - j1284 \text{ ohms}$$

$$\therefore \quad \mathbf{Z}_{in} = (+j1885) + (213 - j1284) = 213 + j601$$
$$= \mathbf{636\ ohms}\underline{/+70.5^\circ}$$

7. Tuned Transformers

The most important application of loose coupling is the use of tuned transformers in radio circuitry. If we consider the source of emf to be a constant voltage source as in Fig. 19.13, the primary winding is designed to form a series resonant circuit with the capacitive reactance of C_p equal to the inductive reactance of the primary winding at the desired resonant frequency. This will result in maximum current in the primary winding at resonance (neglecting for the moment any coupled impedance due to secondary current).*

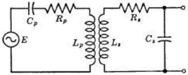


Fig. 19.13. A tuned transformer.

* If a vacuum-tube amplifier is used to feed the primary of a tuned transformer, the vacuum tube represents a constant *current* source providing a signal current equal to the mutual conductance of the tube times the signal input voltage to its grid. Hence C_p must then be connected in *parallel* with the primary winding to obtain maximum primary current (resonant rise of current) at resonance.

form of equivalent circuit for this transformer, representing the self-induced emf by an inductance symbol and the mutually induced emf by a generator symbol. To satisfy Lenz's law, the polarity of the generator must be such that it opposes changes in $\boldsymbol{I}_1$ due to the applied emf $\boldsymbol{E}_1$. Hence, changing the polarity of the generator symbol in the secondary circuit will *not* change the polarity of the voltage $\boldsymbol{I}_s(+j\omega M)$ in the primary circuit. Consequently, Kirchhoff's voltage law equation for the primary loop becomes

$$\boldsymbol{E}_p = \boldsymbol{I}_p(R_p + j\omega L_p) + \boldsymbol{I}_s(+j\omega M)$$

or

$$\boldsymbol{E}_p = \boldsymbol{I}_p\boldsymbol{Z}_p + \boldsymbol{I}_s(+j\omega M) \tag{19.34}$$

where $\boldsymbol{Z}_p$ is the open-circuit impedance of the primary circuit by itself, and ωM is the mutual reactance of the two windings. Note that Lenz's law gives us the same direction for $\boldsymbol{I}_2$ that we chose for our generalized coupling network. [Compare Equation (19.34) with Equation (19.5).] Hence, we write the loop equation for the secondary circuit of Fig. 19.12(c) as

$$0 = \boldsymbol{I}_s(\boldsymbol{Z}_L + R_s + j\omega L_s) + \boldsymbol{I}_p(+j\omega M)$$

or

$$0 = \boldsymbol{I}_s(\boldsymbol{Z}_L + \boldsymbol{Z}_s) + \boldsymbol{I}_p(+j\omega M) \tag{19.35}$$

where $\boldsymbol{Z}_s$ is the open-circuit impedance of the secondary winding by itself. [Compare Equations (19.35) and (19.14).] From Equation (19.35)

$$\boldsymbol{I}_s = \frac{-\boldsymbol{I}_p(+j\omega M)}{\boldsymbol{Z}_s + \boldsymbol{Z}_L} \tag{19.36}$$

Substituting in Equation (19.34),

$$\boldsymbol{E}_p = \boldsymbol{I}_p\boldsymbol{Z}_p - \boldsymbol{I}_p\frac{(+j\omega M)^2}{\boldsymbol{Z}_s + \boldsymbol{Z}_L} \tag{19.37}$$

Equation (19.37) is the equivalent of Equation (19.16). In this case we can go one step further by substituting -1 for j^2. Thus

$$\boldsymbol{E}_p = \boldsymbol{I}_p\boldsymbol{Z}_p + \boldsymbol{I}_p\frac{(\omega M)^2}{\boldsymbol{Z}_s + \boldsymbol{Z}_L} \tag{19.38}$$

and dividing through by $\boldsymbol{I}_p$,

$$\boldsymbol{Z}_{in} = \boldsymbol{Z}_p + \frac{(\omega M)^2}{\boldsymbol{Z}_s + \boldsymbol{Z}_L} \tag{19.39}$$

where $(\omega M)^2/(\boldsymbol{Z}_s + \boldsymbol{Z}_L)$ is the **coupled impedance** *for transformer coupling.* Note that when we substitute -1 for j^2, $(\omega M)^2$ has now become an impedance with a 0° angle.

We can check Equation (19.39) by considering the effect of loading the secondary winding of a pair of magnetically coupled coils. If the secondary is left open-circuit, $\boldsymbol{Z}_L$ is infinitely large and, therefore, the coupled impedance in Equation (19.39) becomes zero. Therefore, the total impedance is simply the primary impedance alone as we would expect. If we connect a resistance across the secondary winding, $\boldsymbol{Z}_L$ will have a 0° angle. Since $\boldsymbol{Z}_s$ is largely the inductive reactance of the secondary winding, the total secondary circuit impedance is inductive. Therefore, when we carry out

Resonant Circuits
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http://mouloudrahmani.com/Electrical/RFMicrowave/ResonantCircuits.html

The resonant circuit is used in electronics systems to selectively pass a certain frequency or group of frequencies from a source to a load while attenuating all other frequencies outside of this passband.

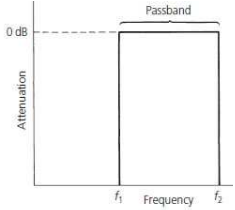


FIG. 2-1. The perfect filter response.

Bandwidth

The bandwidth of any resonant circuit is most commonly defined as being the difference between the upper and lower frequency (f2 −f1) of the circuit at which its amplitude response is 3 dB below the passband response. It is often called the half-power bandwidth.

Q

The ratio of the center frequency of the resonant circuit to its bandwidth is defined as the circuit Q.
$Q = fe/(f2 - f1)$

Shape Factor

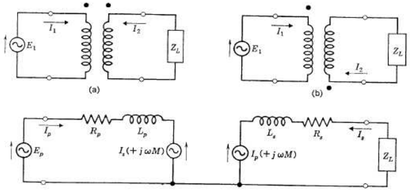
The shape factor of a resonant circuit is typically defined as being the ratio of the 60-dB bandwidth to the 3-dB bandwidth of the resonant circuit. Shape factor is simply a degree of measure of the steepness of the skirts. The smaller the number, the steeper are the response skirts.

Insertion Loss

Whenever a component or group of components is inserted between a generator and its load, some of the signal from the generator is absorbed in those components due to their inherent resistive losses. Thus, not as much of the transmitted signal is transferred to the load as when the load is connected directly to the generator. (No impedance matching function is being performed.) The attenuation that results is called insertion loss and it is usually expressed in decibels (dB).

The voltage division rule (illustrated in Fig. 2-4) states that whenever a shunt element of impedance Zp is placed across the output of a generator with an internal resistance Rs, the maximum output voltage available from this circuit is:
$Vout = (Vin)\, Zp/(Rs + Zp)$

of E_1 in Fig. 19.12, we can reverse the phase of the output voltage by reversing the direction of one of the windings, or simply by reversing the leads to one of the windings. Where it is necessary to keep track of this phase relationship in a circuit diagram, we mark one end of each winding with a dot as shown in Fig. 19.12(a) and (b). If, at a certain moment,



(c)

Fig. 19.12. An equivalent circuit of an air-core transformer.

changing mutual flux induces an instantaneous emf into the primary winding with a polarity such that the dotted end of the winding is positive with respect to the undotted end, then the same mutual flux change must induce an instantaneous emf into the secondary winding with a polarity such that the dotted end of the secondary winding is positive with respect to the undotted end.

Although reversing the secondary polarity will reverse the current through the external circuit, note that I_2 in Fig. 19.12(b) still must flow into the dot end of the secondary winding just as it does in Fig. 19.12(a). Hence, the primary circuit is not affected by any phase reversal we obtain by reversing the secondary leads. In selecting the direction for these current arrows, we must remember that the *same* mutual flux (which is produced by these currents) induces emf's into both primary and secondary windings. So if we draw I_1 pointing into the dot end of the primary winding, we must also draw I_2 pointing into the dot end of the secondary winding.

When we write the Kirchhoff's voltage law (loop) equation for the primary loop, there are three voltages which add up (vectorially) to equal E_1: an IR drop across the resistance of the primary circuit, a *self*-induced emf due to I_1 flowing through the primary winding, and a *mutually* induced emf due to I_2 flowing in the secondary circuit. Figure 19.12(c) shows one

Example 7: If the secondary winding of the transformer of Example 6 has a self-inductance of 0.8 h, what is the coefficient of coupling between the windings?

Solution:

$$K = \frac{M}{\sqrt{L_pL_s}} = \frac{1.67}{\sqrt{5 \times 0.8}} = \mathbf{0.83}$$

One method for experimentally determining mutual inductance is shown in Fig. 19.11. We can measure the total inductance of the series-connected coils on an inductance bridge. Connecting the windings as shown and checking by our hand rule, the flux produced by the current in one winding is in the same direction around the core as the flux produced by the current in the other winding. This increases the total flux, thus increasing the emf induced by a given alternating current; thus increasing the total inductance. All the induced emf's will be in phase. There are four induced emf's: the self-induced emf in the primary, the emf mutually induced in the primary by current in the secondary, the emf mutually induced in the secondary by current in the primary and the self induced emf in the secondary.

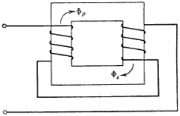


Fig. 19.11. Determining mutual inductance.

$$\therefore \quad E = I(\omega L_p + \omega M + \omega M + \omega L_s)$$

from which $L_T = L_p + L_s + 2M$.

If, however, we reverse the leads to the secondary, the mutually induced emf's in each coil are 180° out of phase with the self-induced emf's resulting in

$$L_T = L_p + L_s - 2M$$

Therefore, we can extend our original Equation (9.6) for two inductances in series to include magnetic coupling between them.

$$\therefore \quad L_T = L_p + L_s \pm 2M \qquad (19.33)$$

The mutual inductance of the magnetically coupled coils in Fig. 19.11 will be one-quarter of the difference between the total inductance readings with the coils connected series aiding and then series opposing.

6. Coupled Impedance

Before we set up the equivalent coupling network for a loosely coupled transformer, we should note a useful practical advantage that transformer coupling has over other forms of coupling networks. For a given direction

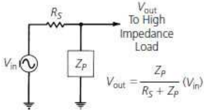


FIG. 2-4. Voltage division rule.

If Zp is a frequencydependent impedance, such as a capacitive or inductive reactance, then Vout will also be frequency dependent and the ratio of Vout to Vin, which is the gain (or, in this case, loss) of the circuit, will also be frequency dependent.

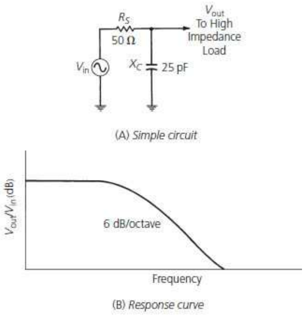


FIG. 2-5. Frequency response of a simple RC (resistor-capacitor) low-pass filter.

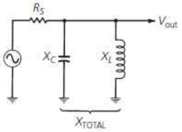


FIG. 2-7. Resonant circuit with two reactive components.

And $$M = L_p \frac{E_s}{E_p} = 5 \times \frac{40}{120} = \mathbf{1.67\ h}$$

Writing Equation (19.28) in terms of the instantaneous emf induced into the secondary,

$$e_s = M \frac{di_p}{dt} \qquad (19.28)$$

And since we are concerned only with sine-wave primary currents,

$$i_p = I_m \sin \omega t \quad \text{and} \quad \frac{di_p}{dt} = \omega I_m \cos \omega t$$

Hence, $$e_s = \omega M I_m \cos \omega t \qquad (19.31)$$

Since the instantaneous emf in Equation (19.31) will be at its peak value when $\cos \omega t = 1$,

$$E_m = 2\pi f M I_m$$

from which $$X_m = \frac{E_{m(\text{sec})}}{I_{m(\text{pri})}} = 2\pi f M \qquad (19.30)$$

Combining Equations (18.1) and (19.28),

$$\frac{N\Phi_m}{t} = \frac{MI_{p_m}}{t}$$

But from Equation (19.27), $\Phi_m = K\Phi_p$

$$\therefore \quad MI_{p_m} = N_s K \Phi_p \qquad (1)$$

If we now reverse the windings and use the original secondary winding as the primary and vice versa, it follows that

$$MI_{s_m} = N_p K \Phi_s \qquad (2)$$

Multiplying Equations (1) and (2),

$$M^2 = K^2 \left(\frac{N_p \Phi_p}{I_{p_m}}\right)\left(\frac{N_s \Phi_s}{I_{s_m}}\right) \qquad (3)$$

But from the definition of self-inductance,

$$E_{av} = \frac{LI_m}{t}$$

and from the definition of the weber,

$$E_{av} = \frac{N\Phi}{t}$$

$$\therefore \quad L = \frac{N\Phi}{I_m}$$

Substituting in Equation (3) gives

$$M = K\sqrt{L_p L_s} \qquad (19.32)$$

5. Mutual Inductance

To develop an equivalent coupling network for an air-core transformer, we require one parameter which is common to both primary and secondary circuits. This parameter is the **mutual inductance** of the pair of coils. In Chapter 9, we found that a coil has a *self*-inductance of one henry when current in that coil *changing* at the rate of one ampere per second induces a cemf of one volt in the coil. Applying the same line of thought to the process of mutual induction, we can say that

> **A pair of magnetically-coupled coils has a mutual inductance of one henry when current changing at the rate of one ampere per second in one coil induces an average emf of one volt in the other coil.**
>
> **The letter symbol for mutual inductance is M.**

Therefore, by definition, $$E_{s_{av}} = \frac{MI_{p_m}}{t} \qquad (19.28)$$

Since the instantaneous current must rise from zero to maximum in one quarter of a cycle of a sine wave,

$$E_{s_{av}} = \frac{MI_{p_m}}{1/4f} = 4fMI_{p_m}$$

But $$E_{av} = \frac{2}{\pi} E_m \qquad (11.15)$$

Therefore $$\frac{2}{\pi} E_{s_m} = 4fMI_{p_m}$$

from which $$E_s = 2\pi fMI_p \qquad (19.29)$$

Rearranging Equation (19.29) gives

$$\frac{E_s}{I_p} = 2\pi fM = X_M \qquad (19.30)$$

where X_M is called the **mutual reactance** *of the magnetically coupled windings.*

> **Example 6:** 120 v, 60 c is applied to the primary of a transformer whose primary inductance is 5 h. The open-circuit secondary voltage is 40 v. Neglecting losses, what is the mutual inductance between the two windings?
>
> *Solution:*
>
> $$E_s = \omega MI_p$$
>
> But $$I_p = \frac{E_p}{\omega L_p}$$
>
> $$\therefore \quad E_s = \frac{M}{L_p} E_p$$

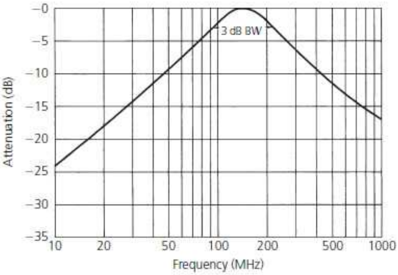


FIG. 2-8. Frequency response of an LC (inductor–capacitor) resonant circuit.

Notice, in Fig. 2-8, that as we near the resonant frequency of the tuned circuit, the slope of the resonance curve increases to 12 dB/octave. This is due to the fact that we now have two significant reactances present and each one is changing at the rate of 6 dB/octave and sloping in opposite directions.

As we move away from resonance in either direction, however, the curve again settles to a 6-dB/octave slope because, again, only one reactance becomes significant.

Loaded Q

The Q of a resonant circuit was defined earlier to be equal to the ratio of the center frequency of the circuit to its 3-dB bandwidth. This “circuit Q,” , is often given the label loaded Q because it describes the passband characteristics of the resonant circuit under actual in-circuit or loaded conditions. The loaded Q of a resonant circuit is dependent upon three main factors.

1 The source resistance (Rs).

2 The load resistance (RL).

3 The component Q.

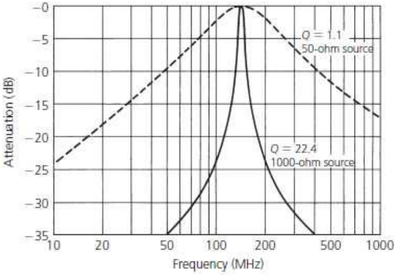


FIG. 2-10. The effect of R_S and R_L on loaded Q.

Raising the source impedance will increase the Q of our resonant circuit.

Mutual Inductance

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http://www.ee.bgu.ac.il/~intrlab/lab_number_7/

http://www.ee.bgu.ac.il/~intrlab/lab_number_7/Mutual%20Inductance%20and%20coupled%20RLC%20circuits.pdf

If an external load were attached to the resonant circuit, the effect would be to broaden or “de-Q” the response curve to a degree that depends on the value of the load resistance.

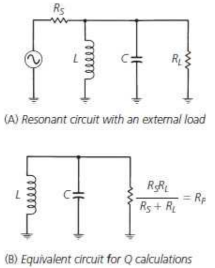


FIG. 2-11. The equivalent parallel impedance across a resonant circuit.

The resonant circuit sees an equivalent resistance of Rs in parallel with RL, as its true load. This total external resistance is, by definition, smaller in value than either Rs or RL, and the loaded Q must decrease. If we put this observation in equation form, it becomes (assuming lossless components):
Q = Rp/Xp

This illustrates that a decrease in Rp will decrease the Q of the resonant circuit and an increase in Rp will increase the circuit Q, and it also illustrates another very important point. The same effect can be obtained by keeping Rp constant and varying Xp.

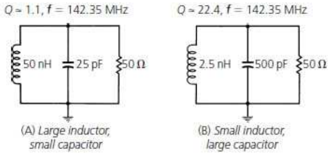


FIG. 2-12. Effect of Q vs. X_p at 142.35 MHz.

In most cases, we only need to involve the Q of the inductor in loaded-Q calculations. The Q of most capacitors is quite high over their useful frequency range, and the equivalent shunt resistance they present to the circuit is also quite high and can usually be neglected. Care must be taken, however, to ensure that this is indeed the case.

Impedance Transformation

Low values of source and load impedance tend to load a given resonant circuit down and, thus, tend to decrease its loaded Q and increase its bandwidth. This makes it very difficult to design a simple LC high-Q resonant circuit for use between two very low values of source and load resistance.

One method of getting around this potential design problem is to make use of one of the impedance transforming circuits shown in Fig. 2-18.

dip much between the peaks and the response is close to what is ideally required. The bandwidth can be estimated as BW = kfo, where k is the coefficient of coupling and fo is the resonant frequency of each circuit.

Figure 3 shows an example of secondary circuit response Vc(f) (voltage measured on capacitor) for different coupling coefficients.

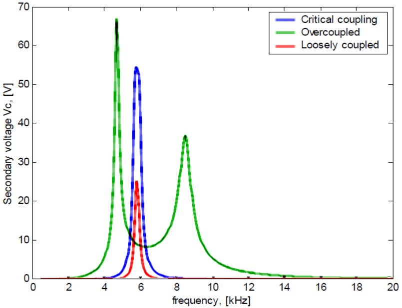


Fig 3

(R = 250 Ohm, C = 5 nF, L = 0.15 H, Vin =5V) inductance (Ze ≈ Z1 + ω2M2·jωC), and an inductance as a capacitance (Ze ≈ Z1 + ω2M2/ jωL).

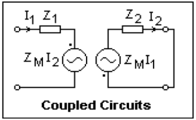


Fig 2

At resonance, the reflected impedance is resistive, and acts to lower the Q of the primary, and thereby to reduce the output. This is counteracted by the increased coupling, which increases the output. The lower Q gives a wider passband. At frequencies lower than exact resonance, the reflected impedance is inductive, which adds to the inductance of the primary and resonates at a lower frequency, producing a peak in the output. At frequencies higher than exact resonance, the reflected impedance is capacitive, which cancels part of the inductance and causes the circuit to resonate at a higher frequency, producing the other peak.

As the coupling is reduced, the response becomes single-peaked at critical coupling, and then decreases as the coupling is made even looser.

The critical coupling coefficient is given by $k_c = \frac{1}{\sqrt{Q_1 \cdot Q_2}}$.

in terms of the Q's of the individual tuned circuits.

Frequently it is assumed that optimum coupling occurs for k ≈ 1.5kc. In this case, the response is double-peaked, but does not

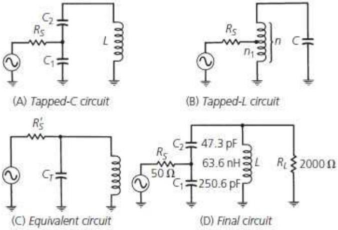


FIG. 2-18. Two methods used to perform an impedance transformation.

For the tapped- C transformer (Fig. 2-18A), we use the formula:

R's = R's (1 + C1/ C2)^2

The equivalent capacitance (CT) that will resonate with the inductor is equal to C1 in series with C2, or:

CT = C1C2/(C1 + C2)

For the tapped-L network of Fig. 2-18B, we use the following formula:

R's = R's (1 + n/ n1)^2

Coupling of Resonant Circuits

In many applications where steep passband skirts and small shape factors are needed, a single resonant circuit might not be sufficient. In situations such as this, individual resonant circuits are often coupled together to produce more attenuation at certain frequencies than would normally be available with a single resonator.

The coupling mechanism that is used is generally chosen specifically for each application, as each type of coupling has its own peculiar characteristics that must be dealt with. The most common forms of coupling are: capacitive, inductive, transformer (mutual), and active (transistor).

Capacitive Coupling

If capacitor C12 of Fig. 2-19 is too large, too much coupling occurs and the frequency response broadens drastically with two response peaks in the filter's passband. If capacitor C12 is too small, not enough signal energy is passed from one resonant circuit to the other and the insertion loss can increase to an unacceptable level. The compromise solution to these two extremes is the point of critical coupling, where we obtain a reasonable bandwidth and the lowest possible insertion loss and, consequently, a maximum transfer of signal power.

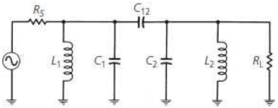


FIG. 2-19. Capacitive coupling.

Thus, the solution from which the frequency response can be obtained is:

$$I_1 = \frac{aV_0}{a^2 - b^2} \text{ and } I_2 = -\frac{bV_0}{a^2 - b^2} \qquad (4)$$

Resonance occurs at the 2 frequencies given by the following equations:

$$\omega_1^2 = \frac{1}{LC \cdot \left[1 - \frac{M}{L}\right]} \text{ and } \omega_2^2 = \frac{1}{LC \cdot \left[1 + \frac{M}{L}\right]}$$

Here, the coupling coefficient k = M/L (for L1=L2).

The behavior of the circuit can be understood qualitatively on the basis of the reflected impedance (or coupled impedance).

A transformer (or inductively coupled circuit) is said to "reflect" impedance in the secondary into the primary circuit. Consider the coupled circuits shown in Fig 2. The positive direction of the currents is chosen into the polarity mark on the generator representing the induced voltages, so that Kirchhoff's equations are

$\mathbf{Z_1 I_1} + \mathbf{Z_M I_2} = \mathbf{V_1}$, and $\mathbf{Z_M I_1} + \mathbf{Z_2 I_2} = 0$. (see Eq.2)

ZM is the mutual impedance jωM, Z1 includes the source impedance, and Z2 the secondary load. These equations may be solved for the equivalent primary impedance

$\mathbf{Z_e} = \mathbf{V_1}/\mathbf{I_1} = \mathbf{Z_1} - \mathbf{Z_M}^2/\mathbf{Z_2}$. (from eqs. (2) and (4))

The reflected impedance is then ω2M2/Z2. Note that a resistance is reflected as a resistance, a capacitance as an

The coupling constant is independent of the number of turns in a coil. The number of turns in a coil determines the magnetic field, which will be produced for a given current. The coupling constant is concerned with how the lines of magnetic force produced by one coil interact with another coil, and hence the coupling constant between two air spaced coils depends only on their physical size and disposition in space. Hence to obtain the best coupling between primary and secondary in an air-cored transformer we can only change the size and spatial relationships of the coils.

Kirchhoff's voltage law equations for the primary and secondary loops are given by

$$\left[R + j\omega L + \frac{1}{j\omega C}\right] \cdot I_1 + j\omega M \cdot I_2 = V_0 \tag{2}$$

$$j\omega M \cdot I_1 + \left[R + j\omega L + \frac{1}{j\omega C}\right] \cdot I_2 = 0$$

(it is assumed that R1+RL1 = R2+RL2 =R and L1 = L2).
We can write these eqs. in the matrix form as follows

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \end{bmatrix} \tag{3}$$

where, $a = \left[R + j\omega L + \frac{1}{j\omega C}\right]$ and $b = j\omega M$.

Following Cramer's rule $I_1 = \frac{\Delta_1}{\Delta}$ and $I_2 = \frac{\Delta_2}{\Delta}$,

where $\Delta_1 = \begin{vmatrix} V_0 & b \\ 0 & a \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & V_0 \\ b & 0 \end{vmatrix}$, and $\Delta = a^2 - b^2$.

There are instances in which overcoupling or undercoupling might serve a useful purpose in a design, such as in tailoring a specific frequency response that a critically coupled filter cannot provide.

In this section, we will only concern ourselves with critical coupling as it pertains to resonant circuit design.

The loaded Q of a critically coupled two-resonator circuit is approximately equal to 0.707 times the loaded Q of one of its resonators. Therefore, the 3-dB bandwidth of a two-resonator circuit is actually wider than that of one of its resonators. Thus, the main purpose of the two-resonator passively coupled filter is not to provide a narrower 3-dB bandwidth, but to increase the steepness of the stopband skirts and, thus, to reach an ultimate attenuation much faster than a single resonator could.

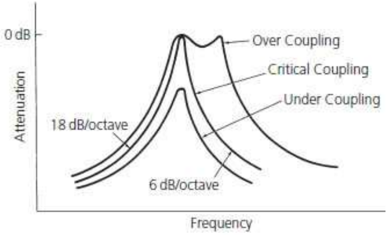


FIG. 2-20. The effects of various values of capacitive coupling on passband response.

Notice that even for the critically coupled case, the response curve is not symmetric around the center frequency but is skewed somewhat. The lower frequency portion of the response plummets down at the rate of 18-dB/octave while the upper slope decreases at only 6-dB/octave.

Below resonance, we have the circuit of Fig. 2-22A. The reactance of the two resonant-circuit capacitors (Fig. 2-19) has increased, and the reactance of the two inductors has decreased to the point that only the inductor is seen as a shunt element and the capacitors can be ignored. This leaves three reactive components and each contributes 6-dB/octave to the response.

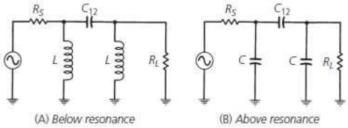


FIG. 2-22. Equivalent circuit of capacitively coupled resonant circuits.

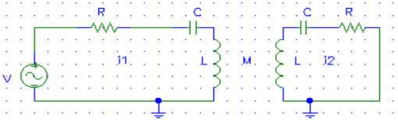
Above resonance, the equivalent circuit approaches the configuration of Fig. 2-22B. Here the inductive reactance has increased above the capacitive reactance to the point where the inductive reactance can be ignored as a shunt element. We now have an arrangement of three capacitors that effectively look like a single shunt capacitor and yields a slope of 6-dB/octave.

Inductive Coupling

Two inductively coupled RLC circuits
Ben-Gurion University

http://www.ee.bgu.ac.il/~intrlab/lab_number_7/Two%20inductively%20coupled%20RLC%20circuits.pdf

Two inductively coupled RLC circuits are shown in Figure 1. Having 2 circuits gives 2 resonant frequencies whose separation depends on the value of the mutual inductance M (the ratio of the voltage in the secondary to the rate of change of primary current with time, and the unit is the henry. This has a reactance at the operating frequency Xm = ωM).



The mutual inductance coupling between primary and secondary can be related to their self-inductance by means of the coupling constant k:

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}}$$

Notice, that since k is defining the relationship between magnetic flux linkages in the circuit, it can never be greater than 1. A value of 1 means that all the flux produced by the primary is linked with the secondary and vice versa. A value of k greater than 1 would mean that more than all of the flux produced by the primary is linked with the secondary.

control and high Q are essential to good response and selectivity.

If the primary circuit is made to resonate at a different frequency from the secondary, audio response is much worse, and considerable distortion is likely. Moreover, the response at mean frequency is less than it would be if the circuits were properly tuned. Air-core transformers are usually made adjustable for tuning and coupling.

Two types of inductively coupled resonant circuits are shown in Fig. 2-23. One type (Fig. 2-23A) uses a series inductor or coil to transfer energy from the first resonator to the next, and the other type (Fig. 2-23B) uses transformer coupling for the same purpose.

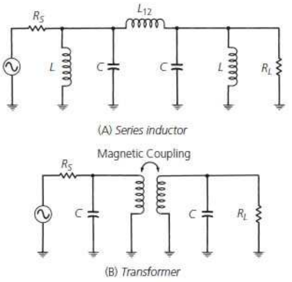


FIG. 2-23. Inductive coupling.

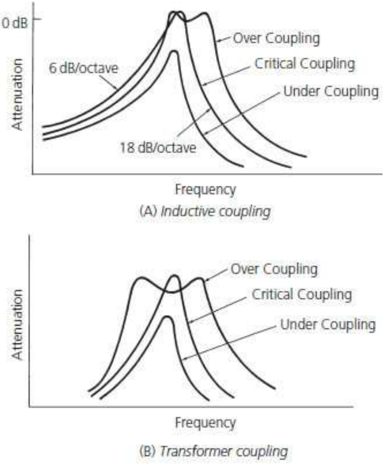


FIG. 2-24. The effects of various values of inductive coupling on passband response.

different values of coupling. If the value of coupling is such that

$$X_M = \sqrt{R_1 R_2}$$

we obtain a condition similar to that of equation 103, in which the maximum power or current is produced in the secondary circuit. Maximum current through condenser C2 gives maximum voltage E2. This value of coupling is known as the critical value. Smaller coefficient of coupling gives a smaller maximum value of E2. Greater coefficient of coupling results in a "double hump" as shown in Fig. 173.

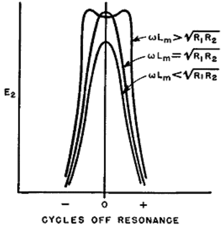


Fig. 173. Response curves for circuit of Fig. 172.

The heights of resonant peaks and frequency distance between peaks depend upon circuit Q and coefficient of coupling k. The double hump curve of Fig. 173 is desirable because, with modulated waves, frequencies in adjacent channels are rejected; yet very little attenuation is offered to audio frequencies which effectively add or subtract from the carrier frequency normally corresponding to resonance. Close tuning

$$k = \frac{L_m}{\sqrt{L_1 L_2}} = \frac{L_m}{\sqrt{\left(L_m + \frac{L_s}{2}\right)^2}} \qquad [104a]$$

$$= \frac{1}{1 + (L_s/2L_m)}$$

If Lm >> Ls

$$k \approx 1 - \frac{L_s}{2L_m} \qquad [104b]$$

Equations 104 (a) and (b) are useful in estimating approximate transformer band width.

A tuned air-core transformer often used in receivers is shown in Fig. 172.

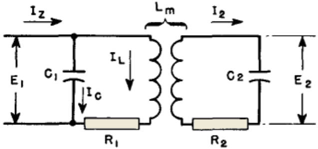


Fig. 172. Tuned air-core transformer.

Here a sinusoidal voltage E1 may be impressed on the primary circuit by a vacuum tube amplifier. Resistances R1 and R2 are usually the inevitable resistance of coils, but occasionally resistance is added to change the circuit response. The value of voltage E2 obtained from this circuit depends on the impressed frequency; in Fig. 173 it is shown for resonance at three

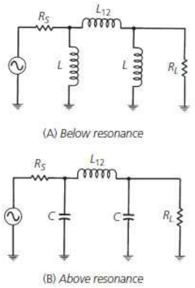


FIG. 2-25. Equivalent circuit of inductively coupled resonant circuits.

states that the mutual reactance XM is the geometric mean between the two values of resistance.

The ratio of mutual inductance to the geometric mean of the primary and secondary self-inductances is the coupling coefficient:

$$k = L_m / \sqrt{(L_1 L_2)} \qquad [104]$$

The value of k is never greater than unity, even when coils are interleaved to the maximum possible extent. Values of k down to 0.01 or lower are common at high frequencies.

Coupling coefficient is related to untuned transformer open- and short-circuit reactance by means of the transformer equivalent circuit shown in Fig. 107(a), p. 147. Assume that the transformer has a 1:1 ratio, and leakage inductance is equally divided between primary and secondary windings. Then if L1 and L2 are the self-inductances of primary and secondary, respectively, Ls is the total leakage inductance (measured in the primary with secondary short-circuited), and Lm the mutual inductance,

$$L_1 = \frac{X_P + X_N}{2\pi f} = \frac{L_s}{2} + L_m$$

$$L_2 = \frac{X_S + X_N}{2\pi f} = \frac{L_s}{2} + L_m$$

From equation 104,

$$0 = Z_2 I_2 + j\omega L_m I_1 \qquad [101]$$

where ω = 2π times operating frequency, and Lm is the mutual inductance between the primary and secondary coils.

From equation 101 we see that the voltage in the secondary coil is numerically equal to ωLmI1, the product of primary current and mutual reactance at the frequency of applied voltage E1. The equivalent impedance of the circuit of Fig. 171 when referred to the primary side is given by

$$Z' = Z_1 + (X_M{}^2/Z_2) \qquad [102]$$

where $X_M = j\omega L_m$.

In the above formulas, the impedances Z1, Z2, and Z' are complex quantities whose real and imaginary terms depend upon the values of resistance, inductance, and capacitance in the circuit. One common practical case arises when the primary resistance is zero, or virtually zero, and the secondary coil is tuned to resonance so that Z2 is a pure resistance R2. Under these conditions, equation 102 reduces to

$$R' = X_M{}^2/R_2 \qquad [103]$$

where R' is the equivalent resistance in the primary.

Equation 103 gives the value of mutual inductance required for coupling a resistance R2 so that it will appear like resistance R' with a maximum power transfer between the two coils, and

RESONANCE

Tony R. Kuphaldt

http://www.allaboutcircuits.com/vol_2/chpt_6/1.html

6.1 An electric pendulum

Capacitors store energy in the form of an electric field, and electrically manifest that stored energy as a potential: static voltage. Inductors store energy in the form of a magnetic field, and electrically manifest that stored energy as a kinetic motion of electrons: current. Capacitors and inductors are flip-sides of the same reactive coin, storing and releasing energy in complementary modes. When these two types of reactive components are directly connected together, their complementary tendencies to store energy will produce an unusual result.

If either the capacitor or inductor starts out in a charged state, the two components will exchange energy between them, back and forth, creating their own AC voltage and current cycles. If we assume that both components are subjected to a sudden application of voltage (say, from a momentarily connected battery), the capacitor will very quickly charge and the inductor will oppose change in current, leaving the capacitor in the charged state and the inductor in the discharged state: (Figure 6.1)

The capacitor will begin to discharge, its voltage decreasing. Meanwhile, the inductor will begin to build up a “charge” in the form of a magnetic field as current increases in the circuit: (Figure 6.2)

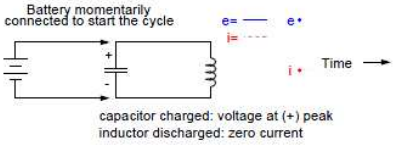


Figure 6.1: Capacitor charged: voltage at (+) peak, inductor discharged: zero current.

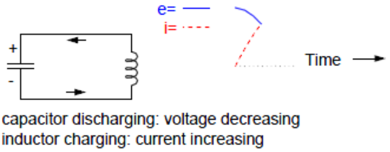


Figure 6.2: Capacitor discharging: voltage decreasing, Inductor charging: current increasing. The inductor, still charging, will keep electrons flowing in the circuit until the capacitor hasbeen completely discharged, leaving zero voltage across it: (Figure 6.3)

Air-Core Transformers
Reuben Lee

http://www.vias.org/eltransformers/lee_electronic_transformers_07b_22.html

Air-Core Transformers

Transformers considered hitherto have had iron or ferrite cores. A class of transformers is widely used in radio-frequency circuits without cores or with small slugs of powdered iron. In a transformer with an iron core, the exciting current required for inducing the secondary voltage is a small percentage of the load component of current. In an air-core transformer all the current is exciting current and induces a secondary voltage proportional to the mutual inductance.

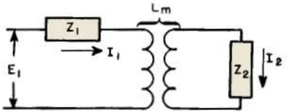


Fig. 171. General case of inductive coupling.

Consider the circuit of Fig. 171 in which Z1 is complex and includes the self-inductance of the primary coil. Likewise, secondary impedance Z2 is complex and includes the self-inductance of the secondary coil. With a sinusoidal voltage applied, Kirchhoff's laws give the following:

$$E_1 = Z_1 I_1 + j\omega L_m I_2 \qquad [100]$$

F.E. Terman, "**Radio Engineers' Handbook**," London, McGraw-Hill, 1st ed., Sep. 1950. P.71

http://www.itermoionici.it/letteratura_files/Radio-Engineers-Handbook.pdf

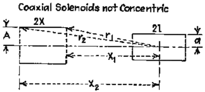


FIG. 42.—Coaxial solenoids not concentric.

15. Mutual Inductance between Single-layer Solenoids. *Coaxial Coils Not Concentric.*—When the configuration is as shown in Fig. 42, the mutual inductance is given by

$$M = 0.02505 \frac{a^2 A^2 n_1 n_2}{4lx} (K_1 k_1 + K_3 k_3 + K_5 k_5) \text{ microhenrys} \tag{85}$$

where a = smaller radius, measured from the axis of the coil to the center of the wire, in.

A = larger radius, measured in the same way, in.

$2l$ = length of coil of smaller radius = number of turns times pitch of winding, in.

$2x$ = length of coil of larger radius, measured in the same way, in.

n_1 and n_2 = total number of turns on the two coils.

$$K_1 = \frac{2}{A^2}\left(\frac{x_2}{r_2} - \frac{x_1}{r_1}\right), \qquad k_1 = 2l$$

$$x_1 = D - x, \qquad r_1 = \sqrt{x_1^2 + A^2}$$

$$x_2 = D + x, \qquad r_2 = \sqrt{x_2^2 + A^2}$$

$$K_3 = \frac{1}{2}\left(\frac{x_1}{r_1^5} - \frac{x_2}{r_2^5}\right), \qquad k_3 = a^2 l\left(3 - \frac{4l^2}{a^2}\right)$$

$$K_5 = -\frac{A^2}{8}\left[\frac{x_1}{r_1^9}\left(3 - \frac{4x_1^2}{A^2}\right) - \frac{x_2}{r_2^9}\left(3 - \frac{4x_2^2}{A^2}\right)\right]$$

$$k_5 = a^4 l\left(\frac{5}{2} - 10\frac{l^2}{a^2} + 4\frac{l^4}{a^4}\right)$$

D = axial distance between centers of coils, in.

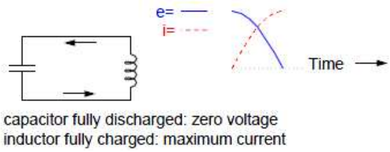


Figure 6.3: Capacitor fully discharged: zero voltage, inductor fully charged: maximum current. The inductor will maintain current flow even with no voltage applied. In fact, it will generate a voltage (like a battery) in order to keep current in the same direction. The capacitor, being the recipient of this current, will begin to accumulate a charge in the opposite polarity as before: (Figure 6.4)

When the inductor is finally depleted of its energy reserve and the electrons come to a halt, the capacitor will have reached full (voltage) charge in the opposite polarity as when it started: (Figure 6.5) Now we're at a condition very similar to where we started: the capacitor at full charge and zero current in the circuit. The capacitor, as before, will begin to discharge through the inductor, causing an increase in current (in the opposite direction as before) and a decrease in voltage as it depletes its own energy reserve: (Figure 6.6)

Eventually the capacitor will discharge to zero volts, leaving the inductor fully charged with

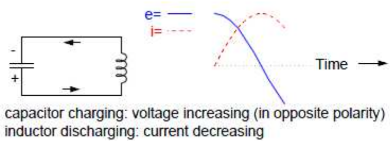


Figure 6.4: Capacitor charging: voltage increasing (in opposite polarity), inductor discharging: current decreasing.

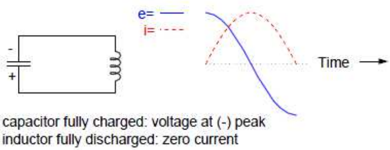
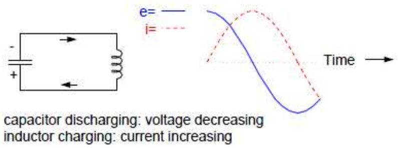


Figure 6.5: Capacitor fully charged: voltage at (-) peak, inductor fully discharged: zero current.



Mutual Inductance,	M :	703	µH
Min Inductance Value,	Lm :	1333	µH
Max Inductance Value,	LM :	4144	µH

When the coils can not rotate but only move along their axis (coaxial coils), assuming a distance D between the two coils centres, we have:

Mutual Inductance,	Md :	10.4	µH
Total Inductance,	Ld :	2759	µH

The total inductance value for this case is computed assuming a positive mutual inductance (coils not rotated 180°).

References:

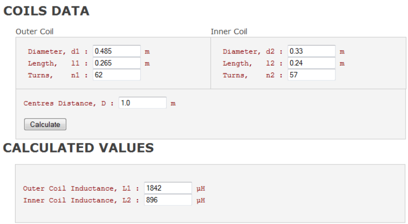
[1] R. Lundin, "A Handbook Formula for the Inductance of a Single-Layer Circular Coil," Proc. IEEE, vol. 73, no. 9, pp. 1428-1429, Sep. 1985.
[2] F.E. Terman, "Radio Engineers' Handbook," London, McGraw-Hill, 1st ed., Sep. 1950.

rotating a coil 180° gives a variation of the total inductance of 4Lm.

Two coaxial coils If the two coaxial coils are moved along their axis instead (D<>0), the mutual inductance decreaes, eventually reaching zero at infinity, but does not change sign. This means that there is less variation in the total inductance just moving the coils with respect to rotating them.

The form below computes the main parameters for the two types of variometer described above. The self-inductances are calculated using the formulas in [1], while the mutual inductances use the formulas from [2], with some minor corrections.

These formulas agree very well with the results of some electromagnetic simulations I have done, see the page for details.


COILS DATA

Outer Coil

Diameter, d1 :	0.485	m
Length, l1 :	0.265	m
Turns, n1 :	62	

Inner Coil

Diameter, d2 :	0.33	m
Length, l2 :	0.24	m
Turns, n2 :	57	

Centres Distance, D : 1.0 m

Calculate

CALCULATED VALUES

Outer Coil Inductance, L1 :	1842	µH
Inner Coil Inductance, L2 :	896	µH

In a typical variometer the two coils are concentric (i.e. D=0) and the overall inductance is varied rotating the inner coil; in this case we have:

Figure 6.6: Capacitor discharging: voltage decreasing, inductor charging: current increasing. full current through it:

(Figure 6.7)

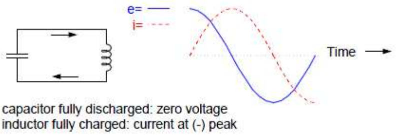


Figure 6.7: Capacitor fully discharged: zero voltage, inductor fully charged: current at (-) peak.

The inductor, desiring tomaintain current in the same direction, will act like a source again, generating a voltage like a battery to continue the flow. In doing so, the capacitor will begin to charge up and the current will decrease in magnitude: (Figure 6.8)

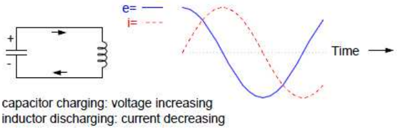


Figure 6.8: Capacitor charging: voltage increasing, inductor discharging: current decreasing.

Eventually the capacitor will become fully charged again as the inductor expends all of its energy reserves trying to maintain current. The voltage will once again be at its positive peak and the current at zero. This completes one full cycle of the energy exchange between the capacitor and inductor: (Figure 6.9)

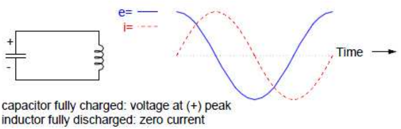


Figure 6.9: Capacitor fully charged: voltage at (+) peak, inductor fully discharged: zero current.

This oscillation will continue with steadily decreasing amplitude due to power losses from stray resistances in the circuit, until the process stops altogether. Overall, this behavior is akin to that of a pendulum: as the pendulum mass swings back and forth, there is a transformation of energy taking place from kinetic (motion) to potential (height), in a similar fashion to the way energy is transferred in the capacitor/inductor circuit back and forth in the alternating forms of current (kinetic motion of electrons) and voltage (potential electric energy).

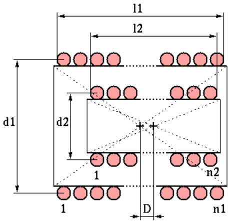
At the peak height of each swing of a pendulum, the mass briefly stops and switches directions. It is at this point that potential energy (height) is at a maximum and kinetic energy (motion) is at zero. As the mass swings back the other way, it passes quickly through a point where the string is pointed

Variometer Design
Claudio Girardi
http://www.qsl.net/in3otd/variodes.html

Variometer Design

A variometer consists usually of two coaxial coils, connected in series, where the coils relative position can be varied in some way. If L1 and L2 are the self-inductances of the first and second coil and Lm is the mutual inductance between the two, the total inductance can be written as $L_{tot}=L1+L2\pm 2Lm$. The mutual inductance is defined as the flux linked by the turns of an inductance when the other carries a unit current; this of course depends not only on the coil length and diameter but also on their relative position.
For two coaxial coils, like the ones in the picture, the mutual inductance is at maximum when they are also concentrical, i.e. D=0.

If the two concentrical coils are rotated, so that their axes are not parallel any more, the mutual inductance decreases, reaching zero (almost) when the angle is 90°. Continuing the rotation beyond 90° the mutual inductance increases again, but this time with the opposite sign. So, according to the above formula,



straight down. At this point, potential energy (height) is at zero and kinetic energy (motion) is at maximum. Like the circuit, a pendulum's back-and-forth oscillation will continue with a steadily dampened amplitude, the result of air friction (resistance) dissipating energy. Also like the circuit, the pendulum's position and velocity measurements trace two sine waves (90 degrees out of phase) over time: (Figure 6.10)

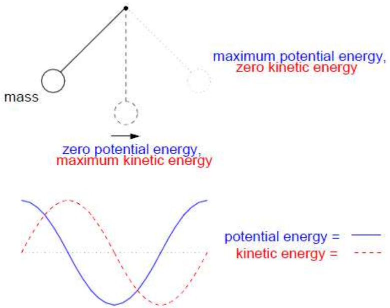


Figure 6.10: Pendelum transfers energy between kinetic and potential energy as it swings low to high.

In physics, this kind of natural sine-wave oscillation for a mechanical system is called Simple Harmonic Motion (often abbreviated as "SHM"). The same underlying principles govern both the oscillation of a capacitor/inductor circuit and the action of a pendulum, hence the similarity in effect. It is an interesting property of any pendulum that its periodic time is governed by the length of the string holding the mass, and not the weight of the mass itself. That is why a pendulum will keep swinging at the same frequency as the oscillations decrease in amplitude. The oscillation rate is independent of the amount of energy stored in it.

The same is true for the capacitor/inductor circuit. The rate of oscillation is strictly dependent on the sizes of the capacitor and inductor, not on the amount of voltage (or current) at each respective peak in the waves. The ability for such a circuit to store energy in the form of oscillating voltage and current has earned it the name tank circuit. Its property of maintaining a single, natural frequency regardless of how much or little energy is actually being stored in it gives it special significance in electric circuit design.

However, this tendency to oscillate, or resonate, at a particular frequency is not limited to circuits exclusively designed for that purpose. In fact, nearly any AC circuit with a combination of capacitance and inductance (commonly called an "LC circuit") will tend to manifest unusual effects when the AC power source frequency approaches that natural frequency. This is true regardless of the circuit's intended purpose.

If the power supply frequency for a circuit exactly matches the natural frequency of the circuit's LC combination, the circuit is said to be in a state of resonance. The unusual effects will reach maximum in this condition of resonance. For this reason, we need to be able to predict what the resonant frequency will be for various combinations of L and C, and be aware of what the effects of resonance are.

$$\Phi = \mu_0 \frac{NI}{l} A. \; L = \frac{N\Phi}{I} = \frac{N}{\cancel{I}} \mu_0 \frac{N\cancel{I}}{l} A$$

$$L = \mu_0 \frac{N^2}{l} A \text{ or } \boxed{L = \mu_0 n^2 A l} \text{ where } n = \frac{N}{l}.$$

(Note how L is independent of the current I.)

be turned into an equation by introducing a constant. Call this constant L, the ***self-inductance*** (or simply ***inductance***) of the coil:

$$N\Phi = LI \quad \text{or} \quad L = \frac{N\Phi}{I}$$

As with mutual inductance, the unit of self-inductance is the henry.

The self-induced emf can now be calculated using Faraday's law:

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta(N\Phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

$$\boxed{\mathcal{E} = -L\frac{\Delta I}{\Delta t}}$$

The above formula is the ***emf due to self-induction***.

Example

Find the formula for the self-inductance of a solenoid of N turns, length l, and cross-sectional area A.

Assume that the solenoid carries a current I. Then the magnetic flux in the solenoid is

• REVIEW:

• A capacitor and inductor directly connected together form something called a tank circuit, which oscillates (or resonates) at one particular frequency. At that frequency, energy is alternately shuffled between the capacitor and the inductor in the form of alternating voltage and current 90 degrees out of phase with each other.

• When the power supply frequency for an AC circuit exactly matches that circuit's natural oscillation frequency as set by the L and C components, a condition of resonance will have been reached.

6.2 Simple parallel (tank circuit) resonance

A condition of resonance will be experienced in a tank circuit (Figure 6.11) when the reactance of the capacitor and inductor are equal to each other. Because inductive reactance increases with increasing frequency and capacitive reactance decreases with increasing frequency, there will only be one frequency where these two reactances will be equal.

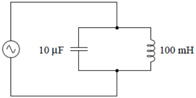


Figure 6.11: Simple parallel resonant circuit (tank circuit).

In the above circuit, we have a 10 µF capacitor and a 100 mH inductor. Since we know the equations for determining the reactance of each at a given frequency, and we're looking for that point where the two reactances are equal to each other, we can set the two reactance formulae equal to each other and solve for frequency algebraically:

$$X_L = 2\pi f L \qquad X_C = \frac{1}{2\pi f C}$$

. . . setting the two equal to each other, representing a condition of equal reactance (resonance) . . .

$$2\pi f L = \frac{1}{2\pi f C}$$

Multiplying both sides by f eliminates the f term in the denominator of the fraction . . .

$$2\pi f^2 L = \frac{1}{2\pi C}$$

Dividing both sides by 2pL leaves f2 by itself on the left-hand side of the equation . . .

$$f^2 = \frac{1}{2\pi 2\pi LC}$$

Taking the square root of both sides of the equation leaves f by itself on the left side . . .

$$f = \frac{\sqrt{1}}{\sqrt{2\pi 2\pi LC}}$$

. . . simplifying . . .

The apparatus used in Experiment EM-11B consists of two coaxial solenoids. A solenoid is essentially just a coil of wire. For a long, tightly-wound solenoid of *n* turns per unit length carrying current *I* the magnetic field over its cross-section is nearly constant and given by $B = \mu_0 n I$. Assume that the two solenoids have the same cross-sectional area *A*. Find a formula for the mutual inductance of the solenoids.

The magnetic flux in the primary coil is

$\Phi_1 = \mu_0 \frac{N_1 I_1}{l_1} A$ where l_1 is the length of the primary coil.

$$M = \frac{N_2 \Phi_2}{I_1} \quad \text{But } \Phi_2 = \Phi_1: \quad M = \frac{N_2 \Phi_2}{I_1} = \frac{N_2}{\cancel{I_1}} \mu_0 \frac{N_1 \cancel{I_1}}{l_1} A; \quad \boxed{M = \mu_0 \frac{N_1 N_2}{l_1} A}$$

(Note how M is independent of the current I_1.)

Self-Inductance

A current-carrying coil produces a magnetic field that links its own turns. If the current in the coil changes the amount of magnetic flux linking the coil changes and, by Faraday's law, an emf is produced in the coil. This emf is called a *self-induced* emf.

Let the coil have N turns. Assume that the same amount of magnetic flux Φ links each turn of the coil. The net flux linking the coil is then $N\Phi$. This net flux is proportional to the magnetic field, which, in turn, is proportional to the current I in the coil. Thus we can write $N\Phi \propto I$. This proportionality can

Let the primary coil have N_1 turns and the secondary coil have N_2 turns. Assume that the same amount of magnetic flux Φ_2 from the primary coil links each turn of the secondary coil. The net flux linking the secondary coil is then $N_2\Phi_2$. This net flux is proportional to the magnetic field, which, in turn, is proportional to the current I_1 in the primary coil. Thus we can write $N_2\Phi_2 \propto I_1$. This proportionality can be turned into an equation by introducing a constant. Call this constant M, the ***mutual inductance*** of the two coils:

$$N_2\Phi_2 = MI_1 \quad \text{or} \quad M = \frac{N_2\Phi_2}{I_1}$$

the unit of inductance is $\dfrac{\text{wb}}{\text{A}} = henry\ (\text{H})$ named after Joseph Henry.

The emf induced in the secondary coil may now be calculated using Faraday's law:

$$\mathcal{E}_2 = -N_2\frac{\Delta\Phi_2}{\Delta t} = -\frac{\Delta\left(N_2\Phi_2\right)}{\Delta t} = -\frac{\Delta\left(MI_1\right)}{\Delta t} = -M\frac{\Delta I_1}{\Delta t}$$

$$\boxed{\mathcal{E}_2 = -M\frac{\Delta I_1}{\Delta t}}$$

The above formula is the ***emf due to mutual induction***.

Example

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

So there we have it: a formula to tell us the resonant frequency of a tank circuit, given the values of inductance (L) in Henrys and capacitance (C) in Farads. Plugging in the values of L and C in our example circuit, we arrive at a resonant frequency of 159.155 Hz.

What happens at resonance is quite interesting. With capacitive and inductive reactances equal to each other, the total impedance increases to infinity, meaning that the tank circuit draws no current from the AC power source! We can calculate the individual impedances of the 10 µF capacitor and the 100mH inductor and work through the parallel impedance formula to demonstrate this mathematically:

$$X_L = 2\pi fL$$

$$X_L = (2)(\pi)(159.155\text{ Hz})(100\text{ mH})$$

$$X_L = 100\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(159.155\text{ Hz})(10\ \mu\text{F})}$$

$$X_C = 100\ \Omega$$

As you might have guessed, I chose these component values to give resonance impedances that were easy to work with (100 even). Now, we use the parallel impedance formula to see what happens to total Z:

$$Z_{\text{parallel}} = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_C}}$$

$$Z_{\text{parallel}} = \frac{1}{\frac{1}{100\ \Omega \angle 90^\circ} + \frac{1}{100\ \Omega \angle -90^\circ}}$$

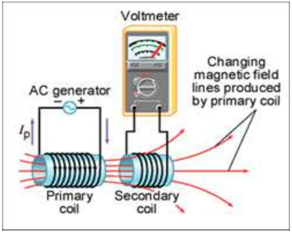
$$Z_{\text{parallel}} = \frac{1}{0.01 \angle -90^\circ + 0.01 \angle 90^\circ}$$

$$Z_{\text{parallel}} = \frac{1}{0} \quad \textit{Undefined!}$$

We can't divide any number by zero and arrive at a meaningful result, but we can say that the result approaches a value of infinity as the two parallel impedances get closer to each other. What this means in practical terms is that, the total impedance of a tank circuit is infinite (behaving as an open circuit) at resonance. We can plot the consequences of this over a wide power supply frequency range with a short SPICE simulation: (Figure 6.12)

The 1 pico-ohm (1 p) resistor is placed in this SPICE analysis to overcome a limitation of SPICE: namely, that it cannot analyze a circuit containing a direct inductor-voltage source loop. (Figure 6.12) A very low resistance value was chosen so as to have minimal effect on circuit behavior.

Mutual Inductance
Dr. David F. Cattell
faculty.ccp.edu/faculty/dcattell/Sp12/.../Mutual%20Inductance.doc



Suppose we hook up an AC generator to a solenoid so that the wire in the solenoid carries AC. Call this solenoid the primary *coil*. Next place a second solenoid connected to an AC voltmeter near the primary coil so that it is coaxial with the primary coil. Call this second solenoid the *secondary coil*. See the figure at the right.

The alternating current in the primary coil produces an alternating magnetic field whose lines of flux *link* the secondary coil (like thread passing through the eye of a needle). Hence the secondary coil encloses a *changing* magnetic field. By Faraday's law of induction this changing magnetic flux induces an emf in the secondary coil. This effect in which changing current in one circuit induces an emf in another circuit is called ***mutual induction***.

the resulting mutual inductance will be equal to the geometric mean of the two individual inductances of the coils.

Also when the inductances of the two coils are the same and equal, L 1 is equal to L 2, the mutual inductance that exists between the two coils will equal the value of one single coil as the square root of two equal values is the same as one single value as shown.

$$M = \sqrt{L_1 L_2} = L$$

Mutual Inductance Example No1

Two inductors whose self-inductances are given as 75mH and 55mH respectively, are positioned next to each other on a common magnetic core so that 75% of the lines of flux from the first coil are cutting the second coil. Calculate the total mutual inductance that exists between the two coils.

$$M = k\sqrt{L_1 L_2}$$

$$M = 0.75\sqrt{75\text{mH} \times 55\text{mH}} = 48.2\text{mH}$$

Mutual Inductance Example No2

When two coils having inductances of 5H and 4H respectively were wound uniformly onto a non-magnetic core, it was found that their mutual inductance was 1.5H. Calculate the coupling coefficient that exists between.

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{1.5}{\sqrt{5 \times 4}} = 0.335 = 33.5\%$$

This SPICE simulation plots circuit current over a frequency range of 100 to 200 Hz in twenty even steps (100 and 200 Hz inclusive). Current magnitude on the graph increases from

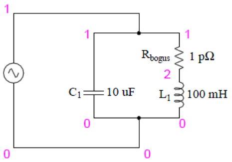


Figure 6.12: Resonant circuit sutitable for SPICE simulation.

```
freq       i(v1)      3.162E-04   1.000E-03   3.162E-03  1.0E-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  9.632E-03 .       .           .           .          *
1.053E+02  8.506E-03 .       .           .           .        * .
1.105E+02  7.455E-03 .       .           .           .      *   .
1.158E+02  6.470E-03 .       .           .           .     *    .
1.211E+02  5.542E-03 .       .           .           .   *      .
1.263E+02  4.663E-03 .       .           .           . *        .
1.316E+02  3.828E-03 .       .           .           .*         .
1.368E+02  3.033E-03 .       .           .          *.          .
1.421E+02  2.271E-03 .       .           .       *   .          .
1.474E+02  1.540E-03 .       .           .    *      .          .
1.526E+02  8.373E-04 .       .         * .           .          .
1.579E+02  1.590E-04 . *     .           .           .          .
1.632E+02  4.969E-04 .       .    *      .           .          .
1.684E+02  1.132E-03 .       .           . *         .          .
1.737E+02  1.749E-03 .       .           .    *      .          .
1.789E+02  2.350E-03 .       .           .       *   .          .
1.842E+02  2.934E-03 .       .           .          *.          .
1.895E+02  3.505E-03 .       .           .           .*         .
1.947E+02  4.063E-03 .       .           .           .  *       .
2.000E+02  4.609E-03 .       .           .           .    *     .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

`tank circuit frequency sweep`

```
v1 1 0 ac 1 sin
c1 1 0 10u
* rbogus is necessary to eliminate a
direct loop
* between v1 and l1, which SPICE can't
handle
rbogus 1 2 1e-12
l1 2 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

left to right, while frequency increases from top to bottom. The current in this circuit takes a sharp dip around the analysis point of 157.9 Hz, which is the closest analysis point to our predicted resonance frequency of 159.155 Hz. It is at this point that total current from the power source falls to zero.

The plot above is produced fromthe above spice circuit file (*.cir), the command (.plot) in the last line producing the text plot on any printer or terminal. A better looking plot is produced by the "nutmeg" graphical post-processor, part of the spice package. The above spice (*.cir) does not require the plot (.plot) command, though it does no harm. The following commands produce the plot below: (Figure 6.13)

```
spice -b -r resonant.raw resonant.cir
( -b batch mode, -r raw file, input is
resonant.cir)
nutmeg resonant.raw
```

From the nutmeg prompt:

```
>setplot ac1 (setplot {enter} for list of
plots)
>display (for list of signals)
>plot mag(v1#branch)
```


However, the above equation assumes zero flux leakage and 100% magnetic coupling between the two coils, L 1 and L 2. In reality there will always be some loss due to leakage and position, so the magnetic coupling between the two coils can never reach or exceed 100%, but can become very close to this value in some special inductive coils.

If some of the total magnetic flux links with the two coils, this amount of flux linkage can be defined as a fraction of the total possible flux linkage between the coils. This fractional value is called the coefficient of coupling and is given the letter k.

Coupling Coefficient

Generally, the amount of inductive coupling that exists between the two coils is expressed as a fractional number between 0 and 1 instead of a percentage (%) value, where 0 indicates zero or no inductive coupling, and 1 indicating full or maximum inductive coupling.

In other words, if $k = 1$ the two coils are perfectly coupled, if $k > 0.5$ the two coils are said to be tightly coupled and if $k < 0.5$ the two coils are said to be loosely coupled. Then the equation above which assumes a perfect coupling can be modified to take into account this coefficient of coupling, k and is given as:

Coupling Factor Between Coils

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad \text{or} \quad M = k\sqrt{L_1 L_2}$$

When the coefficient of coupling, k is equal to 1, (unity) such that all the lines of flux of one coil cuts all of the turns of the second coil, that is the two coils are tightly coupled together,

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

Likewise, the flux linking coil one, L1 when a current flows around coil two, L2 is exactly the same as the flux linking coil two when the same current flows around coil one above, then the mutual inductance of coil one with respect of coil two is defined as M21. This mutual inductance is true irrespective of the size, number of turns, relative position or orientation of the two coils. Because of this, we can write the mutual inductance between the two coils as: M12 = M21 = M.

Then we can see that self inductance characterises an inductor as a single circuit element, while mutual inductance signifies some form of magnetic coupling between two inductors or coils, depending on their distance and arrangement, an hopefully we remember from our tutorials on Electromagnets that the self inductance of each individual coil is given as:

$$L_1 = \frac{\mu_o \mu_r N_1^2 A}{\ell} \quad \text{and} \quad L_2 = \frac{\mu_o \mu_r N_2^2 A}{\ell}$$

By cross-multiplying the two equations above, the mutual inductance, M that exists between the two coils can be expressed in terms of the self inductance of each coil.

$$M^2 = L_1 L_2$$

giving us a final and more common expression for the mutual inductance between the two coils of:

$$M = \sqrt{L_1 L_2} \ \ \text{H}$$

Mutual Inductance Between Coils


```
(magnitude of complex current vector
v1#branch)
```

Incidentally, the graph output produced by this SPICE computer analysis is more generally known as a Bode plot. Such graphs plot amplitude or phase shift on one axis and frequency on the other. The steepness of a Bode plot curve characterizes a circuit's "frequency response," or how sensitive it is to changes in frequency.

• REVIEW:

• Resonance occurs when capacitive and inductive reactances are equal to each other.

• For a tank circuit with no resistance (R), resonant frequency can be calculated with the following formula:

$$f_{resonant} = \frac{1}{2\pi \sqrt{LC}}$$

• The total impedance of a parallel LC circuit approaches infinity as the power supply frequency approaches resonance.

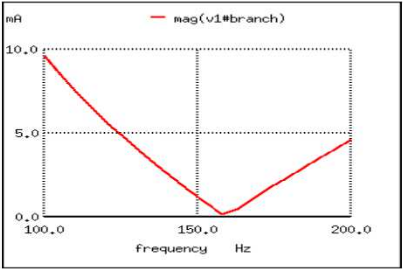


Figure 6.13: Nutmeg produces plot of current I(v1) for parallel resonant circuit.

• A Bode plot is a graph plotting waveform amplitude or phase on one axis and frequency on the other.

6.3 Simple series resonance

A similar effect happens in series inductive/capacitive circuits. (Figure 6.14) When a state of resonance is reached (capacitive and inductive reactances equal), the two impedances cancel each other out and the total impedance drops to zero!

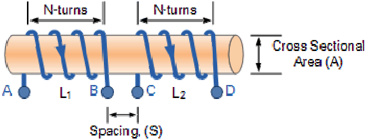
wound one on top of the other over a common soft iron core unity coupling is said to exist between them as any losses due to the leakage of flux will be extremely small. Then assuming a perfect flux linkage between the two coils the mutual inductance that exists between them can be given as.

$$M = \frac{\mu_o \mu_r N_1 N_2 A}{\ell}$$

Where:

- µo is the permeability of free space (4.π.10-7)
- µr is the relative permeability of the soft iron core
- N is in the number of coil turns
- A is in the cross-sectional area in m2
- l is the coils length in meters

Mutual Induction

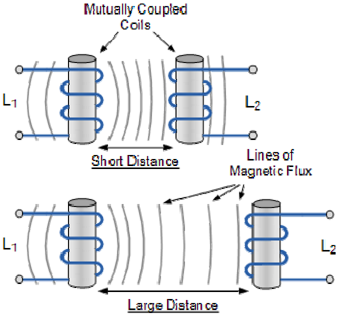


Here the current flowing in coil one, L1 sets up a magnetic field around itself with some of these magnetic field lines passing through coil two, L2 giving us mutual inductance. Coil one has a current of I1 and N1 turns while, coil two has N2 turns. Therefore, the mutual inductance, M12 of coil two that exists with respect to coil one depends on their position with respect to each other and is given as:

coil turns of the second coil inducing a relatively large emf and therefore producing a large mutual inductance value.

Likewise, if the two coils are farther apart from each other or at different angles, the amount of induced magnetic flux from the first coil into the second will be weaker producing a much smaller induced emf and therefore a much smaller mutual inductance value. So the effect of mutual inductance is very much dependant upon the relative positions or spacing, (S) of the two coils and this is demonstrated below.

Mutual Inductance between Coils



The mutual inductance that exists between the two coils can be greatly increased by positioning them on a common soft iron core or by increasing the number of turns of either coil as would be found in a transformer. If the two coils are tightly

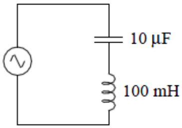


Figure 6.14: Simple series resonant circuit.

At 159.155 Hz:

$Z_L = 0 + j100\ \Omega$ $\qquad$ $Z_C = 0 - j100\ \Omega$

$$Z_{series} = Z_L + Z_C$$

$$Z_{series} = (0 + j100\ \Omega) + (0 - j100\ \Omega)$$

$$Z_{series} = 0\ \Omega$$

With the total series impedance equal to 0 at the resonant frequency of 159.155 Hz, the result is a short circuit across the AC power source at resonance. In the circuit drawn above, this would not be good. I'll add a small resistor (Figure 6.15) in series along with the capacitor and the inductor to keep the maximum circuit current somewhat limited, and perform another SPICE analysis over the same range of frequencies: (Figure 6.16)

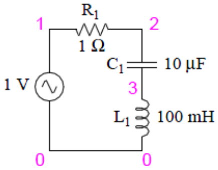


Figure 6.15: Series resonant circuit suitable for SPICE.

```
series lc circuit
v1 1 0 ac 1 sin
r1 1 2 1
c1 2 3 10u
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

As before, circuit current amplitude increases from bottom to top, while frequency increases from left to right. (Figure 6.16) The peak is still seen to be at the plotted frequency point of 157.9 Hz, the closest analyzed point to our predicted resonance point of 159.155 Hz. This would suggest that our resonant frequency formula holds as true for simple series LC circuits as it does for simple parallel LC circuits, which is the case:

Mutual Inductance of Two Coils
Wayne Storr
http://www.electronics-tutorials.ws/inductor/mutual-inductance.html

Mutual Inductance of Two Coils

In the previous tutorial we saw that an inductor generates an induced emf within itself as a result of the changing magnetic field around its own turns, and when this emf is induced in the same circuit in which the current is changing this effect is called Self-induction, (L). However, when the emf is induced into an adjacent coil situated within the same magnetic field, the emf is said to be induced magnetically, inductively or by Mutual induction, symbol (M). Then when two or more coils are magnetically linked together by a common magnetic flux they are said to have the property of Mutual Inductance.

Mutual Inductance is the basic operating principal of transformers, motors, generators and any other electrical component that interacts with another magnetic field. Then we can define mutual induction as the current flowing in one coil induces an emf in an adjacent coil. But mutual inductance can also be a bad thing as "stray" or "leakage" inductance from a coil can interfere with the operation of another adjacent component by means of electromagnetic induction, so some form of electrical screening to a ground potential may be required.

The amount of mutual inductance that links one coil to another depends very much on the relative positioning of the two coils. If one coil is positioned next to the other coil so that their physical distance apart is small, then nearly nearly all of the magnetic flux generated by the first coil will interact with the a frequency point where their two reactive components cancel each other out influencing the characteristics of the circuit. Both circuits have a resonant frequency point.

The difference this time however, is that a parallel resonance circuit is influenced by the currents flowing through each parallel branch within the parallel LC tank circuit. A tank circuit is a parallel combination of L and C that is used in filter networks to either select or reject AC frequencies. Consider the parallel RLC circuit below.

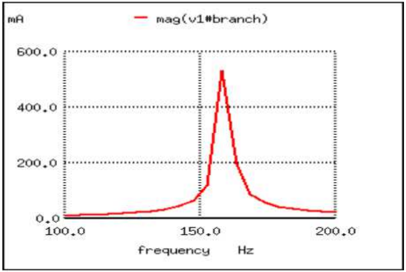


Figure 6.16: Series resonant circuit plot of current I(v1).

$$f_{resonant} = \frac{1}{2\pi\sqrt{LC}}$$

A word of caution is in order with series LC resonant circuits: because of the high currents which may be present in a series LC circuit at resonance, it is possible to produce dangerously high voltage drops across the capacitor and the inductor, as each component possesses significant impedance. We can edit the SPICE netlist in the above example to include a plot of voltage across the capacitor and inductor to demonstrate what happens: (Figure 6.17)

```
series lc circuit
v1 1 0 ac 1 sin
r1 1 2 1
```

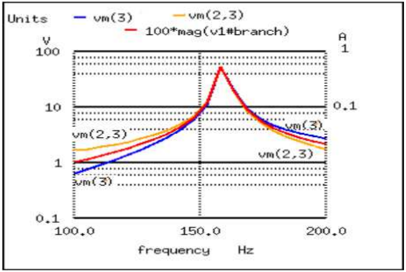
```
c1 2 3 10u
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1) v(2,3) v(3)
.end
```

According to SPICE, voltage across the capacitor and inductor reach a peak somewhere around 70 volts! This is quite impressive for a power supply that only generates 1 volt. Needless to say, caution is in order when experimenting with circuits such as this. This SPICE voltage is lower than the expected value due to the small (20) number of steps in the AC analysis statement (.ac lin 20 100 200). What is the expected value?

Given: f_r = 159.155 Hz, L = 100mH, R = 1

$X_L = 2\pi fL = 2\pi(159.155)(100mH) = j100\Omega$

$X_C = 1/(2\pi fC) = 1/(2\pi(159.155)(10\mu F)) = -j100\Omega$



G + jB with the duality between the two complex impedances being defined as:

Series Circuit	Parallel Circuit
Voltage, (V)	Current, (I)
Resistance, (R)	Conductance, (G)
Reactance, (X)	Susceptance, (B)
Impedance, (Z)	Admittance, (Y)

As susceptance is the reciprocal of reactance, in an inductive circuit, inductive susceptance, BL will be negative in value and in a capacitive circuit, capacitive susceptance, BC will be positive in value. The exact opposite to XL and XC respectively.

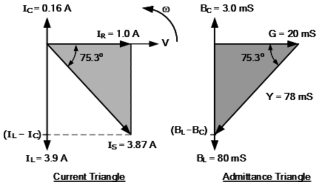
We have seen so far that series and parallel RLC circuits contain both capacitive reactance and inductive reactance within the same circuit. If we vary the frequency across these circuits there must become a point where the capacitive reactance value equals that of the inductive reactance and therefore, XC = XL. The frequency point at which this occurs is called resonance and in the next tutorial we will look at series resonance and how its presence alters the characteristics of the circuit.

Parallel Resonance

The Parallel Resonance Circuit

In many ways a parallel resonance circuit is exactly the same as the series resonance circuit we looked at in the previous tutorial. Both are 3-element networks that contain two reactive components making them a second-order circuit, both are influenced by variations in the supply frequency and both have

Current and Admittance Triangles



Parallel RLC Circuit Summary

In a parallel RLC circuit containing a resistor, an inductor and a capacitor the circuit current IS is the phasor sum made up of three components, IR, IL and IC with the supply voltage common to all three. Since the supply voltage is common to all three components it is used as the horizontal reference when constructing a current triangle.
Parallel RLC networks can be analysed using vector diagrams just the same as with series RLC circuits. However, the analysis of parallel RLC circuits is a little more mathematically difficult than for series RLC circuits when it contains two or more current branches. So an AC parallel circuit can be easily analysed using the reciprocal of impedance called Admittance.

Admittance is the reciprocal of impedance given the symbol, Y. Like impedance, it is a complex quantity consisting of a real part and an imaginary part. The real part is the reciprocal of resistance and is called Conductance, symbol Y while the imaginary part is the reciprocal of reactance and is called Susceptance, symbol B and expressed in complex form as: Y =

Figure 6.17: Plot of Vc=V(2,3) 70 V peak, VL=v(3) 70 V peak, I=I(V1#branch) 0.532 A peak

$$
\begin{aligned}
Z &= 1 + j100 - j100 = 1\ \Omega \\
I &= V/Z = (1\ V)/(1\ \Omega) = 1\ A \\
V_L &= IZ = (1\ A)(j100) = j100\ V \\
V_C &= IZ = (1\ A)(-j100) = -j100\ V \\
V_R &= IR = (1\ A)(1) = 1\ V \\
V_{total} &= V_L + V_C + V_R \\
V_{total} &= j100 - j100 + 1 = 1\ V
\end{aligned}
$$

The expected values for capacitor and inductor voltage are 100 V. This voltage will stress these components to that level and they must be rated accordingly. However, these voltages are out of phase and cancel yielding a total voltage across all three components of only 1 V, the applied voltage. The ratio of the capacitor (or inductor) voltage to the applied voltage is the “Q” factor.

$$Q = V_L/V_R = V_C/V_R$$

• REVIEW:

• The total impedance of a series LC circuit approaches zero as the power supply frequency approaches resonance.

• The same formula for determining resonant frequency in a simple tank circuit applies to simple series circuits as well.

• Extremely high voltages can be formed across the individual components of series LC circuits at resonance, due to high current flows and substantial individual component impedances.

6.4 Applications of resonance

So far, the phenomenon of resonance appears to be a useless curiosity, or at most a nuisance to be avoided (especially if series resonance makes for a short-circuit across our AC voltage source!). However, this is not the case. Resonance is a very valuable property of reactive AC circuits, employed in a variety of applications. One use for resonance is to establish a condition of stable frequency in circuits designed to produce AC signals. Usually, a parallel (tank) circuit is used for this purpose, with the capacitor and inductor directly connected together, exchanging energy between each other. Just as a pendulum can be used to stabilize the frequency of a clock mechanism's oscillations, so can a tank circuit be used to stabilize the electrical frequency of an AC oscillator circuit. As was noted before, the frequency set by the tank circuit is solely dependent upon the values of L and C, and not on the magnitudes of voltage or current present in the oscillations: (Figure 6.18)

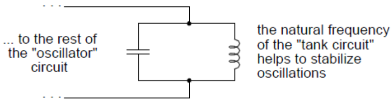


Figure 6.18: Resonant circuit serves as stable frequency source.

Another use for resonance is in applications where the effects of greatly increased or decreased impedance at a particular frequency is desired. A resonant circuit can be used to "block"

7). Total supply current, (IS):

$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{1^2 + (3.9 - 0.16)^2} = 3.87\,(A)$$

8). Conductance, (G):

$$G = \frac{1}{R} = \frac{1}{50} = 0.02S \text{ or } 20mS$$

9). Inductive Susceptance, (BL):

$$B_L = \frac{1}{X_L} = \frac{1}{12.6} = 0.08S \text{ or } 80mS$$

10). Capacitive Susceptance, (BC):

$$B_C = \frac{1}{X_C} = \frac{1}{318.3} = 0.003S \text{ or } 3mS$$

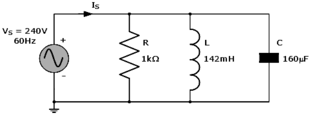
11). Admittance, (Y):

$$Y = \frac{1}{Z} = \frac{1}{12.7} = 0.078S \text{ or } 78mS$$

12). Phase Angle, (φ) between the resultant current and the supply voltage:

$$\cos\phi = \frac{G}{Y} = \frac{20mS}{78mS} = 0.256$$

$$\phi = \cos^{-1}0.256 = 75.3^{\circ} \quad (\text{lag})$$



1). Inductive Reactance, (XL):

$$X_L = \omega L = 2\pi f L = 2\pi.100.0.02 = 12.6\Omega$$

2). Capacitive Reactance, (XC):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.100.5\times 10^{-6}} = 318.3\Omega$$

3). Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{318.3} - \frac{1}{12.6}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0058}} = \frac{1}{0.0788} = 12.7\Omega$$

4). Current through resistance, R (IR):

$$I_R = \frac{V}{R} = \frac{50}{50} = 1.0\,(A)$$

5). Current through inductor, L (IL):

$$I_L = \frac{V}{X_L} = \frac{50}{12.6} = 3.9\,(A)$$

6). Current through capacitor, C (IC):

$$I_C = \frac{V}{X_C} = \frac{50}{318.3} = 0.16\,(A)$$

(present high impedance toward) a frequency or range of frequencies, thus acting as a sort of frequency "filter" to strain certain frequencies out of a mix of others. In fact, these particular circuits are called filters, and their design constitutes a discipline of study all by itself: (Figure 6.19)

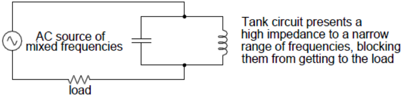


Figure 6.19: Resonant circuit serves as filter.

In essence, this is how analog radio receiver tuner circuits work to filter, or select, one station frequency out of the mix of different radio station frequency signals intercepted by the antenna.

• REVIEW:

• Resonance can be employed to maintain AC circuit oscillations at a constant frequency, just as a pendulum can be used to maintain constant oscillation speed in a timekeeping mechanism.

• Resonance can be exploited for its impedance properties: either dramatically increasing or decreasing impedance for certain frequencies. Circuits designed to screen certain frequencies out of a mix of different frequencies are called filters.

6.5 Resonance in series-parallel circuits

In simple reactive circuits with little or no resistance, the effects of radically altered impedance will manifest at the resonance frequency predicted by the equation given earlier. In a parallel (tank) LC circuit, this means infinite impedance at resonance. In a series LC circuit, it means zero impedance at resonance:

However, as soon as significant levels of resistance are introduced into most LC circuits, this simple calculation for resonance becomes invalid. We'll take a look at several LC circuits with added resistance, using the same values for capacitance and inductance as before: 10 µF and 100 mH, respectively. According to our simple equation, the resonant frequency should be 159.155 Hz. Watch, though, where current reaches maximum or minimum in the following SPICE analyses:

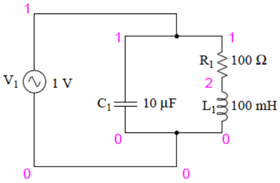


Figure 6.20: Parallel LC circuit with resistance in series with L.

Here, an extra resistor (Rbogus) (Figure 6.22)is necessary to prevent SPICE from encountering trouble in analysis. SPICE

$$\text{Admittance:}\quad Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{(\omega L)} - (\omega C)\right)^2}$$

$$\text{Impedance:}\quad Z = \frac{1}{Y} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{(\omega L)} - (\omega C)\right)^2}}$$

Giving us a power factor angle of:

$$\cos\phi = \frac{G}{Y} \qquad \phi = \cos^{-1}\left(\frac{G}{Y}\right)$$

or

$$\tan\phi = \frac{B}{G} \qquad \phi = \tan^{-1}\left(\frac{B}{G}\right)$$

As the admittance, Y of a parallel RLC circuit is a complex quantity, the admittance corresponding to the general form of impedance Z = R + jX for series circuits will be written as Y = G - jB for parallel circuits where the real part G is the conductance and the imaginary part jB is the susceptance. In polar form this will be given as:

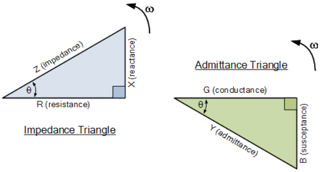
Example No1

A 50Ω resistor, a 20mH coil and a 5uF capacitor are all connected in parallel across a 50V, 100Hz supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit and the phase angle. Also construct the current and admittance triangles representing the circuit.

Parallel RLC Circuit

susceptance, B. This makes it possible to construct an admittance triangle that has a horizontal conductance axis, G and a vertical susceptance axis, jB as shown.

Admittance Triangle for a Parallel RLC Circuit



Now that we have an admittance triangle, we can use Pythagoras to calculate the magnitudes of all three sides as well as the phase angle as shown.

from Pythagoras,

$$Y = \sqrt{G^2 + \left(B_L - B_C\right)^2}$$

$$\text{where: } Y = \frac{1}{Z} \qquad G = \frac{1}{R}$$

$$B_L = \frac{1}{\omega L} \qquad B_C = \omega C$$

Then we can define both the admittance of the circuit and the impedance with respect to admittance as:

can’t handle an inductor connected directly in parallel with any voltage source or any other inductor, so the addition of a series resistor is necessary to “break

```
resonant circuit
v1 1 0 ac 1 sin
c1 1 0 10u
r1 1 2 100
l1 2 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

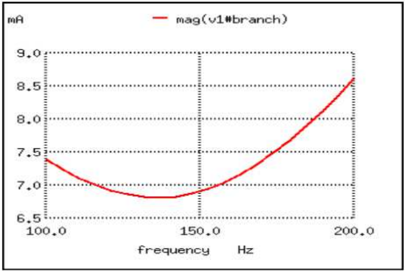


Figure 6.21: Resistance in series with L produces minimum current at 136.8 Hz instead of calculated 159.2 Hz

Minimum current at 136.8 Hz instead of 159.2 Hz!

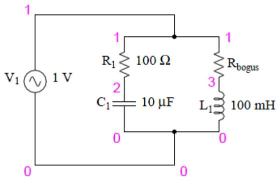


Figure 6.22: Parallel LC with resistance in serieis with C.

up” the voltage source/inductor loop that would otherwise be formed. This resistor is chosen to be a very low value for minimum impact on the circuit’s behavior.

```
resonant circuit
v1 1 0 ac 1 sin
r1 1 2 100
c1 2 0 10u
rbogus 1 3 1e-12
l1 3 0 100m
.ac lin 20 100 400
.plot ac i(v1)
.end
```

Minimum current at roughly 180 Hz instead of 159.2 Hz!

difference between the voltage and the current. The admittance of a parallel circuit is the ratio of phasor current to phasor voltage with the angle of the admittance being the negative to that of impedance.

Conductance (G) :

Conductance is the reciprocal of resistance, R and is given the symbol G.

$$G = \frac{1}{R} \ [S]$$

Conductance is defined as the ease at which a resistor (or a set of resistors) allows current to flow when a voltage, either AC or DC is applied.

Susceptance (B) :

Susceptance is the reciprocal of reactance, X and is given the symbol B.

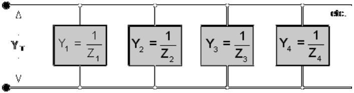
$$B_L = \frac{1}{X_L} \ [S]$$

$$B_C = \frac{1}{X_C} \ [S]$$

In AC circuits susceptance is defined as the ease at which a reactance (or a set of reactances) allows current to flow when a voltage is applied. Susceptance has the opposite sign to reactance so capacitive susceptance BC is positive, +ve in value and inductive susceptance BL is negative, -ve in value.

In AC series circuits the opposition to current flow is impedance, Z which has two components, resistance R and reactance, X and from these two coponents we can construct an impedance triangle. Similarly, in a parallel RLC circuit, admittance, Y also has two components, conductance, G and

$$\frac{1}{Z_T} = Y_T = Y_1 + Y_2 + Y_3 + Y_4 + \ldots\ldots etc$$



The new unit for admittance is the Siemens, abbreviated as S, (old unit mho's ℧, ohm's in reverse). Admittances are added together in parallel branches, whereas impedances are added together in series branches. But if we can have a reciprocal of impedance, we can also have a reciprocal of resistance and reactance as impedance consists of two components, R and X. Then the reciprocal of resistance is called Conductance and the reciprocal of reactance is called Susceptance.

Conductance, Admittance and Susceptance

The units used for conductance, admittance and susceptance are all the same namely Siemens (S), which can also be thought of as the reciprocal of Ohms or ohm-1, but the symbol used for each element is different and in a pure component this is given as:

Admittance (Y) :

Admittance is the reciprocal of impedance, Z and is given the symbol Y.

$$Y = \frac{1}{Z}\ [S]$$

In AC circuits admittance is defined as the ease at which a circuit composed of resistances and reactances allows current to flow when a voltage is applied taking into account the phase

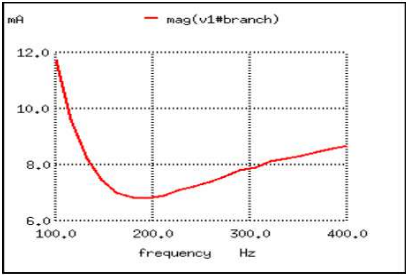


Figure 6.23: Resistance in series with C shifts minimum current from calculated 159.2 Hz to roughly 180 Hz.

Switching our attention to series LC circuits, (Figure 6.24) we experiment with placing significant resistances in parallel with either L or C. In the following series circuit examples, a 1 resistor (R1) is placed in series with the inductor and capacitor to limit total current at resonance. The “extra” resistance inserted to influence resonant frequency effects is the 100 resistor, R2. The results are shown in (Figure 6.25).

And finally, a series LC circuit with the significant resistance in parallel with the capacitor. (Figure 6.26) The shifted resonance is shown in (Figure 6.27)

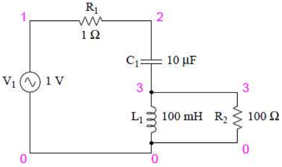


Figure 6.24: Series LC resonant circuit with resistance in parallel with L

```
resonant circuit
v1 1 0 ac 1 sin
r1 1 2 1
c1 2 3 10u
l1 3 0 100m
r2 3 0 100
.ac lin 20 100 400
.plot ac i(v1)
.end
```

Maximum current at roughly 178.9 Hz instead of 159.2 Hz!

Impedance of a Parallel RLC Circuit

$$R = \frac{V}{I_R} \quad X_L = \frac{V}{I_L} \quad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

You will notice that the final equation for a parallel RLC circuit produces complex impedances for each parallel branch as each element becomes the reciprocal of impedance, (1/Z) with the reciprocal of impedance being called Admittance. In parallel AC circuits it is more convenient to use admittance, symbol (Y) to solve complex branch impedances especially when two or more parallel branch impedances are involved (helps with the math's). The total admittance of the circuit can simply be found by the addition of the parallel admittances. Then the total impedance, ZT of the circuit will therefore be 1/YT Siemens as shown.

Admittance of a Parallel RLC Circuit

Since the voltage across the circuit is common to all three circuit elements, the current through each branch can be found using Kirchoff's Current Law, (KCL). Kirchoff's current law or junction law states that "the total current entering a junction or node is exactly equal to the current leaving that node", so the currents entering and leaving node "A" above are given as:

$$\text{KCL:}\ I_S - I_R - I_L - I_C = 0$$

$$I_S - \frac{V}{R} - \frac{1}{L}\int v\,dt - C\frac{dv}{dt} = 0$$

Taking the derivative, dividing through the above equation by C and rearranging gives us the following Second-order equation for the circuit current. It becomes a second-order equation because there are two reactive elements in the circuit, the inductor and the capacitor.

$$I_S - \frac{d^2V}{dt^2} - \frac{dV}{RCdt} - \frac{V}{LC} = 0$$

$$\therefore I_{S(t)} = \frac{d^2V}{dt^2} + \frac{dV}{dt}\frac{1}{RC} + \frac{1}{LC}V$$

The opposition to current flow in this type of AC circuit is made up of three components: XL XC and R and the combination of these three gives the circuit impedance, Z. We know from above that the voltage has the same amplitude and phase in all the components of a parallel RLC circuit. Then the impedance across each component can also be described mathematically according to the current flowing through, and the voltage across each element as.

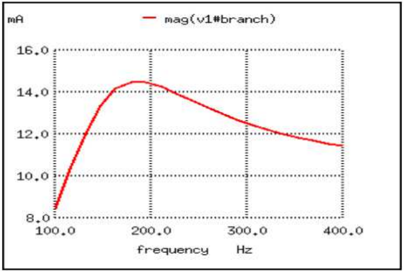


Figure 6.25: Series resonant circuit with resistance in parallel with L shifts maximum currentfrom 159.2 Hz to roughly 180 Hz.

```
resonant circuit
v1 1 0 ac 1 sin
r1 1 2 1
c1 2 3 10u
r2 2 3 100
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

Maximum current at 136.8 Hz instead of 159.2 Hz!

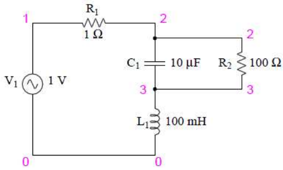


Figure 6.26: Series LC resonant circuit with rsistance in parallel with C.

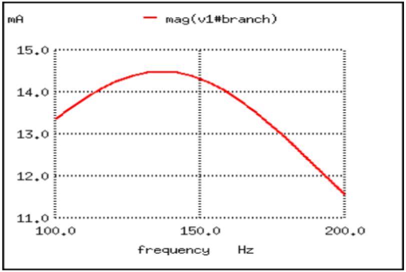
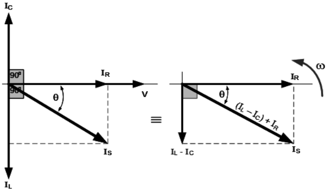


Figure 6.27: Resistance in parallel with C in series resonant circuit shifts curreent maximum from calculated 159.2 Hz to about 136.8 Hz



We can see from the phasor diagram on the right hand side above that the current vectors produce a rectangular triangle, comprising of hypotenuse IS, horizontal axis IR and vertical axis IL - IC Hopefully you will notice then, that this forms a Current Triangle and we can therefore use Pythagoras's theorem on this current triangle to mathematically obtain the magnitude of the branch currents along the x-axis and y-axis and then determine the total current IS of these components as shown.

Current Triangle for a Parallel RLC Circuit

$$I_S^2 = I_R^2 + \left(I_L - I_C\right)^2$$

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2}$$

$$\therefore\ I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

where: $I_R = \frac{V}{R},\quad I_L = \frac{V}{X_L},\quad I_C = \frac{V}{X_C}$

In the above parallel RLC circuit, we can see that the supply voltage, VS is common to all three components whilst the supply current IS consists of three parts. The current flowing through the resistor, IR, the current flowing through the inductor, IL and the current flowing through the capacitor, IC.

But the current flowing through each branch and therefore each component will be different to each other and to the supply current, IS. The total current drawn from the supply will not be the mathematical sum of the three individual branch currents but their vector sum.

Like the series RLC circuit, we can solve this circuit using the phasor or vector method but this time the vector diagram will have the voltage as its reference with the three current vectors plotted with respect to the voltage. The phasor diagram for a parallel RLC circuit is produced by combining together the three individual phasors for each component and adding the currents vectorially.

Since the voltage across the circuit is common to all three circuit elements we can use this as the reference vector with the three current vectors drawn relative to this at their corresponding angles. The resulting vector IS is obtained by adding together two of the vectors, IL and IC and then adding this sum to the remaining vector IR. The resulting angle obtained between V and IS will be the circuits phase angle as shown below.

Phasor Diagram for a Parallel RLC Circuit

The tendency for added resistance to skew the point at which impedance reaches a maximum or minimum in an LC circuit is called antiresonance. The astute observer will notice a pattern between the four SPICE examples given above, in terms of how resistance affects the resonant peak of a circuit:

• Parallel (“tank”) LC circuit:

• R in series with L: resonant frequency shifted down

• R in series with C: resonant frequency shifted up

• Series LC circuit:

• R in parallel with L: resonant frequency shifted up

• R in parallel with C: resonant frequency shifted down

Again, this illustrates the complementary nature of capacitors and inductors: how resistance in series with one creates an antiresonance effect equivalent to resistance in parallel with the other. If you look even closer to the four SPICE examples given, you’ll see that the frequencies are shifted by the same amount, and that the shape of the complementary graphs are mirror-images of each other!

Antiresonance is an effect that resonant circuit designers must be aware of. The equations for determining antiresonance “shift” are complex, and will not be covered in this brief lesson. It should suffice the beginning student of electronics to understand that the effect exists, and what its general tendencies are.

Added resistance in an LC circuit is no academic matter. While it is possible to manufacture capacitors with negligible unwanted resistances, inductors are typically plagued with substantial amounts of resistance due to the long lengths of wire used in their construction. What is more, the resistance of wire tends to increase as frequency goes up, due to a strange phenomenon known as the skin effect where AC current tends to be excluded from travel through the very center of a wire, thereby reducing the wire's effective cross-sectional area. Thus, inductors not only have resistance, but changing, frequency-dependent resistance at that.

As if the resistance of an inductor's wire weren't enough to cause problems, we also have to contend with the "core losses" of iron-core inductors, which manifest themselves as added resistance in the circuit. Since iron is a conductor of electricity as well as a conductor of magnetic flux, changing flux produced by alternating current through the coil will tend to induce electric currents in the core itself (eddy currents). This effect can be thought of as though the iron core of the transformer were a sort of secondary transformer coil powering a resistive load: the less-than-perfect conductivity of the iron metal. This effects can be minimized with laminated cores, good core design and high-grade materials, but never completely eliminated.

One notable exception to the rule of circuit resistance causing a resonant frequency shift is the case of series resistor-inductor-capacitor ("RLC") circuits. So long as all components are connected in series with each other, the resonant frequency of the circuit will be unaffected by the resistance. (Figure 6.28) The resulting plot is shown in (Figure 6.29).

The Parallel RLC Circuit

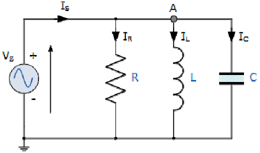
http://www.electronics-tutorials.ws/accircuits/parallel-circuit.html

The Parallel Circuit

The Parallel RLC Circuit is the exact opposite to the series circuit we looked at in the previous tutorial although some of the previous concepts and equations still apply. However, the analysis of parallel RLC circuits can a little more mathematically difficult than for series RLC circuits so in this tutorial about parallel RLC circuits only pure components are assumed in this tutorial to keep things simple.

This time instead of the current being common to the circuit components, the applied voltage is now common to all so we need to find the individual branch currents through each element. The total impedance, Z of a parallel RLC circuit is calculated using the current of the circuit similar to that for a DC parallel circuit, the difference this time is that admittance is used instead of impedance. Consider the parallel RLC circuit below.

Parallel RLC Circuit



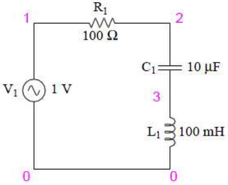


Figure 6.28: Series LC with resistance in series.

```
series rlc circuit
v1 1 0 ac 1 sin
r1 1 2 100
c1 2 3 10u
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

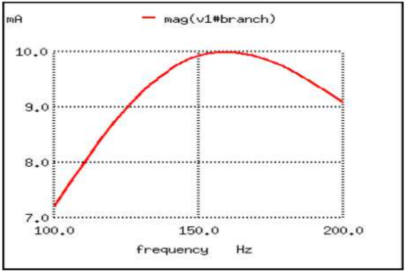


Figure 6.29: Resistance in series resonant circuit leaves current maximum at calculated 159.2 Hz, broadening the curve.

Maximum current at 159.2 Hz once again!

Note that the peak of the current graph (Figure 6.29) has not changed from the earlier series LC circuit (the one with the 1 token resistance in it), even though the resistance is now 100 times greater. The only thing that has changed is the “sharpness” of the curve. Obviously, this circuit does not resonate as strongly as one with less series resistance (it is said to be “less selective”), but at least it has the same natural frequency!

It is noteworthy that antiresonance has the effect of dampening the oscillations of freerunning LC circuits such as tank circuits. In the beginning of this chapter we saw how a capacitor and inductor connected directly together would act something like a pendulum, exchanging voltage and current peaks just like a

$$R_{eq} = \frac{\left(R_L R_C + \frac{L}{C}\right)(R_L + R_C) + \left(\omega L R_C - \frac{R_L}{\omega C}\right)\left(\omega L - \frac{1}{\omega C}\right)}{(R_L + R_C)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$X_{eq} = \frac{\left(\omega L R_C - \frac{R_L}{\omega C}\right)(R_L + R_C) - \left(R_L R_C + \frac{L}{C}\right)\left(\omega L - \frac{1}{\omega C}\right)}{(R_L + R_C)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

When the complex impedances of the branches of the parallel RLC circuit are combined, the equivalent impedance is of the form:

$$Z_{equiv} = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{(R_L + j\omega L)\left(R_C - \frac{j}{\omega C}\right)}{(R_L + R_C) + j\left(\omega L - \frac{1}{\omega C}\right)}$$

When this expression is rationalized and put in the standard form:

$$Z_{eq} = R_{eq} + jX_{eq} = |Z|e^{j\phi}$$

then the impedance in ohms and the phase can be determined. By setting the = 0, the resonant frequency can be calculated. The expressions for these calculations are quite lengthy.

RLC Parallel Expressions

The complex impedances of the parallel RLC circuit takes the form:

$$Z_{equiv} = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{(R_L + j\omega L)\left(R_C - \frac{j}{\omega C}\right)}{(R_L + R_C) + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$Z_{eq} = R_{eq} + jX_{eq} = |Z|e^{j\phi}$$

When rationalized, and the components have the form:

pendulum exchanges kinetic and potential energy. In a perfect tank circuit (no resistance), this oscillation would continue forever, just as a frictionless pendulum would continue to swing at its resonant frequency forever. But frictionless machines are difficult to find in the real world, and so are lossless tank circuits. Energy lost through resistance (or inductor core losses or radiated electromagnetic waves or . . .) in a tank circuit will cause the oscillations to decay in amplitude until they are no more. If enough energy losses are present in a tank circuit, it will fail to resonate at all.

Antiresonance's dampening effect is more than just a curiosity: it can be used quite effectively to eliminate unwanted oscillations in circuits containing stray inductances and/or capacitances, as almost all circuits do. Take note of the following L/R time delay circuit: (Figure 6.30)

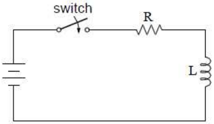


Figure 6.30: L/R time delay circuit

The idea of this circuit is simple: to "charge" the inductor when the switch is closed. The rate of inductor charging will be set by the ratio L/R, which is the time constant of the circuit in seconds. However, if you were to build such a circuit, you might find unexpected oscillations (AC) of voltage across the inductor when the switch is closed. (Figure 6.31)Why is this?

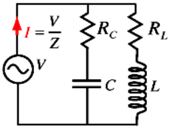
There's no capacitor in the circuit, so how can we have resonant oscillation with just an inductor, resistor, and battery?

All inductors contain a certain amount of stray capacitance due to turn-to-turn and turn-to-core insulation gaps. Also, the placement of circuit conductors may create stray capacitance. While clean circuit layout is important in eliminating much of this stray capacitance, there will always be some that you cannot eliminate. If this causes resonant problems (unwanted AC oscillations), added resistance may be a way to combat it. If resistor R is large enough, it will cause a condition of antiresonance, dissipating enough energy to prohibit the inductance and stray capacitance from sustaining oscillations for very long.

Interestingly enough, the principle of employing resistance to eliminate unwanted resonance is one frequently used in the design of mechanical systems, where any moving object with mass is a potential resonator. A very common application of this is the use of shock absorbers in automobiles. Without shock absorbers, cars would bounce wildly at their resonant frequency after hitting any bump in the road. The shock absorber's job is to introduce a strong antiresonant effect by dissipating energy hydraulically (in the same way that a resistor dissipates energy electrically).

RLC Parallel Circuit
HyperPhysics***** Electricity and Magnetism
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/rlcpar.html

RLC Parallel Circuit



Finding the impedance of a parallel RLC circuit is considerably more difficult than finding the series RLC impedance. This is because each branch has a phase angle and they cannot be combined in a simple way. The impedance of the parallel branches combine in the same way that parallel resistors combine:

$$\frac{1}{Z_{equiv}} = \frac{1}{Z_L} + \frac{1}{Z_C} \quad \text{so} \quad Z_{equiv} = \frac{Z_L Z_C}{Z_L + Z_C}$$

But although the branch impedance magnitudes can be calculated from:

$$Z_L = \sqrt{R_L^2 + \omega^2 L^2} \quad \text{and} \quad Z_C = \sqrt{R_C^2 + \frac{1}{\omega^2 C^2}}$$

they cannot be directly combined as suggested by the expression above because they are different in phase - like vectors in different directions cannot be added directly. This dilemma is most easily solved by the complex impedance method.

RLC Parallel: Complex Impedance Method

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}},$$

where I_{rms} and V_{rms} are rms current and voltage, respectively. The reactances vary with frequency ν, with X_L large at high frequencies and X_C large at low frequencies given as:

$$X_L = 2\pi\nu L, X_C = \frac{1}{2\pi\nu C}.$$

At some intermediate frequency ν_0, the reactances will be equal and cancel, giving Z=R —this is a minimum value for impedance, and a maximum value for I_{rms} results. We can get an expression for by taking $X_L=X_C$. Substituting the definitions of X_L and X_C yields:

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}.$$

ν_0 is the resonant frequency of an RLC series circuit. This is also the natural frequency at which the circuit would oscillate if not driven by the voltage source. At ν_0, the effects of the inductor and capacitor cancel, so that Z=R, and I_{rms} is a maximum. Resonance in AC circuits is analogous to mechanical resonance, where resonance is defined as a forced oscillation (in this case, forced by the voltage source) at the natural frequency of the system.

The receiver in a radio is an RLC circuit that oscillates best at its . A variable capacitor is often used to adjust the resonance frequency to receive a desired frequency and to reject others. is a graph of current as a function of frequency, illustrating a resonant peak in I_{rms} at $\nu_0 = f_0$. The two curves are for two different circuits, which differ only in the amount of resistance in them. The peak is lower and broader for the higher-resistance circuit. Thus higher-resistance circuits do not resonate as strongly, nor would they be as selective in, for example, a radio receiver.

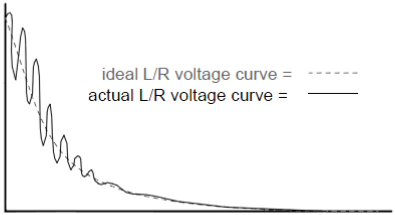


Figure 6.31: Inductor ringing due to resonance with stray capacitance.

• REVIEW:

• Added resistance to an LC circuit can cause a condition known as antiresonance, where the peak impedance effects happen at frequencies other than that which gives equal capacitive and inductive reactances.

• Resistance inherent in real-world inductors can contribute greatly to conditions of antiresonance. One source of such resistance is the skin effect, caused by the exclusion of AC current from the center of conductors. Another source is that of core losses in iron-core inductors.

• In a simple series LC circuit containing resistance (an "RLC" circuit), resistance does not produce antiresonance. Resonance still occurs when capacitive and inductive reactances are equal.

6.6 Q and bandwidth of a resonant circuit

The Q, quality factor, of a resonant circuit is a measure of the "goodness" or quality of a resonant circuit. A higher value for this figure of merit correspondes to a more narrow bandwith, which is desirable in many applications. More formally, Q is the ration of power stored to power dissipated in the circuit reactance and resistance, respectively:

$Q = P_{stored}/P_{dissipated} = I^2X/I^2R$

$Q = X/R$

where: X = Capacitive or Inductive reactance at resonance
R = Series resistance.

This formula is applicable to series resonant circuits, and also parallel resonant circuits if the resistance is in series with the inductor. This is the case in practical applications, as we are mostly concerned with the resistance of the inductor limiting the Q. Note: Some text may show X and R interchanged in the "Q" formula for a parallel resonant circuit. This is correct for a large value of R in parallel with C and L. Our formula is correct for a small R in series with L

A practical application of "Q" is that voltage across L or C in a series resonant circuit is Q times total applied voltage. In a parallel resonant circuit, current through L or C is Q times the total applied current.

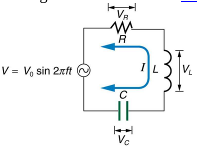
6.6.1 Series resonant circuits

A series resonant circuit looks like a resistance at the resonant frequency. (Figure 6.32) Since the definition of resonance is XL=XC, the reactive components cancel, leaving only the resistance to contribute to the impedance. The impedance is also at a minimum at resonance. (Figure 6.33) Below the resonant frequency, the series resonant circuit looks capacitive

Resonance in RLC Circuits

https://www.boundless.com/physics/textbooks/boundless-physics-textbook/induction-ac-circuits-and-electrical-technologies-22/ac-circuits-162/resonance-in-rlc-circuits-586-3035/

Resonance is the tendency of a system to oscillate with greater amplitude at some frequencies—in an RLC series circuit, it occurs at $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$.
\
Resonance is the tendency of a system to oscillate with greater amplitude at some frequencies than at others. Frequencies at which the response amplitude is a relative maximum are known as the system's resonance frequencies. To study the resonance in an RLC circuit, as illustrated below, we can see how the circuit behaves as a function of the frequency of the driving voltage source.



Combining Ohm's law, $I_{rms}=V_{rms}/Z$, and the expression for impedance Z from

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives

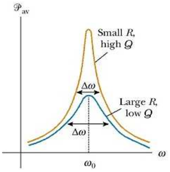


Figure 33.16 Average power versus frequency for a series *RLC* circuit. The width $\Delta\omega$ of each curve is measured between the two points where the power is half its maximum value. The power is a maximum at the resonance frequency ω_0.

The receiving circuit of a radio is an important application of a resonant circuit. One tunes the radio to a particular station (which transmits a specific electromagnetic wave or signal) by varying a capacitor, which changes the resonant frequency of the receiving circuit. When the resonance frequency of the circuit matches that of the incoming electromagnetic wave, the current in the receiving circuit increases. This signal caused by the incoming wave is then amplified and fed to a speaker. Because many signals are often present over a range of frequencies, it is important to design a high-Q circuit to eliminate unwanted signals. In this manner, stations whose frequencies are near but not equal to the resonance frequency give signals at the receiver that are negligibly small relative to the signal that matches the resonance frequency.

since the impedance of the capacitor increases to a value greater than the decreasing inducitve reactance, leaving a net capacitive value. Above resonance, the inductive rectance increases, capacitive reactance decreases, leaving a net inductive component.

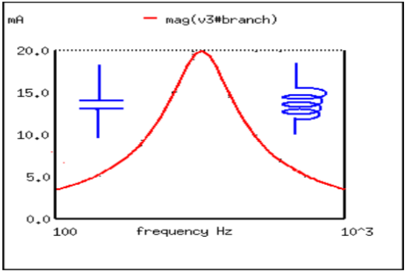


Figure 6.32: At resonance the series resonant circuit appears purely resistive. Below resonance it looks capacitive. Above resonance it appears inductive.

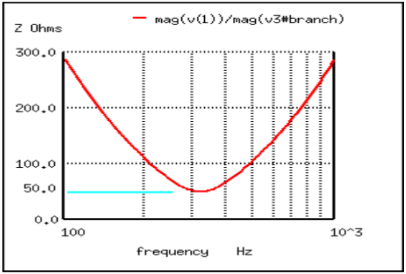


Figure 6.33: Impedance is at a minumum at resonance in a series resonant circuit.

Current is maximum at resonance, impedance at a minumum. Current is set by the value of the resistance. Above or below resonance, impedance increases.

The resonant current peak may be changed by varying the series resistor, which changes the Q. (Figure 6.34) This also affects the broadness of the curve. A low resistance, high Q circuit has a narrow bandwidth, as compared to a high resistance, low Q circuit. Bandwidth in terms of Q and resonant frequency:

Because XL = ωL, XC = 1 /ωC and ω02 = 1/LC, we can express the term (XL - XC)2 as

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}\,(\omega^2 - {\omega_0}^2)^2 \qquad (33.34)$$

Using this result in Equation 33.34 gives

$$\mathcal{P}_{\mathrm{av}} = \frac{(\Delta V_{\mathrm{rms}})^2\,R\omega^2}{R^2\omega^2 + L^2(\omega^2 - {\omega_0}^2)^2} \qquad (33.35)$$

This expression shows that at resonance, when ω = ω0, the average power is a maximum and has the value (ΔVrms)2/R. Figure 33.15b is a plot of average power versus frequency for two values of R in a series RLC circuit. As the resistance is made smaller, the curve becomes sharper in the vicinity of the resonance frequency. This curve sharpness is usually described by a dimensionless parameter known as the quality factor, denoted by Q:

$$Q = \frac{\omega_0}{\Delta\omega}$$

where Δω is the width of the curve measured between the two values of ω for Pav which has half its maximum value, called the half-power points (see Fig. 33.15b.) It is left as a problem (Problem 70) to show that the width at the half-power points has the value Δω = R/L, so

$$Q = \frac{\omega_0 L}{R}$$

The curves plotted in Figure 33.16 show that a high-Q circuit responds to only a very narrow range of frequencies, whereas a low-Q circuit can detect a much broader range of frequencies. Typical values of Q in electronic circuits range from 10 to 100.

curves correspond to three values of R. Note that in each case the current reaches its maximum value at the resonance frequency ω0 . Furthermore, the curves become narrower and taller as the resistance decreases. By inspecting Equation 33.32, we must conclude that, when R = 0, the current becomes infinite at resonance. Although the equation predicts this, real circuits always have some resistance, which limits the value of the current.

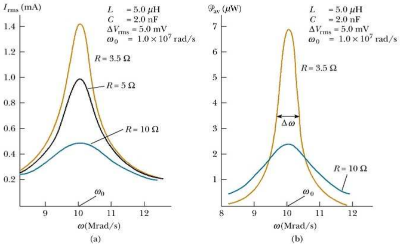


Figure 33.15 (a) The rms current versus frequency for a series RLC circuit, for three values of R. The current reaches its maximum value at the resonance frequency ω0 . (b) Average power versus frequency for the series RLC circuit, for two values of R.

It is also interesting to calculate the average power as a function of frequency for a series RLC circuit. Using Equations 33.30, 33.31, and 33.23, we find that

$$\mathcal{P}_{av} = I_{rms}^2 R = \frac{(\Delta V_{rms})^2}{Z^2} R = \frac{(\Delta V_{rms})^2 R}{R^2 + (X_L - X_C)^2}$$

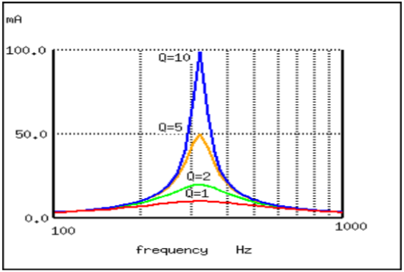


Figure 6.34: A high Q resonant circuit has a narrow bandwidth as compared to a low Q

BW = f_c/Q
Where f_c = resonant frquency
Q = quality factor

Bandwidth is measured between the 0.707 current amplitude points. The 0.707 current points correspond to the half power points since P = I2R, (0.707)2 = (0.5). (Figure 6.35)

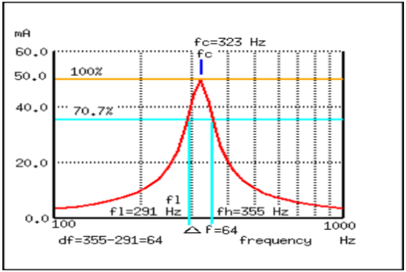


Figure 6.35: Bandwidth, ▪ f ismeasured between the 70.7% amplitude points of series resonant circuit.

BW = Δf = $f_h - f_l$ = f_c/Q

Where f_h = high band edge, f_l = low band edge

$f_l = f_c - \Delta f/2$

$f_h = f_c + \Delta f/2$

Where f_c = center frequency (resonant frequency)

In Figure 6.35, the 100% current point is 50 mA. The 70.7% level is 0707(50 mA)=35.4 mA. The upper and lower band edges read from the curve are 291 Hz for fl and 355 Hz for fh. The bandwidth is 64 Hz, and the half power points are ± 32 Hz of the center resonant frequency:

Resonance in a series RLC circuit

http://www.kshitij-iitjee.com/Resonance-in-a-series-RLC-circuit

A series RLC circuit is said to be in resonance when the current has its maximum value. In general, the rms current can be written

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} \quad (33.31)$$

where Z is the impedance. Substituting the expression for Z from Equation 33.23 into 33.31 gives

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (33.32)$$

Because the impedance depends on the frequency of the source, the current in the RLC circuit also depends on the frequency. The frequency ω0 at which XL – XC = 0 is called the resonance frequency of the circuit. To find ω0 , we use the condition XL = XC , from which we obtain ω0L = 1/ω0C, or

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (33.33)$$

Note that this frequency also corresponds to the natural frequency of oscillation of an LC circuit. Therefore, the current in a series RLC circuit reaches its maximum value when the frequency of the applied voltage matches the natural oscillator frequency—which depends only on L and C. Furthermore, at this frequency the current is in phase with the applied voltage.

A plot of rms current versus frequency for a series RLC circuit is shown in Figure 33.15a. The data assume a constant ΔVrms = 5.0 mV, that L = 5.0 μH, and that C = 2.0 nF. The three

$$BW = \Delta f = f_h - f_l = 355-291 = 64$$
$$f_l = f_c - \Delta f/2 = 323-32 = 291$$
$$f_h = f_c + \Delta f/2 = 323+32 = 355$$

Since BW = fc/Q:

$$Q = f_c/BW = (323\ Hz)/(64\ Hz) = 5$$

6.6.2 Parallel resonant circuits

A parallel resonant circuit is resistive at the resonant frequency. (Figure 6.36) At resonance XL=XC, the reactive components cancel. The impedance is maximum at resonance. (Fig6.6. ure 6.37) Below the resonant frequency, the parallel resonant circuit looks inductive since the impedance of the inductor is lower, drawing the larger proportion of current. Above resonance, the capacitive rectance decreases, drawing the larger current, thus, taking on a capacitive characteristic.

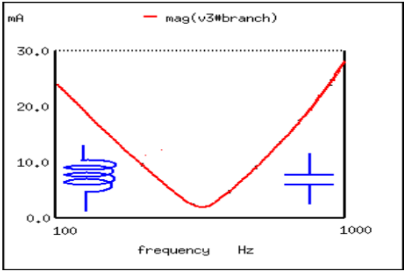


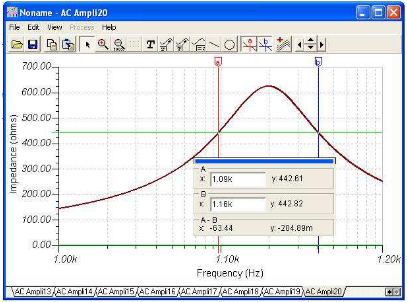
Figure 6.36: A parallel resonant circuit is resistive at resonance, inductive below resonance, capacitive above resonance.

Impedance is maximum at resonance in a parallel resonant circuit, but decreases above or below resonance. Voltage is at a peak at resonance since voltage is proportional to impedance (E=IZ). (Figure 6.37)

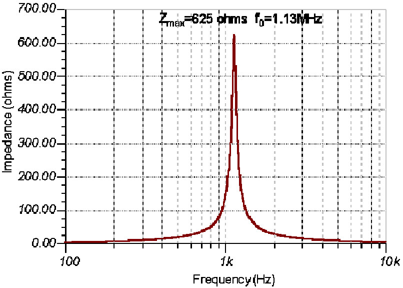
A low Q due to a high resistance in series with the inductor produces a low peak on a broad response curve for a parallel resonant circuit. (Figure 6.38) conversely, a high Q is due to a low resistance in series with the inductor. This produces a higher peak in the narrower response curve. The high Q is achieved by winding the inductor with larger diameter (smaller gague), lower resistance wire.

The bandwidth of the parallel resonant response curve is measured between the half power points. This corresponds to the 70.7% voltage points since power is proportional to E2. ((0.707)2=0.50) Since voltage is proportional to impedance, we may use the impedance curve. (Figure 6.39)

In Figure 6.39, the 100% impedance point is 500
. The 70.7% level is 0707(500)=354
. The upper and lower band edges read from the curve are 281 Hz for fl and 343 Hz for fh. The bandwidth is 62 Hz, and the half power points are ± 31 Hz of the center resonant frequency:



The difference of the A-B cursors is 63.44Hz, which is in very good agreement with the theoretical 63.8Hz result even taking the inaccuracy of the graphic procedure into consideration.



Finally let's examine the bandwidth of this circuit.

The calculated value:

$$\Delta f = \frac{f_0}{Q_0} = \frac{1130}{17.7} = 63.8 Hz$$

Lets confirm it graphically using the diagram.

Zmax = 625 ohm. The impedance limits that define the cutoff frequencies are:

$$Z_1 = Z_2 = \frac{Z_{max}}{\sqrt{2}} = 442\Omega$$

$$BW = \Delta f = f_h - f_l = 343-281 = 62$$
$$f_l = f_c - \Delta f/2 = 312-31 = 281$$
$$f_h = f_c + \Delta f/2 = 312+31 = 343$$

$$Q = f_c/BW = (312 \text{ Hz})/(62 \text{ Hz}) = 5$$

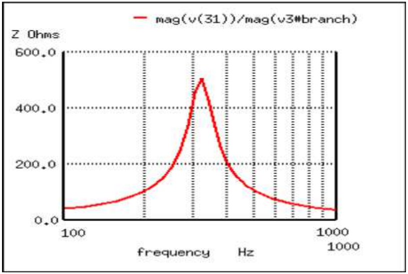


Figure 6.37: Parallel resonant circuit: Impedance peaks at resonance.

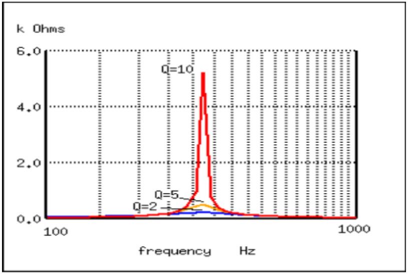
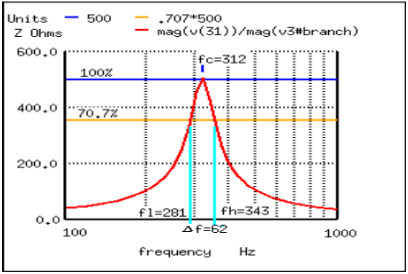
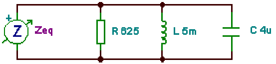


Figure 6.38: Parallel resonant response varies with Q.

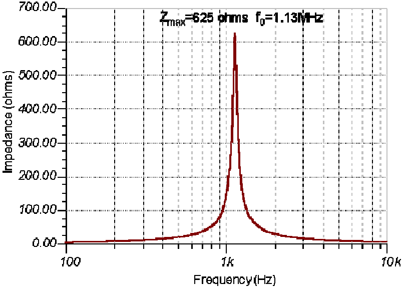


The equivalent parallel resistance: Req = Qo2 RL = 625 ohm

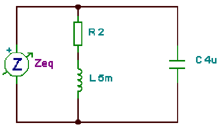
The equivalent parallel circuit:



The impedance diagram:



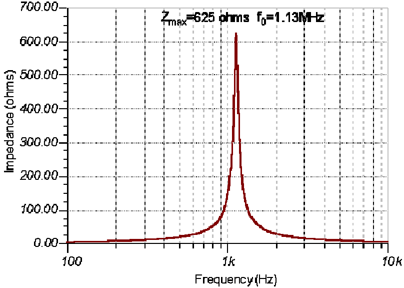
Finally, if we use copy and paste to see both curves on one diagram, we get the following picture where the two curves coincide.



The resonant (Thomson) frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5*4*10^{-9}}} = \frac{10^4}{\sqrt{2}}$$

The impedance diagram is the following:



$$Q_o = \frac{\omega_o L}{R_L} = \frac{10^4}{\sqrt{2}} \frac{5*10^{-3}}{2} = \frac{50}{2*\sqrt{2}} = 12.5*\sqrt{2} = 17.7$$

Figure 6.39: Bandwidth, · f is measured between the 70.7% impedance points of a parallel resonant circuit.

6.7 Contributors

Contributors to this chapter are listed in chronological order of their contributions, from most recent to first. See Appendix 2 (Contributor List) for dates and contact information. Jason Starck (June 2000): HTML document formatting, which led to a much better looking second edition.

The equivalent impedance:

$$Z_{eq} = \frac{\frac{1}{j\omega C}(R + j\omega L)}{\frac{1}{j\omega C} + (R + j\omega L)} = \frac{R + j\ \omega\ L}{1 - \omega^2 LC + j\omega\ C\ R}$$

Let's examine this impedance at the resonant frequency where 1-w02LC=0

We will also assume that the quality factor Qo = woL/ RL>>1.

$$Z_{eq} = \frac{R_L + j\omega_o L}{j\omega_o CR} = R_L \frac{1 + jQ}{j\omega_o CR} \cong \frac{R\omega_o L}{\omega_o CR}$$

At the resonant frequency

Since at resonant frequencyw0L= 1/w0C

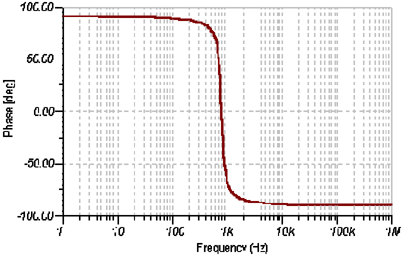
Zeq=Qo2 RL

Since in the pure parallel resonant circuit at the resonant frequency Zeq = R, the real parallel resonant circuit can be replaced by a pure parallel resonant circuit, where:

R = Qo2 RL

Example 3

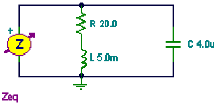
Compare the impedance diagrams of a real parallel and its equivalent pure parallel resonance circuit.



The "pure" parallel circuit above was very easy to examine since all components were in parallel. This is especially important when the circuit is connected to other parts.

However in this circuit, the series loss resistance of the coil was not considered.

Now let's examine the following so called "real parallel resonant circuit," with the series loss resistance of the coil present and learn how we can transform it into a "pure" parallel circuit.



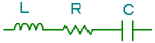
RESONANT CIRCUITS

http://www.tina.com/English/tina/course/28resonant/resonant.htm

Circuits containing R, L, C elements often have special characteristics useful in many applications. Because their frequency characteristics (impedance, voltage, or current vs. frequency) may have a sharp maximum or minimum at certain frequencies these circuits are very important in the operation of television receivers, radio receivers, and transmitters. In this chapter we will present the different types, models and formulas of typical resonant circuits.

SERIES RESONANCE

A typical series resonant circuit is shown in the figure below.



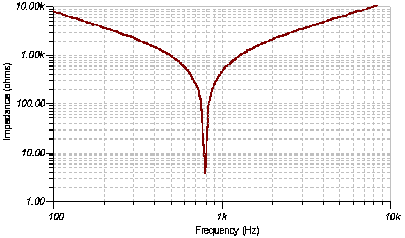
The total impedance:

$$Z = j\omega L + R + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C})$$

In many cases, R represents the loss resistance of the inductor, which in the case of air core coils simply means the resistance of the winding. The resistances associated with the capacitor are often negligible.

The impedances of the capacitor and inductor are imaginary and have opposite sign. At the frequency w0 L= 1/w0C, the total imaginary part is zero and therefore the total impedance is R, having a minimum at the w0frequency. This frequency is called the series resonant frequency.

The typical impedance characteristic of the circuit is shown in the figure below.



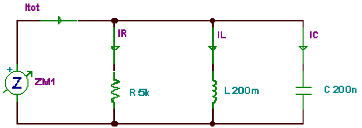
From the w0L= 1/w0Cequation, the angular frequency of the series resonance: or for the frequency in Hz:

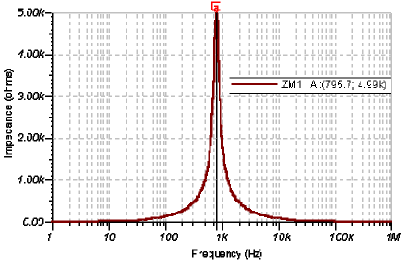
$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{LC}.$$

f0 =

This is the so-called Thomson formula.

If R is small compared to the XL, XC reactance around the resonant frequency, the impedance changes sharply at the series resonant frequencyIn this case we say that the circuit has good selectivity.





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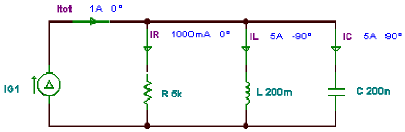
Find the resonant frequency and the resonant quality factor of a pure parallel resonance circuit where R = 5 kohm, L = 0.2 H, C = 200 nF.



The resonant frequency:

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{0.2*0.2*10^{-6}}} = 5000\,{}^{rad}\!/_{s}$$

$$f_0 = \frac{\omega_0}{2\pi} = 795.8\,Hz$$

and the resonant quality factor:

$$Q = \frac{B_C}{G} = \frac{\omega C}{G} = \omega CR = R\sqrt{\frac{C}{L}} = 5*10^3*\sqrt{\frac{0.2*10^{-6}}{0.2}} = 5$$

Incidentally, this quality factor is equal to IL /IR at the resonant frequency.

Now let us draw the impedance diagram of the circuit:

The simplest way is to replace the current source by an impedance meter and run an AC Transfer analysis.

The selectivity can be measured by the quality factor Q If the angular frequency in the formula equals the angular frequency of resonance, we get the resonant quality factor There is a more general definition of the quality factor:

$$Q = 2\pi\frac{\text{energy stored in inductor}}{\text{the energy turned to heat during one period}} = \frac{I^2\omega L}{I^2R} = \frac{\omega L}{R}$$

The voltage across the inductor or capacitor can be much higher then the voltage of the total circuit. At the resonant frequency the total impedance of the circuit is:

Z=R

Assuming that the current through the circuit is I, total voltage on the circuit is

Vtot=I*R

However the voltage on the inductor and the capacitor

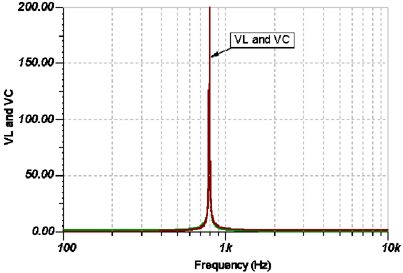
$$V_L = V_C = I\omega_0 L = \frac{I}{\omega_0 C}$$

Therefore

$$\frac{V_L}{V_{tot}} = \frac{V_C}{V_{tot}} = Q_0$$

This means at the resonant frequency the voltages on the inductor and the capacitor are Q0 times greater than the total voltage of the resonant circuit.

The typical run of the VL, VC voltages is shown in the figure below.



Let's demonstrate this via a concrete example.

Example 1

Find the frequency of resonance (f0) and the resonant quality factor (Q0) in the series circuit below, if C=200nF, L=0.2H, R=200 ohms, and R=5 ohms. Draw the phasor diagram and the frequency response of the voltages.

voltage and admittance changes sharply around the resonant frequency. In this case we say the circuit has good selectivity.

Selectivity can be measured by the quality factor Q

$$Q = \frac{B_C}{G} = \frac{\omega C}{G} = \omega CR$$

When the angular frequency equals the angular frequency of resonance, we get the resonant quality factor

$$Q_0 = \frac{B_C}{G} = \frac{\omega_0 C}{G} = \omega_0 CR = R\sqrt{\frac{C}{L}}$$

There is also a more general definition of the quality factor:

$$Q = 2\pi \frac{\text{energy stored in capacitance}}{\text{the energy turned to heat during one period}} = \frac{V^2 \omega_0 C}{V^2 G} = \omega_0 C$$

Another important property of the parallel resonant circuit is its bandwidth. The bandwidth is the difference between the two cutoff frequencies, where the impedance drops from its maximum value to $\frac{1}{\sqrt{2}} = 0.707 \quad (-3dB)\ of \ldots$ the maximum.

It can be shown that the Δf bandwidth is determined by the following simple formula:

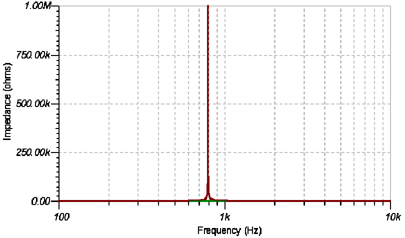
$$\Delta f = \frac{f_0}{Q_0}$$

This formula is also applicable for series resonant circuits.

Let us demonstrate the theory through some examples.

Example 2

linear impedance axis is shown below. Note that viewed with this axis, the impedance appears to be changing even more rapidly near resonance.



The susceptances of the inductance and capacitance are equal but of opposite sign at resonance: BL = BC, 1/w0L = w0C, hence the angular frequency of the parallel resonance:

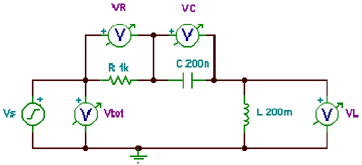
$$\omega_0 = \sqrt{\frac{1}{LC}}$$

determined again by the Thomson formula.

Solving for the resonant frequency in Hz:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

At this frequency the admittance Y = 1/R = G and is at its minimum (i.e., the impedance is maximum). The currents through the inductance and capacitance can be much higher then the current of the total circuit. If R is relatively large, the



$$f_0 = \frac{\omega_0}{2\pi} = 795.8 Hz$$

For R=200 ohms

This is a quite low value for practical resonant circuits, which normally have quality factors over 100. We have used a low value to more easily demonstrate the operation on a phasor diagram.

The current at the resonance frequency I=Vs/R=5m>

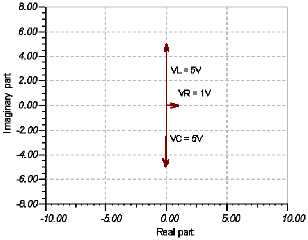
The voltages at current of 5mA: VR = Vs =1 V

meanwhile: VL = VC = I*w0L = 5*10-3 *5000*0.2 = 5V

The ratio between VL, VC,and Vs is equal to the quality factor!

Now let's see the phasor diagram by calling it from the AC Analysis menu of TINA.

We used the Auto Label tool of the diagram window to annotate the picture.



The phasor diagram nicely shows how the voltages of the capacitor and inductor cancel each other at the resonance frequency.

Now let's see VLand VCversus frequency.

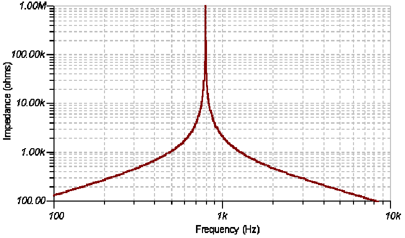
see below the loss resistance of the inductor can be transformed into this resistor.

The total admittance:

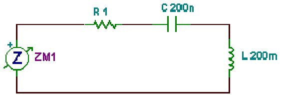
$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = R + j(\omega C - \frac{1}{\omega L})$$

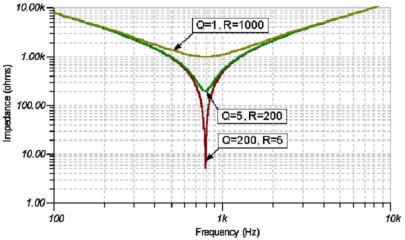
The admittances (called susceptances) of the capacitor and inductor are imaginary and have opposite sign. At the frequency w0C= 1/w0Lthe total imaginary part is zero, so the total admittance is 1/R-its minimum value and the total impedance has its maximum value. This frequency is called the parallel resonant frequency.

The total impedance characteristic of the pure parallel resonant circuit is shown in the figure below:



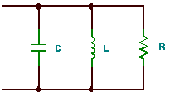
Note that the impedance changes very rapidly around the resonance frequency, even though we used a logarithmic impedance axis for better resolution. The same curve with a



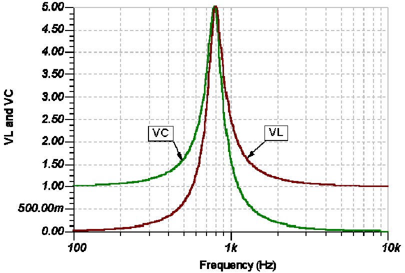


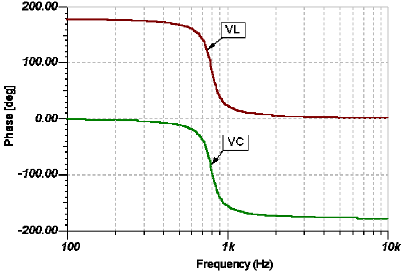
PARALLEL RESONANCE

The pure parallel resonant circuit is shown in the figure below.



If we neglect the loss resistance of the inductor, R represents the leakage resistance of the capacitor. However, as we will





Note that VL starts from zero voltage (because its reactance is zero at zero frequency) while VC starts from 1 V (because its reactance is infinite at zero frequency). Similarly VL tends to 1V and VCto 0V at high frequencies.

Now for R=5 ohms the quality factor is much greater:

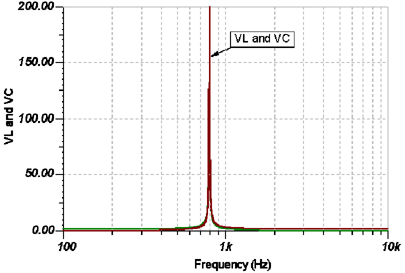
This is a relatively high quality factor, close to the practical achievable values.

The current at the resonance frequency I=Vs/R=0.2A

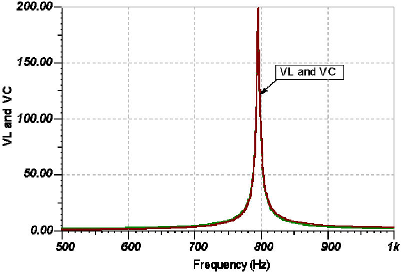
meanwhile: VL = VC = I*w0L = 0.2*5000*0.2 = 200

Again the ratio between the voltages equals the quality factor!

Now let's draw just VL and VC voltages versus frequency. On the phasor diagram, VR would be too small compared to VLand VC



As we can see, the curve is very sharp and we needed to plot 10,000 points to get the maximum value accurately. Using a narrower bandwidth on the linear scale on the frequency axis, we get the more detailed curve below.



Finally let's see the impedance characteristic of the circuit: for different quality factors.

The figure below was created using TINA by replacing the voltage generator by an impedance meter. Also, set up a parameter stepping list for R = 5, 200, and 1000 ohms. To set up parameter stepping, select Control Object from the Analysis menu, move the cursor (which has changed into a resistor symbol) to the resistor on the schematic, and click with the left mouse button. To set a logarithmic scale on the Impedance axis, we have double-clicked on the vertical axis and set Scale to Logarithmic and the limits to 1 and 10k.